

- ▶ PETR CINTULA, ZUZANA HANIKOVÁ, ROSTISLAV HORČÍK, CARLES NOGUERA, *Semilinear non-associative substructural logics: completeness properties and complexity*.

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As presented in the *Handbook of Mathematical Fuzzy Logic* (P. Cintula, P. Hájek, C. Noguera (eds.), Studies in Logic, Mathematical Logic and Foundations, vol. 37 and 38, College Publications, London, 2011), fuzzy logics can be seen as a particular subfamily of substructural logics whose defining property is completeness with respect to linearly ordered algebras, also called *chains*. Following the terminology introduced in a previous paper by Cintula and Noguera (Implicational (Semilinear) Logics I: A New Hierarchy, *Archive for Mathematical Logic* 49:417–446, 2010) these logics and their algebraic counterparts are called *semilinear*, because the subdirectly irreducible members of the corresponding classes of algebras are linearly ordered. Semilinear substructural logics include many prominent examples of non-classical logics. Some of them (e.g. Lukasiewicz infinite-valued logic, Gödel-Dummett logic, etc.) even enjoy better forms of completeness because they are actually complete with respect to special classes of chains, such as finite chains, or chains defined over the real unit interval, or over the rational unit interval.

In this talk we study semilinear non-associative substructural logics. In another talk we show how these systems can be axiomatized; now we focus on their completeness properties and their complexity. Our base logic is the semilinear extension of the non-associative full Lambek calculus SL studied by Galatos and Ono (Cut elimination and strong separation for substructural logics: An algebraic approach, *Annals of Pure and Applied Logic*, 161(9):1097–1133, 2010). We call it SL^ℓ , the logic of totally ordered residuated unital groupoids; moreover we also consider for every set of axioms corresponding to structural rules, $S \subseteq \{e, c, i, o\}$, its associated semilinear logic SL_S^ℓ . We prove that for every logic SL_S^ℓ is complete with respect to the class of all countably infinite dense SL_S -chains, and with respect to the class of all SL_S -chains on $[0, 1]$. Furthermore, since for we can prove the finite embeddability property (FEP) for semilinear lattice-ordered residuated unital groupoids, we also obtain completeness of these logics with respect to the class of their finite chains. Finally, we show that the equational theory (and the quasiequational theory, and the universal theory) of semilinear lattice ordered residuated unital groupoids is coNP-complete.

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