

ON CONDITIONAL MANY-VALUED EVENTS ALGEBRAS

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ABSTRACT.

We investigate probability functions defined over many valued conditional events. Given a conditional event, expressed in a suitable logical language, say $\theta \mid \phi$, we are interested in defining notion of probability P satisfying two sorts of intuitive constraints. The first sort concerns the semantics of the underlying logic which we assume is many-valued. This allows us to model the concept of *conditional many-valued event* which is drawing an increasingly good deal of attention in the literature. The latter sort of constraints we shall be looking at, concerns the properties of the probability function, which we take to represent an agent's subjective degree of belief in the spirit of de Finetti's work. This opens the problem of defining a suitable notion conditional bet. We propose the *rejection constraint* according to which a bet is defined provided that the conditioning event gets a strictly positive truth-value. This, we claim, is a natural requirement for conditionals which express a *causal relation* between the antecedent and the consequent.

In order to deal with these constraints in an algebraic setting, we introduce the notion of *conditional MV-algebra* $A \mid A_{\perp}$. Our construction of $A \mid A_{\perp}$ is similar to Mundici's MV-algebraic tensorial product, and it works as follows: let A be an MV-algebra and let A_{\perp} be its biggest MV-bunch (i.e. A_{\perp} is the biggest subset of A such that $\perp \notin A_{\perp}$, and A_{\perp} is closed under \oplus). Then $A \mid A_{\perp}$ is defined as the quotient algebra $\mathcal{F}(A \times A_{\perp})/I$, where $\mathcal{F}(A \times A_{\perp})$ is the MV-algebra freely generated by the pairs (a, b) in the Cartesian product $A \times A_{\perp}$, and I is the MV-ideal of $\mathcal{F}(A \times A_{\perp})$, generated by those MV-terms ensuring our constraints to be satisfied. In fact, rather than an (internal) algebraic operation to be added to any MV-algebra A , we retain that a conditional can be better described as an *external* operation between A , and A_{\perp} .

The algebraic framework of conditional MV-algebras is a useful tool to study conditional probability on MV-events as unconditional probability of conditional MV-events. Our construction of a conditional MV-algebra in fact gives us a very general and flexible setting to characterize a wide class of conditionals. In this paper we limit ourselves to the case of conditionals interpreted causally but in future work we hope to be able to identify a class of constraints which would eventually allow us to define probability on conditional MV-events in a much wider variety of interpretations.