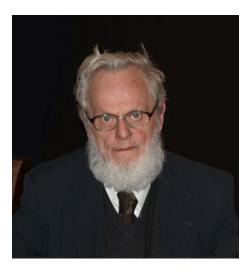# Petr Hájek, obituary

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Picture courtesy of Petr Cintula

Petr Hájek, a renowned Czech logician, passed away on 26 December 2016. He was retired from the Czech Academy of Sciences, where he had been appointed an emeritus, having worked there for more than half a century. His achievements in mathematical logic leave a permanent imprint in set theory, arithmetic, and fuzzy logic, but he also worked in applications of logic. He authored several monographs that capture his scientific interests, some of them remaining sources of knowledge and inspiration to generations of researchers.

Petr Hájek was born in Prague on 6 February 1940. He studied mathematics at the Faculty of Mathematics and Physics of Charles University in Prague from 1957, finishing in 1962 with a master thesis in algebra. He subsequently obtained a postgraduate position at the Institute of Mathematics of the Czechoslovak Academy of Sciences. Apart from his mathematical education, Petr received a profound musical training, eventually graduating from the Academy of Performing Arts in Prague, where he studied the organ with Jiří Reinberger, a Czech organ virtuoso, teacher and composer. For several decades, Petr would serve as organist at the protestant Clement's Church in Prague.

Petr commenced his mathematical studies with the Czech logician Ladislav Svante Rieger, who at that time worked at the Institute of Mathematics. Rieger ran a seminar in mathematical logic; one of the attendees was Petr Vopěnka. Rieger passed away as early as 1963; Vopěnka then started his own seminar in axiomatic set theory at the Faculty of Mathematics and Physics, with Petr Hájek attending. Hájek referred to Vopěnka as his advisor; they would later become coauthors, although they worked from different institutions, as Hájek was not allowed to teach because of his religion.

Vopěnka's set theory seminar brought together a focused group of young researchers, such as Bohuslav Balcar, Tomáš Jech, Karel Hrbáček, Karel Příkrý, Antonín Sochor or Petr Štěpánek. Their research concerned models of set theory and forcing. Hájek's doctoral dissertation, defended in 1965, was titled *Models of set theory with individuals* (see [9]).

The Theory of Semisets, written by Petr Vopěnka and Petr Hájek, was published in 1972 (see [34]). In a somewhat non-classical setting that admits semisets (i.e., proper subclasses of sets) in addition to the usual sets and classes, it presents the methods developed by Vopěnka's school. For Vopěnka, the book meant a goodbye to classical set theory; he later founded another seminar that studied an alternative set theory proposed by himself. Petr Hájek also shifted his interests; after some deliberation, he settled on studying arithmetic (see [28]).

Pavel Pudlák started working with Petr Hájek in mid seventies; another person in the group was Petr's wife Marie, whose thesis concerned arithmetic (see [19]). In the late seventies, Vítězslav Švejdar, joined in, defending his thesis in 1982 (see [30, 31]). A working group in arithmetic, including also Jan Krajíček, formed itself in the Institute of Mathematics, starting a seminar that is still run within the institute. In 1988 Petr submitted a *doctor scientiarum* (DrSc) dissertation *Metamathematics of First-Order Arithmetic* (see [10]), a direct predecessor of Petr's part of the book on arithmetic with the same title, written jointly with Pavel Pudlák a couple of years later (see

[18]).

From mid 1960's, Petr Hájek was involved in the development of the GUHA method of exploratory data analysis, and in theoretical research pertaining to it, which would fall under the umbrella terms of observational calculi and also reasoning under uncertainty. The method was developed and implemented as early as 1965. This work, which ran parallel to Petr's more abstract interests in set theory and arithmetic, later contributed to his interest in fuzzy logic.

GUHA was a pioneering attempt that was as marvellous as it was precocious. Statisticians such as Tomáš Havránek came aboard, and the theory surrounding GUHA was captured in the book *Mechanizing Hypothesis Formation: Mathematical Foundations of a General Theory* (see [16]). Petr gained two doctoral students in the GUHA method: Jiří Ivánek ([23]) and Jan Rauch ([29]). The development of the method occasioned the formation of an application-oriented seminar, which in time broadened its scope and later moved with Petr to the Institute of Computer Science, changing contents according to the shift of Petr's interests. The seminar still continues to be run, giving a lot of attention to fuzzy and substructural logic.

During the eighties Petr Hájek and his colleagues worked on expert systems, seeking to complement the existing GUHA procedures. Again Petr gained students in the area: Julio Valdés (see [33]), and Milan Daniel (see [6]). Theoretical issues on processing uncertainty gave rise to a book, *Uncertain Information Processing in Expert Systems*, written by Petr Hájek, Tomáš Havránek, and Ivan Jiroušek (see [17]). Another important coworkers were Ivan Kramosil and Dagmar Harmancová.

In 1991, Tomáš Havránek, then director of the Institute of Computer Science and Petr Hájek's coauthor and friend, passed away prematurely. It was proposed that Petr Hájek enter as candidate for the position of director; he was elected and assumed office in March 1992, holding it until early 2000. (This would have been impossible during the totalitarian era, because he was not in the good books of the authorities.) He never mitigated his research effort during the period as director; in fact, it was during his years as director that he started research in fuzzy logic, and wrote and published his famous monograph [13].

The early works in fuzzy set theory and fuzzy logic by Lotfi Zadeh ([35]) and Joseph Goguen ([8]) did not pass unnoticed by Petr Hájek, although he was working in different areas at that time. He was thesis opponent to two subsequent students of Aleš Pultr in fuzzy logic: Jan Pavelka defended in 1976 (see [27] for the material), Vilém Novák defended in 1988 (see [26]). Moreover, Vilém Novák published a book in Czech about fuzzy logic ([25]).

Another impetus came in 1991 when Gaisi Takeuti visited Prague, bringing with him his and Satoko Titani's paper [32]. Petr Hájek thought highly of this paper; later he continued the research in the logic (see [13]) and worked on his version of the set theory (see [15]).

Moreover, fuzzy logic constituted by that time, *sui generis*, reasoning under uncertainty, an area that Petr Hájek knew well from his application-oriented research. It was also in 1991 when, taking advantage of his first visit to Francesc Esteva and Lluís Godo in Barcelona, Petr attended the first ECSQAU conference (later called ECSQARU) in Marseille, where he could meet the main European researchers in uncertainty and fuzzy logic. And in fact, one of his early contributions to fuzzy logic was a clarification of the traits that distinguished fuzzy logic within this broader area that contained also probability and possibility theory. He argued that fuzzy logic, like many-valued logic, has a purely formal deductive facet; he stressed the distinction between degrees of truth (involving vague notions, such as *beautiful*) and degrees of belief (probability, possibility, etc.) on potentially crisp notions (see, e.g., [11]).

Starting his work in fuzzy logic, Petr entered a world with a considerable history (see Historical remarks in [13]) and was able to continue some long-term cooperations and long-forged friendships. He already knew Franco Montagna, Matthias Baaz, and Jeff Paris from his arithmetic years. He also enjoyed a longterm cooperation with Francesc Esteva and Lluís Godo, he also knew Siegfried Gottwald and Erich Peter Klement. He was on visiting terms with researchers in Italy pioneering many-valued and fuzzy logic, among them Daniele Mundici, Antonio Di Nola, and Giangiacomo Gerla. These and other connections made it possible to file a successful European COST project in many-valued logic, started in 1995, that established a research network in many-valued logic that, in different incarnations, continues to exist until today.

Since mid 1990's Petr Hájek was working on his "basic logic" BL, intended as the logic of continuous t-norms and their residua. Note that the logics of the two (out of three) prominent continuous t-norms, Łukasiewicz and Gödel logics, were already known before the birth of fuzzy logic. The logic of the remaining prominent continuous t-norm, product t-norm, was introduced in the mid 1990's by Petr together with Godo and Esteva [14]. So BL was a natural next step to study. Although BL was quite an appealing logic, the importance of this work consisted rather in the method he adopted and in the standards he set by this case study; namely, a thorough analysis of the formal deductive system, the algebraic semantics, some extensions, and possible applications. The monograph *Metamathematics of Fuzzy Logic*

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was published in 1998 (see [13]), establishing BL as a formal system quite on a par with the classical and neighbouring formal nonclassical systems. The monograph was a product of several years' effort, its publication marking the end of an era of pioneering work in mathematical fuzzy logic for Petr Hájek. The book left the standard completeness of (propositional) BL w.r.t. the continuous t-norm semantics (the *standard* completeness) as an open problem; it was addressed a bit later in [12] and finally solved affirmatively in [2].

The next decade would see mathematical fuzzy logic in full bloom. In particular, Petr made numerous and important contributions to the field. It is out of scope of this obituary to go through all of them and properly comment on each of them (the interested reader can consult [7]), but very briefly, let us mention the following main topics where Petr outstandingly contributed after his monograph: first-order fuzzy logics, and in particular, model theory, supersound logics, complexity of standard tautologies or of standardly satisfiable formulas and witnessed models; computational complexity of propositional fuzzy logics; logics weaker than BL (MTL, hoop logics, non-commutative BL, flea logics), logics with truth constants for the rationals (rational Pavelka logic); logics of probability, of possibility and of belief, logics with truth hedges, fuzzy modal logics; fuzzy description logic; and mathematical theories (arithmetic, set theory) over fuzzy logic.

The fuzzy logic community that Petr helped to establish has thrived during the millenium years, with new researchers joining in. Again fuzzy logic is an umbrella term; it would include schools of many-valued logic that had many years of existence to their credit. Petr also supervised students in fuzzy logic: Zuzana Haniková ([21]), Petr Cintula ([3]), and Libor Běhounek ([1]). The first two volumes of the *Handbook of Mathematical Fuzzy Logic* (see [4]) were edited by Petr Cintula, Petr Hájek and Carles Noguera, capturing the state of the art with all the new results.

Petr Hájek was a respected teacher and, perhaps one can say, role model for many of us. Research groups formed naturally around him. He served on committees and editorial boards and was a member of professional organizations such as the *Union of Czech Mathematicians and Physicists* or the *Association for Symbolic Logic*. He was also member of the *Learned Society of the Czech Republic*, uniting distinguished professionals across disciplines. His awards and recognition include the Bolzano medal from the Czech Academy of Sciences in 2000, a medal of the Minister of Education of the Czech Republic in 2002, the *De scientiae et humanitate optime meritis* medal from the Academy of Sciences in 2006, the Medal of Merit from the President of the Czech Republic in 2006, the Josef Hlávka medal in 2009,

and the EUSFLAT excellence award in 2013.

Petr Hájek was widowed, his wife Marie having passed away three years earlier. He is survived by his children Marie and Jonáš and his grandson Jonáš. He was a lifelong member of the Evangelical Church of Czech Brethren; he served there as an organist for the greater part of his life. It was not until one visited the community at Clement's Church that one realized fully what Petr's wife Marie had hinted in [20]: that Petr's scientific career was only one facet of his existence, with his involvement in the church community being as profound as his mathematical engagements, and, of course, his family life.

A complete bibliography of Petr Hájek, maintained by the Library of the Institute of Computer Science, is available at `http://hdl.handle.net/11104/0218892`. Edited volumes honouring his work include [5] and [24], the latter including his scientific bibliography [22].

References

[1] Libor Běhounek. *Logical Foundations of Fuzzy Mathematics*. PhD thesis, Charles University in Prague, Faculty of Arts, Prague, 2009.

[2] Roberto Cignoli, Francesc Esteva, Lluís Godo, and Antoni Torrens. Basic fuzzy logic is the logic of continuous t-norms and their residua. *Soft Computing*, 4(2):106–112, 2000.

[3] Petr Cintula. *From Fuzzy Logic to Fuzzy Mathematics*. PhD thesis, Czech Technical University, Faculty of Nuclear Sciences and Physical Egineering, Prague, 2005.

[4] Petr Cintula, Petr Hájek, and Carles Noguera, editors. *Handbook of Mathematical Fuzzy Logic*, volume 1,2. College Publications, 2011.

[5] Petr Cintula, Zuzana Haniková, and Vítězslav Švejdar, editors. *Witnessed Years: Essays in Honour of Petr Hájek*, volume 10 of *Tributes*. College Publications, 2009.

[6] Milan Daniel. *Dempsterova pologrupa a práce s nejistotou v pravidlově orientovaných expertních systémech*. [Dempster semigroup and managing uncertainty in rule-based expert systems.] Doctoral thesis, Institute of Computer Science, 1993.

[7] Francesc Esteva, Lluís Godo, Siegfried Gottwald, and Franco Montagna. Introduction. In Franco Montagna, editor, *Petr Hájek on Mathematical Fuzzy Logic*, volume 6 of *Outstanding Contributions to Logic*, pages 3–20. Springer International Publishing, 2015.

[8] Joseph Amadee Goguen. The logic of inexact concepts. *Synthese*, 19(3–4):325–373, 1969.

[9] Petr Hájek. *Modely teorie množin s individuy.* [Models of set theory with individuals.] Doctoral thesis, Institute of Mathematics, Czechoslovak Academy of Sciences, 1964.

[10] Petr Hájek. *Metamatematika aritmetiky prvního řádu.* [Metamathematics of first-order arithmetic.] DrSc thesis, Institute of Mathematics, Czechoslovak Academy of Sciences, 1988.

[11] Petr Hájek. On Logics of Approximate Reasoning. In M. Masuch and L. Pólos, editors, *Knowledge Representation and Reasoning Under Uncertainty*, volume 808 of *Lecture Notes in Computer Science*, pages 17–29. Springer, 1994.

[12] Petr Hájek. Basic fuzzy logic and BL-algebras. *Soft Computing*, 2(3):124–128, 1998.

[13] Petr Hájek. *Metamathematics of Fuzzy Logic*, volume 4 of *Trends in Logic*. Kluwer, Dordrecht, 1998.

[14] Petr Hájek, Lluís Godo, and Francesc Esteva. A complete many-valued logic with product conjunction. *Archive for Mathematical Logic*, 35(3):191–208, 1996.

[15] Petr Hájek and Zuzana Haniková. A development of set theory in fuzzy logic. In Melvin Chris Fitting and Ewa Orlowska, editors, *Beyond Two: Theory and Applications of Multiple-Valued Logic*, volume 114 of *Studies in Fuzziness and Soft Computing*, pages 273–285. Physica-Verlag, Heidelberg, 2003.

[16] Petr Hájek and Tomáš Havránek. *Mechanizing Hypothesis Formation: Mathematical Foundations of a General Theory.* Springer, Berlin, 1978.

[17] Petr Hájek, Tomáš Havránek, and Radim Jiroušek. *Uncertain Information Processing in Expert Systems.* CRC Press, Boca Raton, 1992.

[18] Petr Hájek and Pavel Pudlák. *Metamathematics of First-Order Arithmetic*. Perspectives in Mathematical Logic. Springer-Verlag, 1993.

[19] Marie Hájková. The lattice of bi-numerations of arithmetic I. *Commentationes Mathematicae Universitatis Carolinae*, pages 281–306, 1971.

[20] Marie Hájková. My life with Petr, Petr's life with me. In P. Cintula, Z. Haniková, and V. Švejdar, editors, *Witnessed Years: Essays in Honour of Petr Hájek*, volume 10 of *Tributes*, pages 9–12. College Publications, 2009.

[21] Zuzana Haniková. *Mathematical and Metamathematical Properties of Fuzzy Logic*. PhD thesis, Charles University in Prague, Faculty of Mathematics and Physics, 2004.

[22] Zuzana Haniková. Petr Hájek: A Scientific Biography. In Franco Montagna, editor, *Petr Hájek on Mathematical Fuzzy Logic*, volume 6 of *Outstanding Contributions to Logic*. Springer, 2015.

[23] Jiří Ivánek. *Příspěvky k matematické teorii generování hypotéz.* [Contributions to the mathematical theory of hypothesis generation.] Doctoral thesis, Institute of Mathematics, Czechoslovak Academy of Sciences, 1984.

[24] Franco Montagna, editor. *Petr Hájek on Mathematical Fuzzy Logic*, volume 6 of *Outstanding Contributions to Logic*. Springer International Publishing Switzerland, 2015.

[25] Vilém Novák. *Fuzzy množiny a jejich aplikace.* [Fuzzy sets and their applications.] SNTL Praha, 1986.

[26] Vilém Novák. *The Alternative Mathematical Model of Linguistic Semantics with the Help of First Order Fuzzy Logic*. Doctoral thesis, Charles University in Prague, 1988.

[27] Jan Pavelka. On fuzzy logic I, II, III. *Zeitschrift für Mathematische Logik und Grundlagen der Mathematik*, 25:45–52, 119–134, 447–464, 1979.

[28] Pavel Pudlák. Arithmetic in Prague in the 1970–80s. In P. Cintula, Z. Haniková, and V. Švejdar, editors, *Witnessed Years: Essays in Honour of Petr Hájek*, volume 10 of *Tributes*, pages 27–31. College Publications, 2009.

[29] Jan Rauch. *Logické základy automatické tvorby hypotéz z databází.* [Logical foundations of automated hypothesis generation from databases.] Doctoral thesis, Institute of Mathematics, Czechoslovak Academy of Sciences, 1986.

[30] Vítězslav Švejdar. *Modální logika a interpretovatelnost.* [Modal logic and interpretability.] Doctoral thesis, Institute of Mathematics, Czechoslovak Academy of Sciences, 1982.

[31] Vítězslav Švejdar. Modal analysis of generalized Rosser sentences. *Journal of Symbolic Logic*, 48(4):986–999, 1983.

[32] Gaisi Takeuti and Satoko Titani. Fuzzy logic and fuzzy set theory. *Archive for Mathematical Logic*, 32(1):1–32, 1992.

[33] Julio J. Valdés. *Algebraic and Logical Foundations of Uncertainty Processing in Rule-Based Expert Systems of Artificial Intelligence.* Doctoral thesis, Institute of Mathematics, Czechoslovak Academy of Sciences, 1987.

[34] Petr Vopěnka and Petr Hájek. *The Theory of Semisets.* Academia Praha/North-Holland Publishing Company, 1972.

[35] Lotfi A. Zadeh. Fuzzy sets. *Information and Control*, 8(3):338–353, 1965.