On the Multimodal Logic of Elementary Normative Systems

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Abstract. We introduce Multimodal Logics of Normative Systems as a contribution to the development of a general logical framework for reasoning about normative systems over logics for Multi-Agent Systems. Given a multimodal logic L, with standard Kripke semantics, for every modality $\Box_i$ and normative system η, we expand the language adding a new modality $\Box_i^\eta$ with the intended meaning of $\Box_i^\eta \phi$ being "ϕ is obligatory in the context of the normative system η over the logic L". In this expanded language we define the Multimodal Logic of Normative Systems over L, for any given set of normative systems N, and give a sound and complete axiomatisation for this logic, proving transfer results in the case that L and N are axiomatised by sets of Sahlqvist or shallow modal formulas.

Keywords. Multimodal Logics, Normative Systems, Multi-Agent Systems, Model Theory, Sahlqvist Formulas,

1. Introduction

Recent research on the logical foundations of Multi-Agent Systems (MAS) has centered its attention in the study of normative systems. MAS could be regarded as a type of dialogical system, in which interactions among agents are realized by means of message interchanges, all these interactions taking place within an institution. The notion of electronic institution is a natural extension of human institutions by permitting not only humans but also autonomous agents to interact with one another. Institutions are used to regulate interactions where participants establish commitments and to facilitate that these commitments are upheld, the institutional conventions are devised so that those commitments can be established and fulfilled (see [1] for a general reference of the role of electronic institutions to regulate agents interactions in MAS). Over the past decade, normative systems have been promoted for the coordination of MAS and the engineering of societies of self-interested autonomous software agents. In this context there is an increasing need to find a general logical framework for the study of normative systems over the logics for MAS.

Given a set of states S and a binary accessibility relation R on S, a normative system η on the structure (S, R) could be understood as a set of constraints $\eta \subseteq R$ on the transitions between states, the intended meaning of $(x, y) \in \eta$ being "the transition from state x to state y is not legal according to normative system η". Several formalisms have

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been introduced for reasoning about normative systems over specific logics, two examples are worth noting: Normative ATL (NATL), proposed in [2] and Temporal Logic of Normative Systems (NTL) in [3]. NATL is an extension to the Alternating-Time Temporal Logic of Alur, Henzinger and Kupferman (see [4]), NATL contains cooperation modalities of the form $<< \eta : C >> \phi$ with the intended interpretation that "C has the ability to achieve ϕ within the context of the normative system η". NTL is a conservative generalization of the Branching-Time Temporal Logic CTL (see [5]). In NTL, the path quantifiers A ("on all paths...") and E ("on some path...") are replaced by the indexed deontic operators O_η ("it is obligatory in the context of the normative system η that...") and P_η ("it is permissible in the context of the normative system η that...").

For our purpose of developing logical models for MAS, it would be worth to work in a generalization to arbitrary logics of the approaches taken in [2] and [3]. The Multimodal Logics of Normative Systems introduced in this article are a contribution to define such a general logical framework. There are some advantages of using these logics for reasoning about MAS: it is possible to compare whether a normative system is more restrictive than the other, check if a certain property holds in a model of a logic once a normative system has restricted its accessibility relation, model the dynamics of normative systems in institutional settings, define a hierarchy of normative systems (and, by extension, a classification of the institutions) or present a logical-based reasoning model for the agents to negotiate over norms.

We have restricted our attention to multimodal logics with Kripke semantics, outlining at the end of the paper how these results could be applied to other formalisms of common use in modelling MAS, such as Hybrid Logics. Our definition of normative system is intensional, but the languages introduced permit to work with extensional definitions like the one in [3]. We present completeness and canonicity results for logics with normative systems that define elementary classes of modal frames, we have called them *Elementary Normative Systems (ENS)*. On the one hand, the choice of ENS seems the more natural to start with, because elementary classes of frames include a wide range of formalisms used in describing MAS, modelling different aspects of agenthood, some Temporal Logics, Logics of Knowledge and Belief, Logics of Communication, etc. On the other hand, at the moment, we are far from obtaining a unique formalism which addresses all the features of MAS at the same time, but the emerging field of combining logics is a very active area and has proved to be successful in obtaining formalisms which combine good properties of the existing logics. In our approach, we regard the Logic of Normative Systems over a given logic L, as being the fusion of logics obtained from L and a set of normative systems over L, this model-theoretical construction will help us to understand better which properties are preserved under combinations of logics over which we have imposed some restrictions and to apply known transfer results (for a recent account on the combination of logics, we refer to [6]).

This paper is structured as follows. In Section 2 we introduce the notion of *Elementary Normative System (ENS)*, a kind of normative system that defines elementary classes of modal frames, and we study the Multimodal Logics of Elementary Normative Systems, proving completeness, canonicity and some transfer results in the case that the logic L and the normative system N are axiomatised by sets of Sahlqvist or shallow modal formulas. In section 3, we give an example to illustrate how our framework can work in Multiprocess Temporal Structures, and we show that we can axiomatise with elementary classes a wide range of formalisms used in describing MAS, modelling different aspects of agenthood: some Temporal Logics, Logics of Knowledge and Belief, Logics of Communication, etc. and to which we can apply also our framework. In Section 4 we present some related work and compare our results with the ones obtained by other approaches. Finally, Section 5 is devoted to future work.

2. Elementary Normative Systems on Multimodal Languages

We begin the section by introducing the notion of First-order Normative System and its corresponding counterpart in modal languages, Elementary Normative Systems. Let L be a first-order language whose similarity type is a set $\{R_i : i \in I\}$ of binary relational symbols. Given an L-structure Ω with domain A, we denote by Ω^* the following set of sequences of elements of A:

$$\Omega^* = \left\{(a_0, \ldots, a_m) : \forall j < m, \exists i \in I \text{ such that } a_j R_i^{\Omega} a_{j+1}\right\}$$

We say that a formula $\phi(x_0, \ldots, x_k) \in L$ is a *First-Order Normative System* iff for every L-structure Ω,

$$\{(a_0, \ldots, a_k) : \Omega \models \phi[a_0, \ldots a_k]\} \subseteq \Omega^*.$$

A *modal similarity type* $\tau = \langle F, \rho \rangle$ consists of a set F of modal operators and a map $\rho : F \rightarrow \omega$ assigning to each $f \in F$ a finite arity $\rho(f) \in \omega$. A propositional modal language of type τ is defined in the usual way by using propositional variables, the operators in F and the boolean connectives $\wedge, \vee, \neg, \rightarrow, \leftrightarrow, \top, \bot$.

Given a set of modal formulas Σ, the *frame class defined by* Σ is the class of all frames on which each formula in Σ is valid. A frame class is *modally definable* if there is a set of modal formulas that defines it, and it is said that the frame class is *elementary* if it is defined by a first-order sentence of the frame correspondence language (the first-order language with equality and one binary relation symbol for each modality). An *Elementary Normative System* (ENS) is a propositional modal formula that defines an elementary class of frames and such that its translation is a First-Order Normative System.

From now on, we assume that our modal languages have standard Kripke semantics and its modal similarity types have only a countable infinite set of monadic modalities $\{\Box_i : i \in I\}$ and a countable infinite set of propositional variables. We introduce a new set of symbols Θ to denote normative systems. Given a modal language of similarity type τ, for every $\eta \in \Theta$, let τ^η be the similarity type whose modalities are $\{\Box_i^\eta : i \in I\}$. For every set of formulas Γ, let us denote by Γ^η the set of formulas of type τ^η obtained from Γ by substituting every occurrence of the modality $\Box_i$ by $\Box_i^\eta$. We define the operators $\Diamond_i$ in the usual way, $\Diamond_i \phi \equiv \neg \Box_i \neg \phi$ and we introduce the corresponding $\Diamond_i^\eta$. For the sake of clarity from now on we will denote by η both the term which indexes the modality and the formula that expresses the normative system.

Given a logic L and a set of normative systems N over L, for every $\eta \in N$, let us denote by $L(\eta)$ the smallest normal logic of similarity type τ^η which includes $L^\eta \cup \{\eta\}$. We define the *Multimodal Logic of Normative Systems* over L and N, denoted by L^N, as being the smallest normal logic in the expanded language which contains L, N and every L^η. We now present a sound and complete axiomatisation and prove some transfer results in the case that L is axiomatised by a set of Sahlqvist formulas and N is a set of Sahlqvist formulas.

Definition 1. (Sahlqvist formulas) *A modal formula is* positive (negative) *if every occurrence of a proposition letter is under the scope of an even (odd) number of negation signs. A* Sahlqvist antecedent *is a formula built up from* $\top, \bot$, *boxed atoms of the form* $\Box_{i_1} \ldots \Box_{i_l} p$, *for* $i_j \in I$ *and negative formulas, using conjunction, disjunction and diamonds. A* Sahlqvist implication *is a formula of the form* $\phi \to \varphi$, *when* ϕ *is a Sahlqvist antecedent and* φ *is positive. A* Sahlqvist formula *is a formula that is obtained from Sahlqvist implications by applying boxes and conjunction, and by applying disjunctions between formulas that do not share any propositional letters.*

Observe that $\bot$ and $\top$ are both Sahlqvist and ENS formulas. Intuitively speaking, $\bot$ is the trivial normative system, in $\bot$ every transition is forbidden in every state and in $\top$ every action is legal. In the sequel we assume that for every set N of ENS, $\top \in N$.

Theorem 2. *Let* L *be a normal modal logic axiomatised by a set* Γ *of Sahlqvist formulas and* N *a set of ENS Sahlqvist formulas, then:*

1. $\Gamma^N = \Gamma \cup N \cup \bigcup \{\Gamma^\eta : \eta \in N\}$ *is an axiomatisation of* L^N.
2. L^N *is complete for the class of Kripke frames defined by* Γ^N.
3. L^N *is canonical.*
4. *If* L *and* L^η *are consistent, for every* $\eta \in N$, *and **P** is one of the following properties:*
   - *Compactness*
   - *Interpolation Property*
   - *Halldén-completeness*
   - *Decidability*
   - *Finite Model Property*[2]

   then L^N *has **P** iff* L *and* $L(\eta)$ *have **P**, for every* $\eta \in N$.

Proof: $1 - 3$ follows directly from the Sahlqvist's Theorem. The main basic idea of the proof of 4 is to apply the Sahlqvist's Theorem to show first that for every $\eta \in N$, the smallest normal logic of similarity type τ^η which includes $\Gamma^\eta \cup \{\eta\}$ is $L(\eta)$, is a complete logic for the class of Kripke frames defined by $\Gamma^\eta \cup \{\eta\}$ and is canonical (observe that this logic is axiomatised by a set of Sahlqvist formulas). Now, since for every Elementary Normative System $\eta \in N$ we have introduced a disjoint modal similarity type τ^η, we can define the fusion of the logics $\bigoplus < L(\eta) : \eta \in N >$. It is enough to check that $L^N = \bigoplus < L(\eta) : \eta \in N >$ (remark that $L^\top = L$) and using transfer results for fusions of consistent logics (see for instance [7] and [8]) we obtain that L^N is a conservative extension and that decidability, compactness, interpolation, Hállden-completeness and the Finite Model Property are preserved. $\Box$

We study now the relationships between normative systems. It is interesting to see how the structure of the set of all the ENS over a logic L (we denote it by $N(L)$) inherits

[2] For the transfer of the Finite Model Property it is required that there is a number n such that each $L(\eta)$ has a model of size at most n.

its properties from the set of first-order counterparts. A natural relationship could be defined between ENS, the relationship of being one *less restrictive* than another, let us denote it by $\preceq$. Given η, η', it is said that $\eta \preceq \eta'$ iff the first-order formula $\phi_{\eta'} \to \phi_\eta$ is valid (when for every $\eta \in N$, ϕ_η is the translation of η). The relation $\preceq$ defines a partial order on $N(L)$ and the pair $(N(L), \preceq)$ forms a complete lattice with least upper bound $\perp$ and greatest lower bound $\top$ and the operations $\wedge$ and $\vee$.

Now we present an extension of the Logic of Elementary Normative Systems over a logic L with some inclusion axioms and we prove completeness and canonicity results. Given a set N of ENS, let I^{N^+} be the following set of formulas:

$$\left\{\Box_{i_1} \ldots \Box_{i_l} p \to \Box^\eta_{i_1} \ldots \Box^\eta_{i_l} p : i_j \in I, \eta \in N\right\}$$

and I^{N^*} the set:

$$\left\{\Box^{\eta'}_{i_1} \ldots \Box^{\eta'}_{i_l} p \to \Box^\eta_{i_1} \ldots \Box^\eta_{i_l} p : i_j \in I, \eta \preceq \eta', \eta, \eta' \in N\right\}$$

Corollary 3. *Let L be a normal modal logic axiomatised by a set Γ of Sahlqvist formulas and N a set of ENS Sahlqvist formulas, then:*

1. $\Gamma^{N^+} = \Gamma^N \cup I^{N^+}$ *is an axiomatisation of the smallest normal logic with contains L^N and the axioms I^{N^+}, is complete for the class of the Kripke frames defined by Γ^{N^+} and is canonical. We denote this logic by L^{N^+}.*
2. $\Gamma^{N^*} = \Gamma^N \cup I^{N^*} \cup I^{N^+}$ *is an axiomatisation of the smallest normal logic with contains L^N and the axioms $I^{N^*} \cup I^{N^+}$, is complete for the class of the Kripke frames defined by Γ^{N^*} and is canonical. We denote this logic by L^{N^*}.*
3. *If L^N is consistent, both L^{N^+} and L^{N^*} are consistent.*

Proof: Since for every $i_j \in I$ every $\eta, \eta' \in N$, the formulas $\Box_{i_1} \ldots \Box_{i_l} p \to \Box^\eta_{i_1} \ldots \Box^\eta_{i_l} p$ and $\Box^{\eta'}_{i_1} \ldots \Box^{\eta'}_{i_l} p \to \Box^\eta_{i_1} \ldots \Box^\eta_{i_l} p$ are Sahlqvist, we can apply Theorem 2. In the case that L^N is consistent, consistency is guaranteed by the restriction to pairs $\eta \preceq \eta'$ and for the fact that η and η' are ENS. □

It is worth to remark that Corollary 3 allows us to see that our framework could also be applied to deal with an extensional definition of normative systems (for example like the one presented in [3], where normative systems are defined to be subsets of the accessibility relation with certain properties), taking $N = L$ in the statement of Corollary 3, the logics L^{N^+} and L^{N^*} have the desired properties. Observe also that for every frame $(S, R_i, R^\eta_i)_{i \in I, \eta \in N}$ of the logic L^{N^*},

$$R^\eta_{i_1} \circ \ldots \circ R^\eta_{i_l} \subseteq R_{i_0} \circ \ldots \circ R_{i_l},$$

and for $\eta \preceq \eta'$, $R^\eta_{i_1} \circ \ldots \circ R^\eta_{i_l} \subseteq R^{\eta'}_{i_1} \circ \ldots \circ R^{\eta'}_{i_1}$, where $\circ$ is the composition relation.

We end this section introducing a new class of modal formulas defining elementary classes of frames, the shallow formulas (for a recent account of the model theory of elementary classes and shallow formulas we refer the reader to [9]).

Definition 4. *A modal formula is* shallow *if every occurrence of a proposition letter is in the scope of at most one modal operator.*

It is easy to see that every closed formula is shallow and that the class of Sahlqvist and shallow formulas don't coincide: $\Box_1(p \vee q) \to \Diamond_2(p \wedge q)$ is an example of shallow formula that is not Sahlqvist. Analogous results to Theorem 2 and Corollary 3 hold for shallow formulas, and using the fact that every frame class defined by any finite set of shallow formulas admits polynomial filtration, by Theorem 2.6.8 of [9], if L is a normal modal logic axiomatised by a finite set Γ of shallow formulas and N is a finite set of ENS shallow formulas, then the frame class defined by Γ^N has the finite model property and has a satisfiability problem that can be solved in NEXPTIME.

3. Multiprocess Temporal Frames and other examples

Different formalisms have been introduced in the last twenty years in order to model particular aspects of agenthood (Temporal Logics, Logics of Knowledge and Belief, Logics of Communication, etc). Logics of ENS defined above are combinations of different logics, and consequently, they reflect different aspects of agents and the agent multiplicity. We show in this section that several logics proposed for describing Multi-Agents Systems are axiomatised by a set of Sahlqvist or shallow formulas and therefore we could apply our results to the study of their normative systems. We introduce first the basic temporal logic of transition systems, not because it is specially interesting in itself, but because is the logic upon which other temporal logics are built and because it is a clear and simple example of how the ENS framework can work.

Given a modal similarity type τ, a τ-frame $\Xi = (S, R_0, \ldots, R_k)$ is a *multiprocess temporal frame* if and only if $\bigcup_{i \leq k} R_i$ is serial. Observe that a τ-frame $\Xi = (S, R_0, \ldots, R_k)$ is a multiprocess temporal frame if and only if the formula

$$\Diamond_0 \top \vee \ldots \vee \Diamond_k \top \text{ (MPT)}$$

is valid in Ξ. Let us denote by $MPTL$ the smallest normal logic containing axiom (MPT). For every nonempty tuple $(i_1, \ldots, i_l)$ such that for every $j \leq l$, $i_j \leq k$, consider the formula $\Box_{i_1} \ldots \Box_{i_l} \bot$. Observe that every formula of this form is shallow and ENS. We state now without proof a result on the consistency of normative systems over $MPTL$ that will allow us to use the logical framework introduced in the previous section.

Proposition 5. *Let X be a finite set of formulas of the form $\Box_{i_1} \ldots \Box_{i_l} \bot$ and let η be the conjunction of all the formulas in X. Then, if $\bot \notin X$ and the following property holds:*

$$\text{If } \Box_{i_1} \ldots \Box_{i_l} \bot \notin X, \text{ there is } j \leq k \text{ such that } \Box_{i_1} \ldots \Box_{i_l} \Box_j \bot \notin X.$$

the logic $MPTL^\eta$ is consistent, complete for the class of Kripke frames defined by $\{MPT, \eta\}$, canonical, has the finite model property and has a satisfiability problem that can be solved in NEXPTIME.

Now we give an example of logic to which our framework could be applied. In a multi-agent institutional environment, in order to allow agents to successfully interact with other agents, they share the dialogic framework (see [10]). The expressions of the communication language in a dialogic framework are constructed as formulas of the type $\iota(\alpha_i : \rho_i, \alpha_j : \rho_j, \phi, \tau)$, where ι is an illocutionary particle, α_i and α_j are agent terms, ρ_i

and ρ_j are role terms and τ is a time term. A scene is specified by a graph where the nodes of the graph represent the different states of the conversation and the arcs connecting the nodes are labelled with illocution schemes that make scene state change.

Several formalisms for modelling interscene exchanges between agents have been introduced using multi-modal logics. In [11] the authors provide an alternating offers protocol to specify commitments that agents make to each other when engaging in persuasive negotiations using rewards. Specifically, the protocol details, how commitments arise or get retracted as a result of agents promising rewards or making offers. The protocol also standardises what an agent is allowed to say or what it can expect to receive from its opponent which, in turn, allows it to focus on making the important negotiation decisions.

The logic introduced in [11] is a multimodal logic in which modalities $\Box_\phi$ for expressions ϕ of the communication language are introduced. The semantics are given by means of Multiprocess Temporal Frames. Therefore, we can use our framework to analyse different protocols over this multimodal logic, regarding protocols as normative systems. Some examples of those protocols are formalised by formulas of the following form $\Box_{\phi_1} \ldots \Box_{\phi_l} \bot$, for example with the formula $\Box_{Offer(i,x)} \Box_{Offer(i,y)} \bot$, for $x \neq y$ we can express that it is not allowed to agent i to do two different offers one immediately after the other.

In general, a normal multimodal logic can be characterized by axioms that are added to the system K_m, the class of *Basic Serial Multimodal Logics* is characterized by subsets of axioms of the following form, requiring that AD is full:

- $\Box_i p \to \Diamond_i p$ AD(i)
- $\Box_i p \to p$ AT(i)
- $\Box_i p \to \Box_j p$ AI(i)
- $p \to \Box_i \Diamond_j p$ AB(i,j)
- $\Box_i p \to \Box_j \Box_k p$ A4(i,j,k)
- $\Diamond_i p \to \Box_j \Diamond_k p$ A5(i,j,k)

An example of Kripke frame of $MPTL$ in which none of the previous axioms is valid is $\Xi = (\{0,1,2\}, \{(0,1),(2,0)\}, \{(1,2)\})$. In particular, our example shows that the Multimodal Serial Logic axiomatised by $\{AD(i) : i \leq k\}$, is a proper extension of $MPTL$. Observe that any logic in the class BSML is axiomatised by a set of Sahlqvist formulas, therefore we could apply the framework introduced before to compare elementary normative systems on these logics.

Another type of logics axiomatised by Sahlqvist formulas are many Multimodal Epistemic Logics. Properties such as positive or negative introspection can be expressed by $\Box_i p \to \Box_i \Box_k p$ and $\neg\Box_i p \to \Box_i \neg\Box_i p$ respectively. And formulas like $\Box_i p \to \Box_j p$ allow us to reason about multi-degree belief.

The Minimal Temporal Logic K_t is axiomatised by the axioms $p \to HFp$ and $p \to GPp$ which are also Sahlqvist formulas. Some important axioms such as linearity $Ap \to GHp \wedge HGp$, or density $GGp \to Gp$, are Sahlqvist formulas, and we can express the property that the time has a beginning with an ENS. By adding the nexttime modality, X, we have an ENS which expresses that every instant has at most one immediate successor.

4. Related work

Several formalisms have been introduced for reasoning about normative systems over specific logics: Normative ATL (NATL), proposed in [2] and Temporal Logic of Normative Systems (NTL) in [3]. NATL is an extension to the Alternating-Time Temporal Logic of Alur, Henzinger and Kupferman (see [4]), NATL contains cooperation modalities of the form $<< \eta : C >> \phi$ with the intended interpretation that "C has the ability to achieve ϕ within the context of the normative system η". NTL is a conservative generalization of the Branching-Time Temporal Logic CTL (see [5]). In NTL, the path quantifiers A ("on all paths...") and E ("on some path...") are replaced by the indexed deontic operators O_η ("it is obligatory in the context of the normative system η that...") and P_η ("it is permissible in the context of the normative system η that..."). In our article we have extended these approaches to deal with arbitrary multimodal logics with standard Kripke semantics. Our definition of normative system is intensional, but the languages introduced permit to work with extensional definitions like the ones in [3] and [2].

A general common framework that generalizes both, the results from [3], [2] and the results we present here can be found in [12]. There we go beyond logics with standard Kripke semantics, defining normative systems over polyadic logics that satisfy the two conditions below:

1. For every modality f in the logic similarity type F, the semantics of $f(p_0, \dots, p_{\rho(f)})$ is a monadic first-order formula build from predicates $P_0, \dots, P_{\rho(f)}$, the relational symbols $\left\{\overline{R_f} : f \in F\right\}$ and equality.
2. For every modality f in the logic similarity type F, there is a derived connective $\Box_f$ such that $\Box_f p$ expresses $\forall x(t\overline{R_f}x \rightarrow Px)$ and is closed under the necessitation rule: If ϕ is a theorem of the logic, then $\Box_f \phi$ is also a theorem of the logic. This second condition corresponds to the notion of *normality*.

In [13], D. M. Gabbay and G. Governatori introduced a multi-modal language where modalities were indexed. Their purpose was the logical formalization of norms of different strength and the formalization of normative reasoning dealing with several intensional notions at once. The systems were combined using Gabbay's fibring methodology. Our approach is different from their, because our main purpose is the comparison between normative systems at the same level, over a fixed logic. Our approaches also differ in the methodologies in use. It could be interesting to combine both perspectives to model the dynamics of norms in hierarchical systems.

5. Future work

In our paper we have dealt only with multimodal languages with monadic modalities, but using the results of Goranko and Vakarelov in [14], on the extension of the class of Sahlqvist formulas in arbitrary polyadic modal languages to the class of inductive formulas, it is possible to generalize our results to polyadic languages. We could apply also our results to different extended modal languages, such as reversive languages with nominals (in [14], the elementary canonical formulas in these languages are characterized) or Hybrid Logic (in [9], Hybrid Sahlqvist formulas are proved to define elementary classes of frames).

We will proceed also to the study of computational questions for the multimodal logics introduced, such as model checking. This kind of results will give us a very useful tool to compare normative systems and to answer some questions, for example, about the existence of normative systems with some given properties. It is known that, if L is a multimodal logic interpreted using Kripke semantics in a finite similarity type, given a finite model and a formula ϕ, there is an algorithm that determines in time $O(|M| \times |\phi|)$ whether or not $M \models \phi$ (see [15], p. 63), using this fact, since we build our formalisms for normative systems by means of fusions, complexity results for fusion of logics could be applied (see for instance [16]).

Most state-of-the-art SAT solvers today are based on different variations of the Davis-Putnam-Logemann-Loveland (DPLL) procedure (see [17] and [18]). Because of their success, both the DPLL procedure and its enhancements have been adapted to handle satisfiability problems in more expressive logics than propositional logic. In particular, they have been used to build efficient algorithms for the Satisfiability Modulo Theories (SMT) problem. Future work will include the study of the (SMT) problem for a Multimodal Logic of Normative Systems L: given a formula ϕ, determine whether ϕ is L -satisfiable, i.e., whether there exists a model of L that is also a model of ϕ.

Using the framework introduced in [19] it would be possible to integrate fusions of logics on a propositional framework. In [19], an Abstract DPLL, uniform, declarative framework for describing DPLL-based solvers is provided both for propositional satisfiability and for satisfiability modulo theories. It could be interesting to work with a Quantified Boolean formulas engine instead of the usual SAT engines used by several SMT solvers, in order to deal with formulas that axiomatise Logics for Multi-Agent Systems.

Given a normative system, it is important also to be able to efficiently check why it is not modelling what we originally want. We could benefit from recent advances in system diagnosis using Boolean Satisfiability and adapt it to our framework. See for instance [20], where efficient procedures are developed to extract an unsatisfiable core from an unsatisfiability proof of the formula provided by a Boolean Satisfiability (SAT) solver.

Acknowledgements

The author wishes to express her thanks to Carles Sierra, Pablo Noriega and the reviewers of this paper for their helpful comments. The author was partially supported by Project TIN2006-15662-C02-01.

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