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On paraconsistent extensions of degree-preserving Gödel logics with an involution

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In this paper we study paraconsistent logics arising from Gödel fuzzy logic expanded with an involutive negation $\mathrm{G}_\sim$, introduced in [4], as well as from its finite-valued extensions $\mathrm{G}_{n\sim}$. It is well-known [2] that Gödel logic G coincides with its degree-preserving companion $\mathrm{G}^\leq$ (since G has the deduction-detachment theorem), but this is not the case for $\mathrm{G}_\sim$. In fact, $\mathrm{G}_\sim$ and $\mathrm{G}^\leq_\sim$ are different logics, and moreover, while $\mathrm{G}^\leq_\sim$ is explosive w.r.t. Gödel negation $\neg$, it is paraconsistent w.r.t. the involutive negation $\sim$.
Among the logics between $\mathrm{G}^\leq_\sim$ and classical logic (CPL) there are the ones defined by matrices $\langle \mathbf{A}, F\rangle$ where $\mathbf{A}$ is a $G_\sim$-algebra and F is a lattice filter of $\mathbf{A}$. In particular we consider the logics over $[0,1]_{G_\sim}$ with order filters

$$\mathrm{G}^{[a}_\sim = \langle [0,1]_{G_\sim}, F_{[a}\rangle \text{ and } \mathrm{G}^{(a}_\sim = \langle [0,1]_{G_\sim}, F_{(a}\rangle$$

where $F_{[a} = \{x \in [0,1] : x \geq a\}$ for all $a \in (0,1]$, and $F_{(a} = \{x \in [0,1] : x > a\}$ for all $a \in [0,1)$. We prove that there are only three different $\sim$-paraconsistent logics among them.

Proposition 1. *Among the logics* $\{\mathrm{G}^{[a}_\sim\}_{a\in(0,1]}$ *and* $\{\mathrm{G}^{(a}_\sim\}_{a\in[0,1)}$*, there are only three different* $\sim$*-paraconsistent logics:* $\mathrm{G}^{[a}_\sim$ *for any* $a \in (0,1/2)$*,* $\mathrm{G}^{[1/2}_\sim$*, and* $\mathrm{G}^{(0}_\sim$*.*

In the second part of the paper we consider the finite-valued Gödel logics with an involutive negation $\mathrm{G}_{n\sim}$ and their degree-preserving counterparts $\mathrm{G}^\leq_{n\sim}$. Actually, it is easy to check that $\mathrm{G}_{3\sim}$ and $\mathrm{G}_{4\sim}$ are respectively logically equivalent to the 3-valued and 4-valued Łukasiewicz logics. As in the $[0,1]$-valued case, $\mathrm{G}^\leq_{n\sim}$ are $\sim$-paraconsistent and thus it makes sense to study the paraconsistent logics between $\mathrm{G}^\leq_{n\sim}$ and CPL. In fact, using a similar argument that in [3], it can be shown that any logic L between $\mathrm{G}^\leq_{n\sim}$ and CPL is defined by a family of matrices $\langle \mathbf{A}, F\rangle$ where $\mathbf{A}$ is a finite direct product of finite $\mathrm{G}_{n\sim}$-chains and F is a lattice filter of $\mathbf{A}$ compatible with L.

In particular, we study which ones are ideal and saturated paraconsistent. Roughly speaking, we call a logic L *saturated paraconsistent* when it is maximally paraconsistent,[1] while a logic is called *ideal paraconsistent* in [1] when it is also maximal w.r.t. to classical logic CPL (with the same signature).
Before introducing the main result related to this question, let us consider three particular matrix logics:

- J_3, defined by the matrix $\langle \mathbf{VG}_{3\sim}, \{1/2, 1\}\rangle$
- J_4, defined by the matrix $\langle \mathbf{VG}_{4\sim}, \{1/3, 2/3, 1\}\rangle$
- $\mathsf{J}_3 \times \mathsf{J}_4$, defined by the matrix $\langle \mathbf{VG}_{3\sim} \times \mathbf{VG}_{4\sim}, \{1/2, 1\} \times \{1/3, 2/3, 1\}\rangle$

where $\mathbf{VG}_{n\sim}$ is the $\mathrm{G}_{n\sim}$-algebra over the universe $\{0, 1/(n-1), \ldots, 1\}$.

Theorem 1. *Let n be an integer number such that $n > 4$ and let L be an extension of $\mathrm{G}^{\leq}_{n\sim}$.*

1. *If n is an even number, then L is saturated $\sim$-paraconsistent iff L is ideal $\sim$-paraconsistent iff $L = \mathsf{J}_4$.*
2. *If n is an odd number, then L is saturated $\sim$-paraconsistent iff $L = \mathsf{J}_3$, $L = \mathsf{J}_4$ or $L = \mathsf{J}_3 \times \mathsf{J}_4$.*
3. *If n is an odd number, then L is ideal $\sim$-paraconsistent iff $L = \mathsf{J}_3$ or $L = \mathsf{J}_4$.*

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[1] That is, when any proper extension is no longer paraconsistent.