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THE MANAGEMENT OF UNCERTAINTY IN THE MILORD SYSTEM

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ABSTRACT

The objective of this paper is to describe the management of uncertainty in the MILORD system. MILORD is an expert systems building tool consisting of two inference engines, an explanation module and a rule editor. The system allows to perform three different calculi of uncertainty on an expert defined term set of linguistic statements about certainty. Each calculus corresponds to specific conjunction and disjunction operators. The internal representation of each linguistic statement is a fuzzy interval on the interval [0,1]. The different calculi of uncertainty applied to the elements in the term set give another fuzzy interval as a result . A term from the term set is assigned to the resultant fuzzy interval by means of a linguistic approximation process keeping thereby closed the calculus of uncertainty. One of the main advantages of this approach is that once the linguistic statements have been defined by the expert, the system computes and stores the matrices corresponding to the different conjunction and disjunction operators for all the pairs of linguistic statements. Therefore, when MILORD is applied, *the propagation and combination of uncertainty is performe d by simply accessing the precomputed matrices.*

KEYWORDS : **Management of Uncertainty, Expert Systems, Fuzzy Logic,Linguistic Approximation, Linguistic CertaintyValue, T-Norm, T-Conorm**

1 Introduction

This paper describes MILORD, an expert systems building tool containing two inference engines (forward and backward) with uncertain reasoning capabilities based on fuzzy logic. MILORD allows to express the degree of certainty by means of expert-defined linguistic statements and gives to the user the possibility to choose among three different calculi of uncertainty corresponding to three different models of the AND, OR and IMPLICATION connectives.

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The switching between the two engines is transparent to the user. MILORD has two types of control strategies : one consists in a lookahead technique that allows to detect in advance whether or not the linguistic certainty value of a conclusion will reach a minimal threshold acceptance value. The other concerns the selection of rules according to several criteria. MILORD also contains a limited but useful explanation module as well as a rule editor that are not described in this paper.

2 The Knowledge Representation

The knowledge base consists of facts and rules. The facts are LISP atoms associated with a linguistic certainty value. A non evaluated fact will have the value NIL and therefore it is very fast to check if a given fact is known, i.e. if a certainty value has been assigned to it.

Every rule has a set of conditions which when evaluated with a certain degree of linguistic certainty, lead to a conclusion whose degree of linguistic certainty depends on the degrees of the conditions. The rules are externally represented as follows :

(RULE rule-number (IF conditions) [vc]
(THEN conclusions))

where [vc] is the linguistic certainty value of the rule.

In order to enable a fast access to the rules, MILORD translates the above list into the following internal representation that uses the LISP property lists :

RULE-N ===> VAL [vc] IF $(p_1 \cdots p_N)$ THEN $(c_1 \cdots c_M)$

where VAL, IF and THEN are properties of the atom RULE. The access to the conditions and conclusions of a rule is then an access to the properties of an atom.

The internal representation of the rules builds for each conclusion a property list which is the list of rules that deduce this conclusion together with the linguistic certainty value of each rule, i.e. :

CONCLUSION ===> RULES $((rule_1\ vc_1) \cdots (rule_k\ vc_k))$

where the rules in this list are listed in decreasing order of their linguistic certainty values. This ordering will be used by the lookahead control strategy that will be described later.

3 Forward, Backward and their combination

The forward reasoning starts with a set of given facts and its goal is to deduce an hypothesis whose linguistic certainty value reaches a given acceptance threshold. If the forward reasoning gets to an hypothesis whose certainty value is below the threshold, the backward resoning is called in order to try to increase this certainty value by considering, through a lookahead process, other rule-paths that would conclude the same hypothesis with a higher certainty.

3.1. The Lookahead Prospection Technique

MILORD applies a prospection process from the hypothesis towards the external (non deducible) facts in such a way that at any time it checks if the certainty value of the hypothesis can reach the acceptance threshold value. If not, it will consider a new hypothesis. Let us now briefly describe such process with the following default operators, for the AND, OR and "===>" connectives, to perform the calculus of uncertainty (although the lookahead process is independent of the operators used) :

$$v(A\ \text{AND}\ B) = \text{MIN}(v(A), v(B))$$
$$v(C_{R1}\ \text{OR}\ C_{R2}) = \text{MAX}(v(C_{R1}), v(C_{R2}))$$
$$v(C) = \text{MIN}(v(R), v(P))$$

where A and B are conditions of a same premise, C_{R1} and C_{R2} represent the same conclusion deduced by the two rules R1 and R2, and C is the conclusion of rule R whose premise is P.

The above operators are used respectively in the evaluation of the satisfaction of the premise, in the combination of several rules with the same conclusion and in the propagation of the uncertainty from the premise to the conclusion of a rule.

The lookahead process in the backward reasoning starts assuming that all the non evaluated conditions of the rules leading to the same conclusion, have the highest linguistic certainty value among the ordered set of linguistic values defined by the expert. This allows to compute the highest possible certainty value that this conclusion could reach. If this value is higher than the acceptance threshold the backward reasoning proceeds asking the user to assign a linguistic certainty value to the non evaluated non deducible conditions one by one. Each time a condition gets its value, it is propagated to the conclusion using the above formulae, and if its certainty value is still higher than the threshold, the process proceeds asking for the value of the next non deducible condition and so on until either the certainty value of the conclusion falls bellow the threshold (in which case MILORD calls back the forward reasoning mode to deduce another hypothesis), or all the non deducible conditions have been assigned a certainty value. As far as the deducible conditions are concerned, the lookahead process is applied recursively to each one of them as described and its certainty value is also propagated towards the conclusion in order to keep checking if its certainty value is higher than the threshold in which case the process resumes . If not,the forward reasoning mode will try to deduce a new hypothesis.

If the user initially gives a set of hypothesis instead of a set of facts, MILORD calls the backward reasoning mode with one of the hypothesis and tries to validate it with a linguistic certainty value higher than the threshold, using exactly the same process described above. If it fails, it tries another hypothesis, and so on until either one of them succeeds or all of them fail.

3.2. The Rule Selection Criteria

The set of criteria to select rules has to be easily modifiable because the efficiency of any criterium depends on each particular application. In MILORD it is very easy, for the user, to modify or introduce criteria . The selection among a given set of criteria can, in some cases, be done automatically. For example, if a knowledge base only contains rules which have only one conclusion, any criterium based on the number of conclusions would not be considered. The criteria that, in addition to metarules, are available in MILORD are :

a) The order of the rules

b) The linguistic certainty values

c) The number of conditions

d) The number of conclusions

e) The rule most recently used

f) The rule containing the most recently deduced fact in its premise

Furthermore, the user can combine several criteria according to a given priority. Let us see an example :

R1 : CONDITION_1 , CONDITION_2 ===> [absolutely-true] CONCLUSION_1

R2 : CONDITION_2 , CONDITION_3 ===> [almost-true] CONCLUSION_2

R3 : CONDITION_4 ===> [quite-true] CONCLUSION_1

The extreme values corresponding to the following ordered criteria are :

1) MAXIMUM CERTAINTY VALUE : absolutely-true
2) MAXIMUM NUMBER OF CONCLUSIONS : 1
3) MINIMUM NUMBER OF CONDITIONS : 1

In this case the system will try to select a rule, among the applicable ones , having a certainty value equal to "absolutely-true", having one condition and one conclusion. If there is no rule satisfying these criteria it will drop the last one (number of conditions) and so on until one or more rules are obtained. If several rules have been obtained, the user can use the rest of the criteria to end up with only one rule. In our example, after dropping the last criterium the selected rule is R1.

4. The Management of Uncertain Reasoning.

The numerical approaches to the representation of uncertainty imply hypothesis of independence, mutual exclusiveness, etc. about the information they deal with. On the other hand, they oblige the expert and the user to be unrealistically precise and consistent in the assignment of such numerical values to rules and facts. Furthermore, these approaches are computationally expensive.

Our approach is based on a linguistic characterization of tne uncertainty and follows the work of Bonissone [3]. The linguistic certainty values are terms defined by the expert. The internal representation of each term is a fuzzy number on the interval [0,1] characterized by a parametric representation for computational reasons.

MILORD has been parametrized in order to perform three different calculi of uncertainty operating on the expert defined term set of linguistic certainty values.

4.1. The Calculus of Uncertainty.

It can be shown [4] that Triangular norms (T-norms) and Triangular conorms (T-conorms) are the most general families of two-place functions from [0,1]X[0,1] to [0,1], that respectively satisfy the requirements of conjunction and disjunction operators.

A T-norm T(p,q) performs a conjunction operator, on the degrees of certainty of two or more conditions in the same premise, satisfying the following properties :

$T(0,0)=0$
$T(p,1)=T(1,p)=p$
$T(p,q)=T(q,p)$
$T(p,q)\leq T(r,s)$ if $p\leq r$ and $q\leq s$
$T(p,T(q,r))=T(T(p,q),r)$

A T-conorm S(p,q) computes the degree of certainty of a conclusion derived from two or more rules. It is a disjunction operator satisfying the following properties :

$S(1,1)=1$
$S(0,p)=S(p,0)=p$
$S(p,q)=S(q,p)$
$S(p,q)\leq S(r,s)$ if $p\leq r$ and $q\leq s$
$S(p,S(q,r))=S(S(p,q),r)$

The propagation function P(p,r), giving the certainty value of the conclusion of a rule as a function of the certainty value of the premise and the certainty value of the rule itself, satisfies the properties of a T-norm.

For suitable negation operators N(x) [7], T-norms and T-conorms are dual in the sense of DeMorgan's law

Some usual pairs of dual T-norms and T-conorms are:

$$T_0(x,y)=\begin{cases}0, & \text{if } x,y<1\\ \min(x,y), & \text{otherwise}\end{cases} \qquad S_0(x,y)=\begin{cases}1, & \text{if } x,y>0\\ \max(x,y), & \text{other.}\end{cases}$$

$T_1(x,y)=\max(0,x+y-1)$ $\quad$ $S_1(x,y)=\min(1,x+y)$
(Luckasiewicz)

$T_{1.5}(x,y)=x.y/[2-(x+y-xy)]$ $\quad$ $S_{1.5}(x,y)=(x+y)/(1+xy)$

$T_2(x,y)=x.y$ $\quad$ $S_2(x,y)=x+y-xy$
(Probabilistic)

$T_{2.5}(x,y)=x.y/(x+y-xy)$ $\quad$ $S_{2.5}(x,y)=(x+y-2xy)/(1-xy)$

$T_3(x,y)=\min(x,y)$ $\quad$ $S_3(x,y)=\max(x,y)$
(Zadeh)

It can be shown that they are ordered as follows :

$$T_0\leq T_1\leq T_{1.5}\leq T_2\leq T_{2.5}\leq T_3$$

$$S_3\leq S_{2.5}\leq S_2\leq S_{1.5}\leq S_1\leq S_0$$

In MILORD we have implemented the pairs (T_1,S_1), (T_2,S_2) and (T_3,S_3) following the experimental results obtained by Bonissone [3] which consisted in applying nine T-norms to three different term sets. Bonissone analyzed the sensitivity of each operator with respect to the granularity (number of elements) in the term sets and concluded that only the T-norms T_1, T_2 and T_3 generated sufficiently different results for term sets that do not have more than nine elements. On the other hand, according to the results of Miller [6] concerning the span of absolute judgement, it is unlikely that any expert or user would consistently qualify uncertainty using more than nine different terms.

5. The Linguistic Certainty Values

MILORD allows the expert to define the term set of linguistic certainty values which constitutes the verbal scale that he and the users will use to express their degree of confidence in the rules and facts respectively. Recent psychological studies [1], have shown the feasibility of such verbal scales : " ... A verbal scale of probability expresions is a compromise between people's resistance to the use of numbers and the necessity to have a common numerical scale" (Beyth-Maron 1982); " ... people asked to give numerical estimations on a common-day situation err most of the time and in a non consistent way. Furthermore, they are unable to appreciate their judgements imprecision (errors are by far bigger than the maximum error accepted as possible by the subjects themselves). Nevertheless, judgements embodied in linguistic descriptors appear consistent in this same situation ... " (Freksa 1981) [5].

Each linguistic value is represented internally by a fuzzy interval (fuzzy number) i.e. the membership function of a fuzzy set on the real line, or, more precisely, on the truth space represented by the interval [0,1]. These membership functions can be interpreted as the meanings of the terms in the term set. The conjunction and disjunction operators applied to these functions will produce another membership function as a result that will have to be matched to a term in the term set, in order to keep the term set closed. This can be done by a linguistic approximation process that will be described later (see Bonissone [2] for an extensive study of the linguistic approximation process).

5.1. A Default Term Set and its Representation

Although the expert can define its own term set together with its internal representation, MILORD provides the following default term set :

{ FALSE , ALMOST_FALSE , MAYBE , ALMOST_TRUE , TRUE }

Each term T_i is represented by a membership function $\mu_i(x)$, for x in the interval [0,1].

For computational reasons, each membership function is represented by four parameters $T_i=(a_i,b_i,c_i,d_i)$, corresponding to the following trapezoidal function :

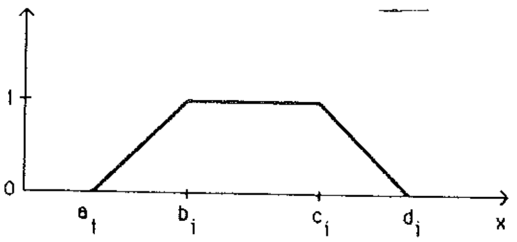


FIGURE 1

The five element default term set has the following representation :

FALSE	= (0,0,0,0)
ALMOST_FALSE	= (0,0,.25,.40)
MAYBE	= (.25,.40,.60,.75)
ALMOST_TRUE	= (.60,.75,1,1)
TRUE	= (1,1,1,1)

corresponding to the following functions :

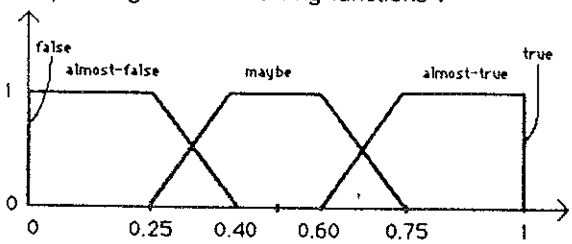


FIGURE 2

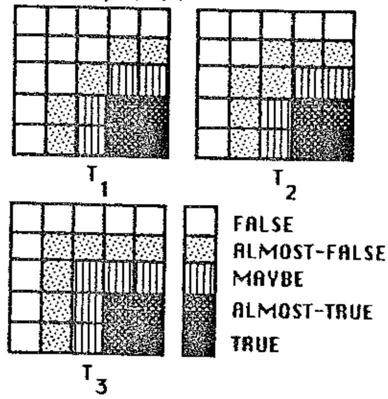
In order to be able to evaluate the T-norms T_1,T_2,T_3 and the T-conorms S_1 ,S_2,S_3 on the elements of the term set, we have applied the following formulæ according to the arithmetic rules on fuzzy numbers

Given two fuzzy intervals I = (a,b,c,d) and I' = (a',b',c',d'), we have :

I+I' = (a+a',b+b',c+c',d+d')
I-I' = (a-d',b-c',c-b',d-a')
I*I' = (aa',bb',cc',dd')
min(I,I') = (min(a,a'),min(b,b'),min(c,c'),min(d,d'))
max(I,I') = (max(a,a'),max(b,b'),max(c,c'),max(d,d'))

5.2. The Linguistic Approximation

A linguistic approximation process is performed in order to find a term (linguisctic value) in the term set whose "meaning" (membership function) is the closest (according to a given metric) to the "meaning" (membership function) of the result of the conjunction or disjunction operation performed on any two linguistic values of the term set. This allows to maintain closed the operations for any T-norm and T-conorm. The problem is, therefore, that of computing a distance between two trapezoidal membership functions. In order to do so, we have adopted a simple solution consisting on the computation of a weighted euclidean distance of two features of the functions : the first moment and the area under the function. The next figure shows the results obtained with the selected T-norms T_1,T_2,and T_3 on the default term set of figure 2 :



6. Concluding Remarks

We have described some aspects of the MILORD system and in particular its management of uncertainty. The most relevant features of our approach are the representation of uncertainty by means of expert-defined linguistic statements and the use of the certainty values to guide the search tree by means of a lookahead prospection technique.

The main advantage of this approach is that once the linguistic values have been defined by the expert, the system computes and stores the matrices corresponding to the different conjunction and disjunction operations on all the pairs of terms in the term set. Later, when MILORD is run on a particular application, the propagation and combination of uncertainty is performed by simply accessing these pre-computed matrices.

The gain in speed with respect to the most common numerical approaches is remarkable, for example, a rule with N conditions in its premise will need N-1 accesses to a matrix to obtain the linguistic certainty value of the premise, and one additional access to combine this value with that of the rule itself in order to obtain the linguistic certainty value of the conclusion.

The easiness for the expert and the user in expressing linguistically their confidence in the rules and facts it is also a remarkable feature.

MILORD is being used in the development of the PNEUMON-IA expert system for diagnosis and treatment of pneumoniæ.

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