

# ManyVal 2019

Book of Abstracts

Bucharest, Romania

November 1-3, 2019

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# On paraconsistent extensions of degree-preserving Gödel logics with an involution

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In this paper we study paraconsistent logics arising from Gödel fuzzy logic expanded with an involutive negation  $G_{\sim}$ , introduced in [4], as well as from its finite-valued extensions  $G_{n\sim}$ . It is well-known [2] that Gödel logic  $G$  coincides with its degree-preserving companion  $G^{\leq}$  (since  $G$  has the deduction-detachment theorem), but this is not the case for  $G_{\sim}$ . In fact,  $G_{\sim}$  and  $G_{\sim}^{\leq}$  are different logics, and moreover, while  $G_{\sim}^{\leq}$  is explosive w.r.t. Gödel negation  $\neg$ , it is paraconsistent w.r.t. the involutive negation  $\sim$ .

Among the logics between  $G_{\sim}^{\leq}$  and classical logic (CPL) there are the ones defined by matrices  $\langle \mathbf{A}, F \rangle$  where  $\mathbf{A}$  is a  $G_{\sim}$ -algebra and  $F$  is a lattice filter of  $\mathbf{A}$ . In particular we consider the logics over  $[0, 1]_{G_{\sim}}$  with order filters

$$G_{\sim}^{[a]} = \langle [0, 1]_{G_{\sim}}, F_{[a]} \rangle \text{ and } G_{\sim}^{(a)} = \langle [0, 1]_{G_{\sim}}, F_{(a)} \rangle$$

where  $F_{[a]} = \{x \in [0, 1] : x \geq a\}$  for all  $a \in (0, 1)$ , and  $F_{(a)} = \{x \in [0, 1] : x > a\}$  for all  $a \in [0, 1)$ . We prove that there are only three different  $\sim$ -paraconsistent logics among them.

**Proposition 1.** *Among the logics  $\{G_{\sim}^{[a]}\}_{a \in (0,1)}$  and  $\{G_{\sim}^{(a)}\}_{a \in [0,1)}$ , there are only three different  $\sim$ -paraconsistent logics:  $G_{\sim}^{[a]}$  for any  $a \in (0, 1/2)$ ,  $G_{\sim}^{[1/2]}$ , and  $G_{\sim}^{(0)}$ .*

In the second part of the paper we consider the finite-valued Gödel logics with an involutive negation  $G_{n\sim}$  and their degree-preserving counterparts  $G_{n\sim}^{\leq}$ . Actually, it is easy to check that  $G_{3\sim}$  and  $G_{4\sim}$  are respectively logically equivalent to the 3-valued and 4-valued Łukasiewicz logics. As in the  $[0, 1]$ -valued case,  $G_{n\sim}^{\leq}$  are  $\sim$ -paraconsistent and thus it makes sense to study the paraconsistent logics between  $G_{n\sim}^{\leq}$  and CPL. In fact, using a similar argument that in [3], it can be shown that any logic  $L$  between  $G_{n\sim}^{\leq}$  and CPL is defined by a family of matrices  $\langle \mathbf{A}, F \rangle$  where  $\mathbf{A}$  is a finite direct product of finite  $G_{n\sim}$ -chains and  $F$  is a lattice filter of  $\mathbf{A}$  compatible with  $L$ .

In particular, we study which ones are ideal and saturated paraconsistent. Roughly speaking, we call a logic  $L$  *saturated paraconsistent* when it is maximally paraconsistent,<sup>1</sup> while a logic is called *ideal paraconsistent* in [1] when it is also maximal w.r.t. to classical logic CPL (with the same signature).

Before introducing the main result related to this question, let us consider three particular matrix logics:

- $J_3$ , defined by the matrix  $\langle \mathbf{VG}_{3\sim}, \{1/2, 1\} \rangle$
- $J_4$ , defined by the matrix  $\langle \mathbf{VG}_{4\sim}, \{1/3, 2/3, 1\} \rangle$
- $J_3 \times J_4$ , defined by the matrix  $\langle \mathbf{VG}_{3\sim} \times \mathbf{VG}_{4\sim}, \{1/2, 1\} \times \{1/3, 2/3, 1\} \rangle$

where  $\mathbf{VG}_{n\sim}$  is the  $G_{n\sim}$ -algebra over the universe  $\{0, 1/(n-1), \dots, 1\}$ .

**Theorem 1.** *Let  $n$  be an integer number such that  $n > 4$  and let  $L$  be an extension of  $G_{n\sim}^{\leq}$ .*

1. *If  $n$  is an even number, then  $L$  is saturated  $\sim$ -paraconsistent iff  $L$  is ideal  $\sim$ -paraconsistent iff  $L = J_4$ .*
2. *If  $n$  is an odd number, then  $L$  is saturated  $\sim$ -paraconsistent iff  $L = J_3$ ,  $L = J_4$  or  $L = J_3 \times J_4$ .*
3. *If  $n$  is an odd number, then  $L$  is ideal  $\sim$ -paraconsistent iff  $L = J_3$  or  $L = J_4$ .*

**Acknowledgments** Esteva and Godo acknowledge partial support by the Spanish FEDER/MINECO project TIN2015-71799-C2-1-P (RASO). J. Gispert acknowledges partial support by the Spanish MINECO/FEDER projects MTM2016-74892 and MDM-2014-044, and grant 2017-SGR-95 of Generalitat de Catalunya.

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<sup>1</sup>That is, when any proper extension is no longer paraconsistent.