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The logical style painting classifier based on Horn clauses and explanations (ℓ-SHE)

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Abstract This paper presents a logical Style painting classifier based on evaluated Horn clauses, qualitative colour descriptors and Explanations (ℓ-SHE). Three versions of ℓ-SHE are defined, using rational Pavelka logic, and expansions of Gödel logic and product logic with rational constants: RPL, $G(\mathbb{Q})$ and $\sqcap(\mathbb{Q})$, respectively. We introduce a fuzzy representation of the more representative colour traits for the Baroque, the Impressionism, and the Post-Impressionism art styles. The ℓ-SHE algorithm has been implemented in Swi-Prolog and tested on 90 paintings of the QArt-Dataset and on 247 paintings of the Paintings-91-PIB dataset. The percentages of accuracy obtained in the QArt-Dataset for each ℓ-SHE version are: 73.3% (RPL), 65.6% ($G(\mathbb{Q})$) and 68.9% ($\sqcap(\mathbb{Q})$). Regarding the Paintings-91-PIB dataset, the percentages of accuracy obtained for each ℓ-SHE version are: 60.2% (RPL), 48.2% ($G(\mathbb{Q})$) and 57.0% ($\sqcap(\mathbb{Q})$). Our logic definition for the Baroque style has obtained the highest accuracy in both datasets, for all the ℓ-SHE versions (the lowest Baroque case gets 85.6% of accuracy). An important feature of the classifier is that it provides reasons regarding why a painting belongs to a certain style. The classifier also provides reasons about why outliers of

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one art style may belong to another art style, giving a second classification option depending on its membership degrees to these styles.

Keywords qualitative colour, art, fuzzy logics, Horn clause, logic programming, classifier, explainable AI.

1 Introduction

Classification tasks in AI have been recently fostered by machine learning algorithms (i.e. neural networks, support vector machines, deep learning, etc.). In the literature, research works that deal with the challenge of classifying paintings in art styles are the following: traditional Chinese paintings were classified using colour and support vector machines (SVM) [23]; 2-way classification of paintings by Renoir/Monet, Pollock/Ernst, Dalí/Ernst, Renoir/ Rothko, and Dalí/Kandinsky were categorised using signature styles (computer vision statistical features) and SVMs [34, 35]; deep neural networks achieved a separation of image content from style, which allowed to recast the content of one image in the style of another image [17]; deep neural networks were also trained on object recognition for style categorisation of artworks [24] and obtained 81.45% accuracy for Baroque paintings, 82.15% for the Impressionism style and 74.51% for the Post-Impressionism style. However, although machine learning methods provide high categorisation accuracies, they need great amounts of training data and they usually cannot provide reasons to users regarding why an item is classified in a category. Providing reasons for a decision is very important in human-machine interactions, because users expect intelligent systems to explain themselves in a rational or human-like way when they take decisions. Moreover, although the expressiveness of deep neural networks is the reason they succeed, it also causes them to learn uninterpretable solutions that could have counter-intuitive properties [37]: (i) the individual units in the learning algorithm does not contain semantic information; and (ii) the stability of neural networks can be affected by small perturbations to their inputs (adversarial examples). Our research differs from all these approaches in that it does not use machine learning, but logic representation and moreover, it can generate explanations of the outliers (i.e., items classified in a wrong category) using qualitative concepts (as individual units with semantic meaning), which are used for the classification as features.

Logical reasoning has been also associated with image interpretation: non-monotonic reasoning has been applied to image description [30]; description logics have been used to interpret digital images by describing each object by its colour and qualitative shape and by its main spatial features (location, relative orientation and topology) which allows to infer new object categories (i.e. doors) by reasoning [11], etc. Fuzzy descriptions logics have been also applied to image interpretation: a fuzzy spatial relation ontology have been developed to deal with brain structures in 3D magnetic resonance images [22]; a fuzzy logic-based colour histogram analysis for discriminating benign skin lesions from malignant melanomas in dermoscopy images has been proposed [3]; fuzzy logics have been also used in landslide identification and classification [2]; a general type-2 fuzzy logic method for edge detection has also been applied to colour format images [18, 31]; a fuzzy description logics-based reasoning framework has been developed which reasons over an extracted description of an outdoor image and it handles the underlying vagueness in a formal way providing well-defined reasoning services [9]. The work presented in this paper differs from all these logic approaches for image interpretation in that it uses qualitative features of colour and do classify images of paintings into art styles, also providing explanations when there are reasons to believe that a painting may fit in two styles. In the literature, some research works tend to follow the *explainable AI* principle too. Recently, research works have appeared which provide reasons for a concept/object to be classified in a category: when categorising leaves [4] or when categorising places, movies and wines [10]. In addition, Sørmo et al. [36] consider different theories of explanation from the philosophy and cognitive science communities. Using these studies, the authors present a framework for explanation in case-based reasoning. Moreover, the SWALE project[1] studies creative explanation of anomalous events. The current paper follows also this *explainable AI* principle.

Qualitative descriptors have been shown to be successful in managing incomplete, imprecise and ambiguous information [6,16] when reasoning. In addition, they use linguistic concepts which align with human perception and can be easily used to generate narratives that explain the reasoning process in order to give feedback to the users. Regarding human-machine interaction, qualitative descriptors [12] and conceptual spaces [28,4] have proven to be successful in providing human understandable narratives of scenes. In the literature, few works have used qualitative colours for image interpretation. Semantic categories (e.g. *warm*, *cold*) and colour names have shown to be effective for painting retrieval in databases [27]. Qualitative colour descriptors have been used to categorise painting styles using machine learning techniques (i.e. KNN and SVMs) and the results obtained an accuracy of 75% for a dataset of 70 paintings [14]. Later, this approach was extended by *QArt-Learn* [15] adding quantitative global features to the qualitative colour descriptors and the accuracy obtained was 65% for 252 paintings. However, as far as the authors are concerned, there are no research works that integrate qualitative descriptors and logics for art style categorisation which also can provide explanations of decisions and outliers. This is the main contribution of this paper, that is, the definition of a ℓogical **S**tyle painting classifier based on **H**orn clauses and **E**xplanations (**ℓ-SHE**). This paper extends the pilot study [8] which formalises distinctive colour traits for the Baroque, the Impressionism and the Post-Impressionism styles, and introduces an evaluated Horn clause based on these colour features as a categorisation of each style. ℓ-SHE has been tested using the above-mentioned fuzzy propositional logics using 90 painting of the QArt-dataset and on a wider and different dataset, the Paintings-91-PIB dataset containing 247 paintings.

The rest of the paper is organised as follows. Section 2 introduces the Qualitative Colour Descriptor (QCD) as preliminaries. Section 3 presents the colour traits that characterise the Baroque, the Impressionism and the Post-Impressionism styles in the literature and it explains how these traits are obtained. Section 4 presents three different logics which can be used to categorise the art styles, rational Pavelka logic and expansions of Gödel logic and product logic with rational constants, and explains how the definitions for each art style are parameterised using the QArt-dataset. Section 5 describes in detail the ℓ-SHE categorisation. Section 6 presents and discusses the results obtained when classifying the 90 images in the QArt-Dataset with the three art style painting classifiers defined: ℓ-SHE$^{\text{RPL}}$, ℓ-SHE$^{G(\mathbb{Q})}$ and ℓ-SHE$^{\sqcap(\mathbb{Q})}$. Section 8 shows and analyses the results obtained when classifying the 247 images in the Paintings-91-PIB dataset with the three classifiers. Finally, in Section 9 conclusions and future work are presented.

[1] http://www.cs.indiana.edu/ leake/projects/swale.

2 Preliminaries

This section introduces the Qualitative Colour Descriptor model (QCD) and shows how the colour frequencies of any digital image are extracted and expressed as facts for reasoning using Prolog Horn clauses. The datasets used in this paper are also introduced here: the QArt-Dataset and the Painting-91-BIP.

The Qualitative Colour Descriptor model (QCD) was defined by Falomir et al. [13]. It extracts the colour coordinates of each pixel of any digital image and it describes it using the Hue Saturation and Ligthness (HSL) colour space.

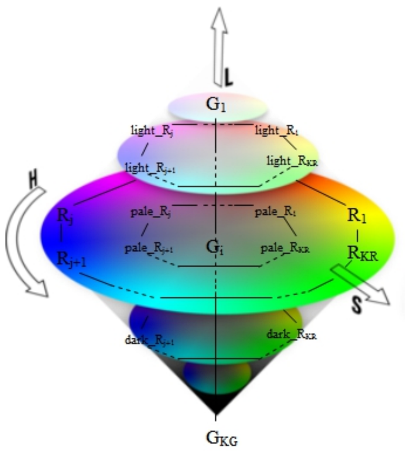

Fig. 1 HSL colour space and QCD discretisation according to the QCRS. The colour version of the figure is available on the online version of this paper.

The HSL colour space is described by 3 coordinates $(uH, uL, uS) \in [0,360] \times [0,100] \times [0,100] \subseteq \mathbb{N}^3$, where $\mathbb{N}$ stands for the set of natural numbers:

$\underline{uH}$: The hue refers to the pure spectrum colours and corresponds to the dominant colour as perceived by a human. The uH takes any value from the interval $[0,360]$, that is, $0 \leq uH \leq 360$.

$\underline{uS}$: The saturation refers to the relative purity or the amount of white light mixed with hue. The uS takes any value from the interval $[0,100]$, i.e., $0 \leq uS \leq 100$.

$\underline{uL}$: The luminance corresponds to the amount of light in a colour. The uL takes any value from the interval $[0,100]$, that is, $0 \leq uL \leq 100$.

For each pixel in a digital image, the Qualitative Colour Descriptor model (QCD) [13] extracts its HSL colour coordinates and it obtains its corresponding colour name according to the following Qualitative Colour Reference System (QCRS) (see Fig. 1), which discretises the Hue, Saturation and Lightness (HSL) colour space as follows:

$$QCRS = \{uH, uS, uL, QC_{NAME1...5}, QC_{INT1...5}\},$$

where uH, uS and uL stands for the previous definitions, and the colour names $QC_{NAME1...5}$ and its corresponding HSL interval values $QC_{INT1...5}$ are shown by Table 1. The QCRS was calibrated using machine learning on data obtained from surveys to people [13,33].

The QCD model considers 37 labels for colour names [13], which are grouped into 5 sets according to their spatial properties in the colour space:

$QC_{NAME_1} = \{$*black, light_grey, grey, dark_grey, white*$\}$,
$QC_{NAME_2} = \{$*red, orange, yellow, green, turquoise, blue, purple, pink*$\}$,
$QC_{NAME_3} = \{$*pale_red, pale_orange, pale_yellow, pale_green, ..., pale_pink*$\}$,
$QC_{NAME_4} = \{$*light_red, light_orange, light_yellow, light_green, ..., light_pink*$\}$, and
$QC_{NAME_5} = \{$*dark_red, dark_orange, dark_yellow, dark_green, ..., dark_pink*$\}$.

Let $QC_{NAME1...5} = \{$*black, red, orange, ..., dark_purple, dark_pink*$\} = \{QC_i \mid 1 \leq i \leq 37\}$ be the set of all the 37 colour names considered by the QCD model, where each QC_i denotes a colour name for $1 \leq i \leq 37$. In order to determine a colour name, QC_i, for the colour displayed by a pixel, the QCD considers the Qualitative Colour Reference System (QCRS) (see Fig. 1), which can also be expressed as the following function:

$$f_{QCRS}(uH, uL, uS)\colon\ [0,360] \times [0,100] \times [0,100] \to QC_{NAME1...5}$$
$$(uH, uL, uS) \in [0,360] \times [0,100] \times [0,100] \mapsto QC_i \in QC_{NAME1...5},$$

defined by Table 1.

In order to obtain a colour description of the images in the datasets, each fine-art painting image (Img) is described by applying computer vision techniques [15] which extract a colour vector histogram: $(f_1(Img), f_2(Img), \ldots, f_{37}(Img)) \in \mathbb{N}^{37}$, where $f_i(Img)$ corresponds to the number of pixels labeled as QC_i in Img. Let $T(Img)$ be number of pixels in Img, we define the frequency of the colour QC_i, $F_i(Img)$, as $f_i(Img)/T(Img)$ for $1 \leq i \leq 37$. Note that for any image Img, $f_i(Img), F_i(Img) \geq 0$ for $1 \leq i \leq 37$.

We transform the colour traits in each painting to Prolog facts with the following sintaxis:

$$colour\_painting(P, QC_i, F_i),$$

where P corresponds to the digital image identifier (provided by the chosen dataset), $QC_i \in QC_{NAME1...5}$, F_i is defined as indicated above and $1 \leq i \leq 37$.

The QArt-Dataset contains 90 images (30 Baroque paintings, 30 Impressionist paintings and 30 Post-Impressionist paintings) and we have used its colour histograms describing each image to obtain the parameters in ℓ-SHE. For each art style, the QArt-Dataset considers two representative authors: Velázquez and Vermeer for the Baroque style, Monet and Renoir for the Impressionism style, and Gauguin and van Gogh for the Post-Impressionism style (see Fig. 2 for some examples). From the Painting-91 dataset introduced in [25], Falomir et al. [15] extracted the paintings of the six authors considered in the QArt-Dataset, 252 in total. This paper considers a total of 247 of images from the Painting-91 dataset: 74 for the Baroque style (39 by Velázquez and 35 by Vermeer), 82 for the Impressionism style (46 by Renoir and 36 by Monet), and 91 for the Post-Impressionism style (40 by Van Gogh and 51 by Gauguin). This paper renames this new dataset as Painting-91-BIP (Fig. 6). Section 8 uses the Painting-91-BIP dataset in order to test ℓ-SHE. In order to name the images in both datasets, the QArt-Dataset and the Painting-91-BIP dataset, we use the notation established in each dataset. Examples of paintings described by the Prolog facts are shown in Fig. 3, 4 and 5.

Table 1 The definition of the function f_{QCRS}.

uH	uL	uS	$f_{QCRS}(uH, uL, uS)$ $QC_{NAME1...5}$
[0, 360]	(0,20]	[0,20]	*black*
	(20,40]		*dark_grey*
	(40,60]		*grey*
	(60,80]		*light_grey*
	(80,100]		*white*
[0, 20] ∪ (335, 360]	(40,55]	(50,100]	*red*
(20, 50]			*orange*
(50, 80]			*yellow*
(80, 160]			*green*
(160, 200]			*turquoise*
(200, 239]			*blue*
(239, 297]			*purple*
(297, 335]			*pink*
[0, 20] ∪ (335, 360]	(40,55]	(20,50)	*pale_red*
(20, 50]			*pale_orange*
(50, 80]			*pale_yellow*
(80, 160]			*pale_green*
(160, 200]			*pale_turquoise*
(200, 239]			*pale_blue*
(239, 297]			*pale_purple*
(297, 335]			*pale_pink*
[0, 20] ∪ (335, 360]	(55,100]	(50,100]	*ligth_red*
(20, 50]			*ligth_orange*
(50, 80]			*ligth_yellow*
(80, 160]			*ligth_green*
(160, 200]			*ligth_turquoise*
(200, 239]			*ligth_blue*
(239, 297]			*ligth_purple*
(297, 335]			*ligth_pink*
[0, 20] ∪ (335, 360]	(20,40]	(50,100)	*dark_red*
(20, 50]			*dark_orange*
(50, 80]			*dark_yellow*
(80, 160]			*dark_green*
(160, 200]			*dark_turquoise*
(200, 239]			*dark_blue*
(239, 297]			*dark_purple*
(297, 335]			*dark_pink*

3 Art Style Representation based on Fuzzy Qualitative Colour Descriptors

In this section we present a representation of the characteristic colour traits of the Baroque, the Impressionism, and the Post-Impressionism styles using fuzzy sets. The fuzzy sets are defined using the frequencies of the colours.

The literature explains that **Baroque paintings** show mainly indoor scenes where lighting is exaggerated by contrasting *dark* colours to *light/pale* colours [32]. Regarding colour features in the **Impressionist style**, the literature [29,26] explains that the development of synthetic pigments provided artists with vibrant shades of *blue* and *green*, among others. Moreover, the Impressionists captured the effects of sunlight by painting *en plein air* (outdoors), and thereby the *blue* of the sky, *light* colours and *grey* shadows are common colour traits in this style. In contrast to the Impressionism style, the **Post-Impressionist style** [20] breaks the tendency of representing colours as appearing in reality [21]. The Post-Impressionists looked for expressiveness using colours arbitrarily [1]. Thus colours with pure hues (i.e., *vivid colours*) are present in the Post-Impressionist paintings.

B

I

PI

Fig. 2 Extract from the QArt-Dataset: Paintings corresponding to the Baroque style (B), Impressionist style (I) and Post-Impressionist style (PI). All rights by Wikimedia commons, public domain. The colour version of this figure is available on the online version of this paper.

```
colour_painting(v10, black, 0.362).
colour_painting(v10, dark_turquoise, 0.056).
colour_painting(v10, dark_green, 0.025).
colour_painting(v10, dark_grey, 0.117).
colour_painting(v10, dark_orange, 0.022).
...
colour_painting(v10, light_green, 0.014).
colour_painting(v10, light_grey, 0.054).
colour_painting(v10, light_orange, 0.010).
...
colour_painting(v10, pale_yellow, 0.0128).
colour_painting(v10, pale_green, 0.046).
colour_painting(v10, turquoise, 0.0004).
colour_painting(v10, white, 0.021).
```

Extracted Prolog facts corresponding to this painting ($v10$, QArt-Dataset).

Fig. 3 Extracted Prolog facts from *Equestrian Portrait of Prince Balthasar Charles* by Velázquez. All rights under © creative commons, public license. The colour version of this figure is available on the online version of this paper.

```
colour_painting(rn3, black, 0.206).
colour_painting(rn3, dark_grey, 0.213).
colour_painting(rn3, dark_orange, 0.034).
...
colour_painting(rn3, green, 0.151).
colour_painting(rn3, light_grey, 0.1245).
colour_painting(rn3, light_orange, 0.033).
...
colour_painting(rn3, pale_orange, 0.068).
colour_painting(rn3, pale_green, 0.046).
colour_painting(rn3, white, 0.034).
```

Extracted Prolog facts corresponding to this painting (rn_3, QArt-Dataset).

Fig. 4 Extracted Prolog facts from *Luncheon of the Boating Party* by Renoir. All rights under © creative commons, public license. The colour version of this figure is available on the online version of this paper.

Hence, with the goal of defining the different colour features for the art styles selected, we extend the QCD model.

Definition 1 The following is an extension of the QCD model:

```
colour_painting(vg12, black, 0.003).
colour_painting(vg12, dark_green,0.001).
colour_painting(vg12, dark_grey, 0.015).
...
colour_painting(vg12, grey, 0.018).
colour_painting(vg12, light_orange, 0.057).
colour_painting(vg12, light_yellow, 0.009).
...
colour_painting(vg12, pale_orange, 0.089).
colour_painting(vg12, pale_green, 0.002).
colour_painting(vg12, white, 0.017).
```

Extracted Prolog facts corresponding to the digital image ($vg12$, QArt-Dataset).

Fig. 5 Extracted Prolog facts from *Sunflowers* by van Gogh. All rights under © creative commons, public license. The colour version of this figure is available on the online version of this paper.

B

I

PI

Fig. 6 Extract from the Painting-91-BIP dataset. All rights under © creative commons, public license. The colour version of this figure is available on the online version of this paper.

$dark\_colours = \{black, dark\_red, dark\_orange, dark\_yellow, dark\_green, dark\_turquoise, dark\_blue, dark\_purple, dark\_pink, dark\_grey\}$,
$pale\_colours = \{pale\_red, pale\_orange, pale\_yellow, pale\_green, pale\_turquoise, pale\_blue, pale\_purple, pale\_pink, grey\}$,
$light\_colours = \{white, light\_red, light\_orange, light\_yellow, light\_green, light\_turquoise, light\_blue, light\_purple, light\_pink\}$,
$grey\_hue = \{grey, pale\_grey, light\_grey, dark\_grey\}$,
$red\_hue = \{red, pale\_red, light\_red, dark\_red\}$,
$orange\_hue = \{orange, pale\_orange, light\_orange, dark\_orange\}$,
$yellow\_hue = \{yellow, pale\_yellow, light\_yellow, dark\_yellow\}$,
$green\_hue = \{green, pale\_green, light\_green, dark\_green\}$,
$turquoise\_hue = \{turquoise, pale\_turquoise, light\_turquoise, dark\_turquoise\}$,
$blue\_hue = \{blue, pale\_blue, light\_blue, dark\_blue\}$,
$purple\_hue = \{purple, pale\_purple, light\_purple, dark\_purple\}$, and
$pink\_hue = \{pink, pale\_pink, light\_pink, dark\_pink\}$.

Vivid colours, warm hues, and cold hues are also defined:

$warm\_hue = \{red\_hue, orange\_hue, yellow\_hue\}$,
$vivid\_colours = \{red, orange, yellow, green, turquoise, blue, purple, pink\}$, and

cold_hue={*green_hue,turquoise_hue,blue_hue,purple_hue,pink_hue*}.

Considering these colour features outlined by the art experts and the extension of the QCD presented above, we propose to use the following distinctive colour traits for the Baroque style:

darkness_level: the accumulative sum of the frequencies of *dark_colours*.
no_paleness_level: the total frequency of colours that are not *pale_colours*.
contrast_level: the total frequency of *dark* and *pale* colours bounded to 1.

Regarding the Impressionism style, four characteristic colour features are proposed:

bluish_level: the total frequency of the QCs extracted as having blue hue (see Definition 1).
greyish_level: the total frequency of the QCs extracted as having grey hue (see Definition 1).
diversityofHues: all the QCs in a painting are grouped according to their hues (see Definition 1) and they are related to the total number of hues in QCD, which is 11 ($|vivid\_colours \cup \{black, white\}| = 11$).
diversityofQCDs: the relation between the amount of qualitative colours (including all their pale-, light-, and dark- variants) in a painting, and the total number of QCs possible (i.e., 37).

Two distinctive colour traits for the Post-Impressionism are suggested:

vividness_level: the total frequency of the QCs extracted as having pure hue (see Definition 1).
warm_colours_level: the total frequency of the QCs extracted as having warm hue (see Definition 1).

4 Art Style Categorisation based on Evaluated Horn Clauses

Since the distinctive colour traits presented in Section 3 can be regarded in a natural way as fuzzy notions, in this section we introduce one evaluated Horn clause for each painting style using the rational Pavelka logic, and other propositional fuzzy languages expanded with truth-constants. These evaluated Horn clauses give a categorisation of the different painting styles. First we recall the syntax and semantics of these formal languages, and then introduce a propositional variable for each main colour trait of the different painting styles. Finally we show how the rational parameters of the evaluated Horn clauses are obtained using the data on qualitative colours and frequencies extracted from the QArt-Dataset.

Definition 2 (Syntax and semantics of continuous t-norm based propositional fuzzy logics [5, Chapter I, Definition 1.1.13]) The language of continuous t-norm based propositional fuzzy logics contains a set of propositional variables *Var*, the binary connectives in the set $\{\rightarrow, \&, \wedge, \vee, \leftrightarrow\}$, the unary connective $\neg$, and the truth-constants $\overline{0}, \overline{1}$. Let $[0,1] \subseteq \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers, a $[0,1]$-*evaluation* e is a mapping $e : Var \rightarrow [0,1]$. Let $*$ be a continuous t-norm, an evaluation e extends uniquely to an evaluation e^* of the set of well-formed formulas as usual.

For the sake of simplicity, no distinction between e and e^* is made and the notation is simplified to e in both cases. For each rational number $r \in [0,1]$, we consider the truth-constant $\overline{r}$ so that $e^*(\overline{r}) = r$. In order to categorise the three art styles considered, we use rational Pavelka logic (RPL for short), and expansions with rational constants of Gödel logic ($G(\mathbb{Q})$ for short) and product logic ($\sqcap(\mathbb{Q})$ for short). Let φ, ψ be two formulas, we recall the interpretation of & and $\rightarrow$ in RPL:

$e(\varphi \& \psi) = max\{0, e(\varphi) + e(\psi) - 1\}$, and
$e(\varphi \rightarrow \psi) = min\{1 - e^*(\varphi) + e^*(\psi), 1\}$.

Regarding $G(\mathbb{Q})$, let us remember that:

$e(\varphi \& \psi) = min\{e(\varphi), e(\psi)\}$, and
$$e(\varphi \rightarrow \psi) = \begin{cases} 1 & \text{if } \varphi \leq \psi \\ \psi, & \text{otherwise.} \end{cases}$$

Finally, with respect to $\sqcap(\mathbb{Q})$, let us recall that:

$e(\varphi \& \psi) = e(\varphi)e(\psi)$, and
$$e(\varphi \rightarrow \psi) = \begin{cases} 1 & \text{if } \varphi \leq \psi \\ \frac{\psi}{\varphi}, & \text{otherwise.} \end{cases}$$

Next definition is a generalisation of the definition of RPL$\forall$-Horn clause introduced in [7].

Definition 3 (Evaluated Horn clause [7, Definition 10]) An *atomic evaluated formula* (φ, r) is defined as $\overline{r} \rightarrow \varphi$, where $r \in [0,1]$ is a rational number and φ is an atomic formula without truth constants apart from $\overline{0}$ and $\overline{1}$. An *evaluated Horn clause* has the form

$$(\varphi_1, r_1) \& \ldots \& (\varphi_n, r_n) \rightarrow (\varphi, s),$$

where $(\varphi_1, r_1), \ldots, (\varphi_n, r_n)$ and (φ, s) are atomic evaluated formulas.

For the sake of clarity, evaluated Horn clauses are simply named *Horn clauses*. Consider the following propositional variables referring to the colour features defined in Section 3: $darkness\_level, no\_paleness\_level, contrast\_level, bluish\_level, greyish\_level,$ $diversityofHues, diversityofQCDs, vividness\_level, warm\_colours\_level$; and consider also the following propositional variables referring to the styles Baroque, Impressionism and Post-Impressionism, respectively: $baroque, impressionism, post\_impressionism$. We propose the following Horn clauses to categorise the different art styles selected. H_B represents the Baroque style:

$$(darkness\_level, 0.76) \& (no\_paleness\_level, 0.84) \& \\ (contrast\_level, 0.90) \rightarrow (baroque, 1).$$

H_I represents the Impressionist style:

$$(diversityofQCDs, 0.60) \& (diversityofHues, 0.75) \& (bluish\_level, 0.05) \\ \& (greyish\_level, 0.44) \rightarrow (impressionism, 1).$$

And H_{PI} represents the Post-Impressionist style:

$$(vividness\_level, 0.14) \& (warm\_colours\_level, 0.53) \rightarrow (post\_impressionism, 1).$$

Observe that the semantics of the three logics selected in this paper are different, and thus the interpretation of the Horn clauses depends on the logic used. Since their systematization by Hájek [19], these three logics have shown to be some of the most significant and well-known t-norm based logics.

For any digital painting p, it can be associated an evaluation e_p of the variables in the antecedent of the Horn clauses H_B, H_I, H_{PI}. For instance, for painting $v10$ (see Fig. 3), in RPL we obtain that: $e_{v10}(contrast\_level, 0.90) = min\{1 - 0.9 + e_{v10}(contrast\_level), 1\} = min\{1 - 0.9 + 0.87, 1\} = 0.97$. Given a painting p, the antecedent of clause H_B is evaluated using e_p in order to obtain a membership degree for the Baroque style. For the sake of clarity, let us introduce some notation: $B_1(p) = e_p(darkness\_level, 0.76)$ and:

$$
\begin{array}{cc}
B_2(p) = e_p(no\_paleness\_level, 0.84) & I_1(p) = e_p(diversityofQCDs, 0.60) \\
B_3(p) = e_p(contrast\_level, 0.90) & I_2(p) = e_p(diversityofHues, 0.75) \\
PI_1(p) = e_p(vividness\_level, 0.14) & I_3(p) = e_p(bluish\_level, 0.05) \\
PI_2(p) = e_p(warm\_colours\_level, 0.53) & I_4(p) = e_p(greyish\_level, 0.44).
\end{array}
$$

According to the semantics of each logic selected, the membership degrees to the Baroque, the Impressionism and the Post-Impressionism styles are next calculated. For the sake of clarity, throughout this section we focus the presentation on the ℓ-SHE$^{\mathrm{RPL}}$ version. For ℓ-SHE$^{\mathrm{RPL}}$:

$$B(p) = max\{0, B_1(p) + B_2(p) + B_3(p) - 2\}$$

(i.e., $B(p) = e_p((darkness\_level, 0.76)\&(no\_paleness\_level, 0.84)\&(contrast\_level, 0.9))$),

$I(p) = max\{0, I_1(p) + I_2(p) + I_3(p) + I_4(p) - 3\}$, and $PI = max\{0, PI_1(p) + PI_2(p) - 1\}$.

Let us now consider the painting $v10$ in the QArt-Dataset, we take ℓ-SHE$^{\mathrm{RPL}}$ and we show how the membership degree of $v10$ to each art style is obtained. First, the levels of the characteristic colour traits of the Baroque style are obtained:

$B_1(v10) = e_{v10}(darkness\_level, 0.76) = min\{1 - 0.76 + e_{v10}(darkness\_level), 1\} = min\{0.24 + 0.67, 1\} = 0.91$,
$B_2(v10) = e_{v10}(no\_paleness\_level, 0.84) = min\{1 - 0.84 + e_{v10}(no\_paleness\_level), 1\} = min\{0.16 + 0.80, 1\} = 0.96$, and
$B_3(v10) = e_{v10}(contrast\_level, 0.90) = min\{1 - 0.9 + e_{v10}(contrast\_level), 1\} = min\{1 - 0.9 + 0.87, 1\} = 0.97$.

The membership degree to the Baroque style given to $v10$ is $B(v10) = max\{0, B_1(p) + B_2(p) + B_3(p) - 2\} = \{0.91 + 0.96 + 0.97 - 2, 0\} = 0.84$. The other membership degrees are obtained similarly: $I(v10) = 0.78$, and $PI(v10) = 0.53$.

We explain the procedure for the colour trait *darkness_level* of clause H_B. The median and the standard deviation for *darkness_level* are denoted by $\overline{x_{dly}}$ and σ_{dly}, respectively, where dl denotes *darkness_level* and y is substituted by B, I, PI, depending on the art style selected. For instance, $\overline{x_{dlB}}$ denotes the median of the *darkness_level* of the 30 Baroque paintings in the QArt-Dataset. We also define the interval $R_{dly} = [\overline{x_{dly}} - \sigma_{dly}, \overline{x_{dly}} + \sigma_{dly}]$. For each art style y, $\overline{x_{dly}}, \sigma_{dly}, R_{dly}$ are computed from the colour frequencies using the R platform [38] (observe that for any digital painting, each colour trait yields a degree in $[0, 1]$):

darkness_level (dl)	$\overline{x_{dly}}$	σ_{dly}	$R_{dly} = [\overline{x_{dly}} - \sigma_{dly}, \overline{x_{dly}} + \sigma_{dly}]$
Baroque (B)	0.76	0.15	$[0.61, 0.91]$
Impressionism (I)	0.42	0.20	$[0.22, 0.62]$
Post-Impressionism (PI)	0.33	0.19	$[0.14, 0.52]$

Let $a,b,c,d \in [0,1]$, where $b \geq a$ and $d \geq c$, and $R_1 = [a,b], R_2 = [c,d]$ be two intervals, the intersection of R_1 and R_2 is defined as:

$$R_1 \cap R_2 = \begin{cases} \emptyset & \text{if } c > b \text{ or } a > d \\ [max\{a,c\}, min\{b,d\}] & \text{otherwise.} \end{cases}$$

Let $R = [a,b]$ be an interval, the length of R, denoted by $|R|$, is defined as $|R| = b - a$; and by convention the length of the empty set is 0.

Note that $|R_{dlB} \cap R_{dlI}| = 0.009$, that is, the darkness level interval corresponding to the Baroque and the Impressionism styles presents an intersection which corresponds to 2.98% of the length of R_{dlB} and 2.22% of the length of R_{dlI}. This shows that these styles have very few in common regarding the *darkness_level* feature. Moreover, $|R_{dlB} \cap R_{dlPI}| = 0$, but $|R_{dlI} \cap R_{dlPI}| = 0.303$. Thus the *darkness_level* feature is very similar in both styles, the Impressionism and the Post-Impressionism styles. These results suggest that using the level of darkness to categorise Baroque paintings is reasonable, considering the large difference between $\overline{x_{dlB}}$ and both $\overline{x_{dlI}}$ and $\overline{x_{dlPI}}$. They also show that darkness is not a useful colour feature for separating the Impressionism and the Post-Impressionism styles. Furthermore, $\overline{x_{dlB}}$ is much larger than $\overline{x_{dlI}}$ and $\overline{x_{dlPI}}$, and σ_{dlI} and σ_{dlPI} represent around the half part of $\overline{X_{dlI}}$ and $\overline{x_{dlPI}}$, respectively. Considering these, it has been deemed advisable to consider 0.76 as the parameter for *darkness_level*. This simple method avoids hard computation such as other training methods used in machine learning.

5 The ℓ-SHE Categorisation

The aim of this section is to describe the ℓ-SHE algorithm which is intended to generate human-understandable explanations based on colour traits according to the categorisation obtained. We recall that ℓ-SHE has been defined for RPL, $G(\mathbb{Q})$, $\sqcap(\mathbb{Q})$: ℓ-SHE$^{\text{RPL}}$, ℓ-SHE$^{G(\mathbb{Q})}$ and ℓ-SHE$^{\sqcap(\mathbb{Q})}$, respectively. Note that ℓ-SHE categorisations are not crisp, that is, a membership degree for each art style –Baroque, Impressionism and Post-Impressionism– is provided, as detailed in Section 4.

From now on, let p denote any digital painting. We define the *belief degree* for p to belong to an art style as:

$$dbAS(p) = \begin{cases} (B_{st}, B(p)) & \text{if } max\{B(p), I(p), PI(p)\} = B(p) \text{ and } B(p) \neq I(p) \\ (I_{st}, I(p)) & \text{if } max\{B(p), I(p), PI(p)\} = I(p) \\ (PI_{st}, PI(p)) & \text{if } max\{B(p), I(p), PI(p)\} = PI(p) \text{ and } B(p) \neq PI(p) \neq I(p). \end{cases}$$

Note that in the event of a tied membership degree, *dbAS* chooses the most restrictive art style. Since ℓ-SHE has to give a second option in difficult cases, a similarity between membership degrees, *Sim*, is defined:
$Sim_{B,I}(p) = |B(p) - I(p)|$, $Sim_{B,PI}(p) = |B(p) - PI(p)|$, and $Sim_{I,PI}(p) = |I(p) - PI(p)|$, where $Sim_{B,I}(p)$ stands for the closeness between the Baroque and the Impressionism membership degrees of p, and $Sim_{B,PI}(p)$ and $Sim_{I,PI}(p)$ are described analogously. From data analysis obtained in the ℓ-SHE classification of the QArt-Dataset, we considered different values for determining doubt between art styles: $0.10, 0.15, 0.20$ and 0.25. Finally, it was found by experimentation that 0.15 is the best option for this parameter.

For the sake of clarity, throughout this section we introduce only the ℓ-SHE$^{\mathrm{RPL}}$ version. The rest of the ℓ-SHE versions are defined analogously. The ℓ-SHE$^{\mathrm{RPL}}$ algorithm categorises paintings in the three following styles:

(1) If $dbAS = (B_{st}, B(p))$, then "p is a Baroque painting." & $explanation_{\mathrm{RPL}}(B, p)$.
  - If $Sim_{B(p),I(p)} \leq 0.15$, then "Although p is categorised in the Baroque style, there are reasons to believe that it may belong to the Impressionism." & $explanation_{\mathrm{RPL}}(I, p)$.
  - If $Sim_{B(p),PI(p)} \leq 0.15$, then "Although p is categorised in the Baroque style, there are reasons to believe that it may belong to the Post-Impressionist." & $explanation_{\mathrm{RPL}}(PI, p)$.

(2) If $dbAS = (I_{st}, I(p))$, then "p is an Impressionist painting (I)." & $explanation_{\mathrm{RPL}}(I, p)$.
  - If $Sim_{B(p),I(p)} \leq 0.15$, then "Although p is categorised in the Impressionist style, there are reasons to believe that it may belong to the Baroque." & $explanation_{\mathrm{RPL}}(B, p)$.
  - If $Sim_{I(p),PI(p)} \leq 0.15$, then "Although p is categorised in the Impressionist style, there are reasons to believe that it may belong to the Post-Impressionism." & $explanation_{\mathrm{RPL}}(PI, p)$.

(3) If $dbAS = (PI_{st}, PI(p))$, then "p is a Post-Impressionist painting (PI)." & $explanation_{\mathrm{RPL}}(PI, p)$.
  - If $Sim_{B(p),PI(p)} \leq 0.15$, then "Although p is categorised in the Post-Impressionist style, there are reasons to believe that it may belong to the Baroque." & $explanation_{\mathrm{RPL}}(B, p)$.
  - If $Sim_{I(p),PI(p)} \leq 0.15$, then "Although p is categorised in the Post-Impressionist style, there are reasons to believe that it may belong to the Impressionism." & $explanation_{\mathrm{RPL}}(I, p)$.

In addition, explanations for specific characteristics in each art style can also be provided, as explained below. Let us consider the feature *darkness_level* as significant for classifying a painting p into the Baroque style, whenever $B1(p)$ is higher than a threshold T_{B1}. If this is the case, the presence of this feature must appear as an explanation/evidence for p classified into this style. This threshold T_{B1} is calculated as $\overline{x_{B_1}} - \sigma_{B_1}$, where $\overline{x_{B_1}}$ is the mean of $\{B_1(p) \mid p \text{ is a Baroque painting}\}$ and σ_{B_1} is the corresponding standard deviation. Notice that the thresholds are obtained from the truth-value of an implication, and this truth-value depends on the t-norm used. Hence each threshold depends on the selected logic. Thus a superscript indicating the logic used is provided. For instance, the three cases regarding the threshold B_1 are noted as $T_{B1}^{\mathrm{RPL}}, T_{B1}^{G(\mathbb{Q})}, T_{B1}^{\sqcap(\mathbb{Q})}$. Table 5 shows the value of each threshold for the three logics considered.

For the Baroque style, $explanation_{\mathrm{RPL}}(B, p)$ provided by ℓ-SHE are the following:

If $B_1 \geq T_{B1}^{\mathrm{RPL}}$, then "The darkness evidences the Baroque style."
If $B_2 \geq T_{B2}^{\mathrm{RPL}}$, then "Due to the contrast of *dark* and *pale* colours."
If $B_3 \geq T_{B3}^{\mathrm{RPL}}$, then "The lack of *pale* colours evidences this style."

The ℓ-SHE also provides the following explanations, $explanation_{\mathrm{RPL}}(I, p)$, for the Impressionist style:

If $I_1 \geq T_{I1}^{\mathrm{RPL}}$, then "The diversity of qualitative colours evidences the Impressionism style."
If $I_2 \geq T_{I2}^{\mathrm{RPL}}$, then "The variety of hues evidences the Impressionism style."
If $I_3 \geq T_{I3}^{\mathrm{RPL}}$, then "The amount of bluish evidences this style."

Table 2 Thresholds used by the explanations corresponding to each colour trait and logic.

Logic/Thresholds	RPL	$G(\mathbb{Q})$	$\sqcap(\mathbb{Q})$
T_{B1}	0.82	0.94	0.90
T_{B2}	0.90	0.98	0.97
T_{B3}	0.78	0.98	0.95
T_{I1}	0.87	0.89	0.90
T_{I2}	0.89	0.95	0.94
T_{I3}	0.97	1.00	0.87
T_{I4}	0.84	0.92	0.80
T_{PI1}	0.96	0.89	0.47
T_{PI2}	0.76	0.89	0.75

If $I_4 \geq T_{I4}^{\mathrm{RPL}}$, then "The amount of grey colour evidences this style."

The explanations by ℓ-SHE for the Post-Impressionist style, $explanation_{\mathrm{RPL}}(PI, p)$, are the next:

If $PI_1 \geq T_{PI1}^{\mathrm{RPL}}$, then "The presence of vivid colours evidences the Post-Impressionism style."
If $PI_2 \geq T_{PI2}^{\mathrm{RPL}}$, then "The high level of warm colours evidences the Post-Impressionism style."

6 Implementing ℓ-SHE

This section shows the implementation of the ℓ-SHE algorithm and provides some examples of responses produced by ℓ-SHE.

ℓ-SHE has been implemented in Prolog using Swi-Prolog [39] as the testing platform, whereas some thresholds have been obtained using the platform R [38], as indicated previously.

Some of the clauses implemented in Prolog for evaluating RPL formulae are shown next:

```
evaluate_formulaRPL(Formula,Rational,DoB):-
 Formula >= Rational,
 DoB is 1.

evaluate_formulaRPL(Formula,Rational,DoB):-
  Formula < Rational,
  DoB is 1-Rational+Formula.
```

The Prolog implementation clauses for obtaining the degree of believing of a painting P to be classified into the Baroque style is the following:

```
baroque_style(P,DoBa, DoBb, DoBc, DoB):-
  darkness_level(P,_,DLevel, _, _, _, _),
  darknessTh(DT), evaluate_formulaRPL(DLevel,DT,DoBa),
  contrast_level(P,ContrLevel), contrastTh(CT),
  evaluate_formulaRPL(ContrLevel,CT,DoBb),
  no_paleness_level(P,NoPaleness), noPalenessTh(NPT),
  evaluate_formulaRPL(NoPaleness,NPT,DoBc),
  DoBaux is DoBa+DoBb+DoBc-2, DoB is max(0,DoBaux).
```

For the sake of simplicity the exposition is focused on the ℓ-SHE$^{\mathrm{RPL}}$ version (the versions ℓ-SHE$^{G(\mathbb{Q})}$ and ℓ-SHE$^{\sqcap(\mathbb{Q})}$ are implemented analogously).

The clauses used for obtaining the *darkness_level* of a painting are shown next. The Qualitative Colour Descriptors (QCDs, Section 2 and 4) are highlighted in blue:

```
darkness_level(P,DarkQCD,Total,N1,N2,R,Q):-
  find_fuzzy_colours_in_painting(P,CL,QL,N1,Q),
  get_dark_colours(CL,QL,DarkQCD,Total),
  length(DarkQCD,N2),R is (N2/N1)*100.

find_fuzzy_colours_in_painting(P,CL,QL,N,Q):-
  findall(Colour,colour_painting(P,Colour,_),CL),
  findall(Q, colour_painting(P,_,Q),QL),
  length(CL,N), sum_list(QL,Q).
```

```
get_dark_colours([],[],[],0).
get_dark_colours([C|CL],[_|QL],DL,Total):-
  not(dark(Q)),
  get_dark_colours(CL,QL,DL,T), Total is T+0.

get_dark_colours([C|CL],[Q|QL],[[C,Q]|DL],Total):-
  dark(Q),
  get_dark_colours(CL,QL,DL,T), Total is T+Q.
```

The examples highlighted in Section 2 are $v10, rn3$ and $vg12$. Our purpose was to choose 3 possibilities of correct classifications: (i) clear case with membership degree 1 (Figure 5); (ii) a clear case with membership degree different from 1 (Figure 4), and (iii) not clear case, so a second opinion is needed (Figure 3). Thus Figure 5 shows a painting ($vg12$) classified in the Post-Impressionism style with membership degree 1. Figure 4 ($rn3$ painting) shows that ℓ-SHE correctly classifies a painting when the membership degree to Impressionism is 0.891 (not 1). As we will see, here it does not give a second opinion, because it is a clear case since the other membership degrees are 0.475 to Baroque and 0.604 to Post-Impressionism. Figure 3 shows a case where $v10$ painting has a membership degree of 0.875 to Baroque Style. As we will see, here ℓ-SHE provides a second opinion because the membership degree to Impressionism Style is 0.78, close to 0.875.

So an example of the response produced by ℓ-SHE$^{\text{RPL}}$ regarding painting $v10$ is the following:

```
?- baroque_style(v10, DoBa, DoBb, DoBc, DoB).
SHE obtains ...
Darkness level:0.667
Contrast level:0.868
noPaleness level:0.799
DoB = 0.875.
```

And the categorisation reasons provided by ℓ-SHE$^{\text{RPL}}$ for the painting $v10$ (shown in Fig. 3) are:

```
v10 is a Baroque painting. The darkness evidences the Baroque style.
Due to the contrast of dark and pale colours. The lack of pale colours
evidences this style. Although v10 is classified in the Baroque style, there are reasons
to believe that it may belong to the Impressionism. The diversity of qualitative colours
evidences the Impressionism style. The variety of hues evidences the Impressionism style.
The amount of bluish evidences this style.
```

An example of the response produced by ℓ-SHE$^{\text{RPL}}$ regarding painting $rn3$ (Fig. 4) is the following:

```
?- impr_style(rn3,DoB).
SHE obtains ...
Diversity of Hues:0.727
Diversity of QCDs:0.514
Bluish Level:0.105
Greyish Level:0.488
DoB = 0.891.
```

And the explanations provided by ℓ-SHE$^{\text{RPL}}$ for the painting $rn3$ (shown in Fig.4) are:

```
rn3 is an Impressionist painting.
The diversity of qualitative colours evidences the Impressionism style.
The variety of hues evidences the Impressionism style.
The amount of bluish evidences this style.
The amount of grey colour evidences this style.
```

Finally, an example of the response produced by ℓ-SHE$^{\mathrm{RPL}}$ regarding painting $vg12$ (Fig. 5) is the following:

```
?- postimpr_style_g(vg12,DoB).
Vividness level:0.389
Warm Level:0.658
DoB = 1.
```

And the explanations provided for $vg12$ by ℓ-SHE$^{\mathrm{RPL}}$ are:

```
vg12 is a Post-Impressionist painting.
The presence of vivid colours evidences the Post-Impressionism style.
The high level of warm colours evidences the Post-Impressionism style.
```

7 Testing the ℓ-SHE on the QArt-Dataset

This section presents and discusses the results obtained when classifying the 90 images in the QArt-Dataset using the three art style painting classifiers defined: ℓ-SHE$^{\mathrm{RPL}}$, ℓ-SHE$^{G(\mathbb{Q})}$, and ℓ-SHE$^{\sqcap(\mathbb{Q})}$.

The ℓ-SHE$^{\mathrm{RPL}}$ version has been tested using the paintings in the QArt-Dataset. Table 3 shows the confusion matrix obtained for the three art styles. The blue cells correspond to the correct classifications: on the left, correct classifications where ℓ-SHE$^{\mathrm{RPL}}$ is sure (✓); on the right, correct classifications where ℓ-SHE$^{\mathrm{RPL}}$ is not sure and provides an alternative style as a second opinion (**?**). The rest of the cells correspond to the outliers: in each column, the cell on the left indicates the outliers in that ℓ-SHE$^{\mathrm{RPL}}$ does not give a second opinion (✓); and the cell on the right shows the outliers in that the second opinion given by ℓ-SHE$^{\mathrm{RPL}}$ classifies correctly the painting (**?**). The rest of the confusion matrices of this paper use the same notation. In order to clarify, let us indicate that, when the algorithm is doubting, it provides two possible styles as a result. The first option (highest certainty) is the one considered as a correct classification (column **?**). If the second opinion (lowest certainty) is the correct one, it is not counted as a correct classification and it appears in a column corresponding to a different style.

Table 3 Confusion matrix for ℓ-SHE$^{\mathrm{RPL}}$ using the QArt-Dataset.

	Baroque		Impressionism		Post-Impressionism	
	✓	**?**	✓	**?**	✓	**?**
Baroque	24	3	1	0	2	0
Impressionism	3	0	16	5	2	4
Post-Impressionism	1	1	4	6	15	3

From the analysis of the data in Table 3 regarding ℓ-SHE$^{\mathrm{RPL}}$, we can conclude that:

– the highest accuracy is obtained for the Baroque style (90%), and that almost 67% of the outliers (i.e., paintings classified outside its art style) are classified in the Post-

Impressionist style. From the data analysis, we have obtained that the membership degree to the Baroque style of 55.6% of these correct classifications is 1.

- The Impressionism style obtains an accuracy of 70%. In 22.2% of the misclassifications obtained (that is, the outliers), ℓ-SHE$^{\text{RPL}}$ warns that there is evidence to belong to the Impressionist style. In addition, almost 67% of the outliers are classified in the Post-Impressionist style. An example of an outlier in the Impressionist style is *Garden Scene in Brittany* ($rn14$ in the QArt-Dataset, Fig. 8) by Renoir, where $PI(rn14) = 0.877$, while $I(rn14) = 0.875$. In fact ℓ-SHE$^{\text{RPL}}$ points out to the diversity of colours, the variety of hues, the high level of bluish and the use of greys as strong reasons to believe that $rn14$ is an Impressionist painting, although ℓ-SHE$^{\text{RPL}}$ categorises the painting as Post-Impressionist.
- The Post-Impressionism style gets 60% of accuracy rate. In 41.7% of the outliers, ℓ-SHE$^{\text{RPL}}$ warns that there is evidence to believe that a painting belongs to the Post-Impressionist style. Again, separating the Impressionism and Post-Impressionism features becomes difficult, since 83.3% of outliers are categorised as Impressionist paintings. For instance, *Les Alyscamps* ($gg2$ in the QArt-Dataset, Fig. 8) by Gauguin is classified as an Impressionist painting with a membership of $I(gg2) = 0.90$, whereas $PI(gg2) = 0.84$. This misclassification is due to ℓ-SHE$^{\text{RPL}}$ recognises in $gg2$ the totality of the colour features that have been considered as distinctive of the Impressionism style.

rn14, outlier for ℓ-SHE$^{\text{RPL}}$ gg2, outlier for ℓ-SHE$^{\text{RPL}}$ rn2, outlier for ℓ-SHE$^{G(\mathbb{Q})}$ gg6, outlier for ℓ-SHE$^{G(\mathbb{Q})}$ m11, outlier for ℓ-SHE$^{\sqcap(\mathbb{Q})}$

Fig. 7 Examples of outliers or paintings misclassified from the QArt-Dataset. All rights under © creative commons, public license. The colour version of this figure is available on the online version of this paper.

Table 4 shows the confusion matrix corresponding to the art style classification obtained by ℓ-SHE$^{G(\mathbb{Q})}$ regarding the 90 paintings in the QArt-Dataset.

Table 4 Confusion matrix for ℓ-SHE$^{G(\mathbb{Q})}$ using the QArt-Dataset.

	Baroque		Impressionism		Post-Impressionism	
	✓	?	✓	?	✓	?
Baroque	29	1	0	0	0	0
Impressionism	9	3	10	7	1	0
Post-Impressionism	10	2	6	0	12	0

From the analysis of the data in Table 4 regarding ℓ-SHE$^{G(\mathbb{Q})}$, we have obtained the following results.

- Again, the highest accuracy is obtained for the Baroque style (100%). Notice that ℓ-SHE$^{G(\mathbb{Q})}$ provided another possible style for 1 painting although with less certainty, this is why it was not classified into the other categories, but it is not a piece which had a clear diagnostic. Observe also that the membership degree to the Baroque style of 63.3% of these correct classifications is 1.

– The Impressionism style gets 56.7% of accuracy rate. In addition, in 23.1% of the obtained misclassifications, ℓ-SHE$^{G(\mathbb{Q})}$ warns that there is evidence to belong to the Impressionist style. With respect to outliers, 92.3% are classified as Baroque paintings, whereas the rest, 7.7%, are classified as Post-Impressionist paintings. An example of an outlier in this style is *Bal du moulin de la Galette* ($rn2$ in the QArt-Dataset, Fig. 8) by Renoir, where $B(rn2) = 0.68$ and $I(rn2) = 0.51$.
– The Post-Impressionism style gets an accuracy of 40%, and the membership degree to the Post-Impressionist style of 83.3% of these correct classifications is 1. In 11.1% of the misclassifications ℓ-SHE$^{G(\mathbb{Q})}$ warns that there is evidence to believe that a painting belongs to the Post-Impressionist style. Most of the outliers, 66.7%, are classified in Baroque style. An example of an outlier in this style is *Madame Roulin* ($gg6$ in the QArt-Dataset, Fig. 8) by Gauguin, for which $B(gg6) = 0.40$ and $PI(gg6) = 0.13$.

Table 5 shows the confusion matrix obtained by ℓ-SHE$^{\sqcap(\mathbb{Q})}$ for all the art styles in the QArt-Dataset.

Table 5 Confusion matrix for ℓ-SHE$^{\sqcap(\mathbb{Q})}$ using the QArt-Dataset.

	Baroque		Impressionism		Post-Impressionism	
	✓	?	✓	?	✓	?
Baroque	28	0	0	1	1	0
Impressionism	5	5	18	0	2	0
Post-Impressionism	3	1	9	1	15	1

From the analysis of the data in Table 5 regarding ℓ-SHE$^{\sqcap(\mathbb{Q})}$, we have obtained the following results.

– The highest accuracy is obtained for the Baroque style (93.3%), and that the membership degree to the Baroque style of 64.3% of these correct classifications is 1. Moreover, outliers are classified equally in the Impressionist and Post-Impressionist styles.
– The Impressionism style gets 60% of accuracy rate. In addition, in 33.3% of the obtained misclassifications, ℓ-SHE$^{\sqcap(\mathbb{Q})}$ warns that there is evidence to belong to the Impressionist style. Regarding outliers, 86.3% are categorised as Baroque paintings. An example of an outlier in the Impressionist style is *The Waterlily Pond, green harmony* ($m11$ in the QArt-Dataset, Fig. 8) by Monet, where $B(m11) = 1$ and $I(m11) = 0.87$.
– Regarding the Post-Impressionism style, the accuracy obtained is 53.3%. In none of the misclassifications ℓ-SHE$^{\sqcap(\mathbb{Q})}$ warns that there is evidence to believe that a painting belongs to the Post-Impressionist style. Most of the outliers, 71.4%, are classified in Impressionist style. An example of an outlier in this style is *The three graces on the temple of Venus* ($gg2$ in the QArt-Dataset and shown in Fig. 8) by Gauguin, where $I(gg2) = 0.77$ and $PI(gg2) = 0.65$.

Note that the total accuracies obtained with each ℓ-SHE version for the QArt-Dataset (Table 6) do not show a large difference, but ℓ-SHE$^{\mathrm{RPL}}$ is the proposal with highest general accuracy, 73.3%. In addition, although the lowest accuracy for the Baroque style is obtained by ℓ-SHE$^{\mathrm{RPL}}$, let us remark that the highest accuracies for the Impressionist and the Post-Impressionist styles are also obtained by ℓ-SHE$^{\mathrm{RPL}}$. In summary, ℓ-SHE$^{\mathrm{RPL}}$ shows the highest general accuracy and the highest accuracies for two of the three art styles considered. Therefore, we conclude that the ℓ-SHE$^{\mathrm{RPL}}$ is the best proposal for the QArt-Dataset.

Table 6 Percentages of accuracy obtained in the QArt-Dataset for each ℓ-SHE version.

ℓ-SHE version / Art style	ℓ-SHE$^{\mathrm{RPL}}$	ℓ-SHE$^{G(\mathbb{Q})}$	ℓ-SHE$^{\sqcap(\mathbb{Q})}$
Baroque	90	100.0	93.3
Impressionism	70	56.7	60
Post-Impressionism	60	40	53.3
General accuracy	73.3	65.6	68.9

8 Evaluating ℓ-SHE using a different dataset: Paintings-91-PIB

This section presents the performance of the three versions of ℓ-SHE in a larger dataset, Paintings-91-PIB, which contains 247 paintings 74 for the Baroque style (39 by Velázquez and 35 by Vermeer), 82 for the Impressionism style (46 by Renoir and 36 by Monet), and 91 for the Post-Impressionism style (40 by Van Gogh and 51 by Gauguin). See Section 2 for more details. Let us recall that the Painting-91-BIP dataset is slightly unbalanced by author and also with respect to the number of paintings belonging to each style. For this reason, the general accuracy obtained for each ℓ-SHE version has been calibrated: the general accuracy has been obtained as the median of the accuracies of each art style, and not as the quotient between the total of correct classifications and the total of outliers. In this way, the adequacy of ℓ-SHE using RPL, $G(\mathbb{Q})$ and $\sqcap(\mathbb{Q})$ is evaluated again. Notice that the same dataset, QArt-Dataset, was used both to parametrise and test the ℓ-SHE algorithm. Consequently, it was important to test ℓ-SHE with another dataset.

Let us start the analysis with ℓ-SHE$^{\mathrm{RPL}}$. Table 7 shows the confusion matrix obtained with ℓ-SHE$^{\mathrm{RPL}}$ for all the art styles in the Painting-91-BIP dataset.

Table 7 Confusion matrix for ℓ-SHE$^{\mathrm{RPL}}$ using the Painting-91-BIP dataset.

	Baroque		Impressionism		Post-Impressionism	
	✓	?	✓	?	✓	?
Baroque	61	9	4	1	6	2
Impressionism	22	2	33	9	27	6
Post-Impressionism	20	10	22	6	49	10

From the analysis of the data in Table 7 regarding ℓ-SHE$^{\mathrm{RPL}}$ in the Painting-91-BIP dataset, we have obtained the following results.

– The Baroque style gets 86.5% of accuracy rate, and the membership degree to this style of 76.6% of these correct classifications is 1. In 30% of the misclassifications ℓ-SHE$^{\mathrm{RPL}}$ warns that there is evidence to believe that a painting belongs to the Baroque.

With respect to the outliers, 40% are classified as Impressionist paintings and 60% are classified as Post-Impressionist paintings. An example of an outlier in this style is *The Music Lesson* (*jan_vermeer*_12 in the Painting-91-BIP dataset) by Vermeer, for which $B(jan\_vermeer\_12) = 0.74$, $I(jan\_vermeer\_12) = 0.12$ and $PI(jan\_vermeer\_12) = 0.87$.

- The Impressionism obtains an accuracy of only 40.2%. With respect to the outliers, 44.9% are classified as Baroque paintings, whereas the rest, 55.1%, are classified as Post-Impressionist paintings. An example of an outlier in this style is *Woman in a boat* (*pierre_auguste_renoir*_10 in the Painting-91-BIP dataset) by Renoir, for which $PI(pierre\_auguste\_renoir\_10) = 0.87$, whereas $I(pierre\_auguste\_renoir\_10) = 0.72$ and $B(pierre\_auguste\_renoir\_10) = 0$.
- The Post-Impressionism style gets 53.9% of accuracy rate, and the membership degree to this style of 44.9% of these correct classifications is 1. With respect to outliers, 47.6% are classified as Baroque paintings and 52.4% are classified as Impressionist paintings. In addition, in 38.1% of the obtained misclassifications ℓ-SHE$^{\mathrm{RPL}}$ warns that there is evidence to belong to the Post-Impressionist style. An example of an outlier in the Post-Impressionist style is *Arearea* (*paul_gauguin*_4 in the Painting-91-BIP dataset) by Gauguin, for which $PI(paul\_gauguin\_4) = 0.83$, $B(paul\_gauguin\_4) = 0.44$ and $I(paul\_gauguin\_4) = 0.88$.

diego_velazquez_27, outlier for ℓ-SHE$^{\sqcap\mathbb{Q}}$ | jan_vermeer_12, outlier for ℓ-SHE$^{\mathrm{RPL}}$ | pierrer_auguste_renoir_10, outlier for ℓ-SHE$^{\mathrm{RPL}}$ | claude_monet_8, outlier for ℓ-SHE$^{G(\mathbb{Q})}$ | claude_monet_3, outlier for ℓ-SHE$^{\sqcap\mathbb{Q}}$

paul_gauguin_4, outlier for ℓ-SHE$^{\mathrm{RPL}}$ | vincent_van_gogh_10, outlier for ℓ-SHE$^{G(\mathbb{Q})}$ | vincent_van_gogh_41, outlier for ℓ-SHE$^{\sqcap\mathbb{Q}}$

Fig. 8 Examples of outliers or paintings misclassified from the Painting-91-BIP dataset. All rights under © creative commons, public license. The colour version of this figure is available on the online version of this paper.

Let us consider ℓ-SHE$^{G(\mathbb{Q})}$. Table 8 shows the confusion matrix obtained with ℓ-SHE$^{G(\mathbb{Q})}$ for all the art styles in the Painting-91-BIP dataset.

From the analysis of the data in Table 8 regarding ℓ-SHE$^{G(\mathbb{Q})}$ in the Painting-91-BIP dataset, we have obtained the following results.

- Regarding the Baroque style, observe that this style gets 100% of accuracy rate, and we obtain that 71.7% of the Baroque classifications have Baroque membership degree 1.

Table 8 Confusion matrix for ℓ-SHE$^{G(\mathbb{Q})}$ using Painting-91-BIP dataset.

	Baroque		Impressionism		Post-Impressionism	
	✓	?	✓	?	✓	?
Baroque	74	0	0	0	0	0
Impressionism	60	4	7	8	3	0
Post-Impressionism	54	3	10	0	24	0

– The Impressionism style gets 18.3% of accuracy rate. Most of the outliers, 95.5%, are classified as Baroque paintings, and only 4.5% of the misclassifications ℓ-SHE$^{G(\mathbb{Q})}$ warns that there is evidence to believe that a painting belongs to the Impressionist style. An example of an outlier is *Water lilies* (*claude_monet*_8 in the Painting-91-BIP dataset) by Monet: $B(claude\_monet\_8) = 0.37$, $I(claude\_monet\_8) = 0.32$ and $PI(claude\_monet\_8) = 0.02$.
– The Post-Impressionism style gets 26.4% of accuracy, and we obtain that 83.3% of the Post-Impressionist classifications have Post-Impressionist membership degree 1. Again, most of the outliers, 85.1%, are classified in the Baroque style. Besides, none of the misclassifications ℓ-SHE$^{G(\mathbb{Q})}$ warns that there is evidence to believe that a painting belongs to the Post-Impressionist style. An example of an outlier in the Post-Impressionist style is *Two Cut Sunflowers* (*vincent_van_gogh*_10 in the Painting-91-BIP dataset) by van Gogh, for which $B(vincent\_van\_gogh\_10) = 0.41$, $I(vincent\_van\_gogh\_10) = 0.21$ and $PI(vincent\_van\_gogh\_10) = 0.06$.

Let us now analyse ℓ-SHE$^{\sqcap(\mathbb{Q})}$. Table 9 shows the confusion matrix obtained with ℓ-SHE$^{\sqcap(\mathbb{Q})}$ for all the art styles in the Painting-91-BIP dataset.

Table 9 Confusion matrix for ℓ-SHE$^{\sqcap(\mathbb{Q})}$ using the Painting-91-BIP dataset.

	Baroque		Impressionism		Post-Impressionism	
	✓	?	✓	?	✓	?
Baroque	69	2	2	1	0	0
Impressionism	42	3	25	3	9	0
Post-Impressionism	29	2	20	3	35	2

From the analysis of the data in Table 9 regarding ℓ-SHE$^{\sqcap(\mathbb{Q})}$ in the Painting-91-BIP dataset, we have obtained the following results.

– The highest accuracy is obtained for the Baroque style (96.0%), and that the membership degree to this style of 66.2% of these correct classifications is 1. In 33.3% of the misclassifications, ℓ-SHE$^{\sqcap(\mathbb{Q})}$ warns that there is evidence to believe that a painting belongs to the Baroque style. An example of an outlier is *Tercio* (*diego_velazquez*_27 in the Painting-91-BIP dataset) by Velázquez, for wich $B(diego\_velazquez\_27) = 0.53$, $I(diego\_velazquez\_27) = 0.61$ and $PI(diego\_velazquez\_27) = 0.00$.
– The Impressionism style gets 34.2% of accuracy rate. Regarding the outliers, 83.3% are classified as Baroque paintings and 16.7% as Post-Impressionist paintings. Only 9.3% of the misclassifications ℓ-SHE$^{\sqcap(\mathbb{Q})}$ warns that the painting might belong to the Impressionism style. An example of an outlier is *Water lilies* (*claude_monet*_3 in the

Painting-91-BIP dataset) by Monet: $B(claude\_monet\_3) = 1$, $I(claude\_monet\_3) = 0.87$, and $PI(claude\_monet\_3) = 0.02$.

- The Post-Impressionism style obtains 40.7% of accuracy, and that the membership degree to this style of 59.5% of these correct classifications is 1. Regarding outliers, 57.4% are classified in the Baroque style and 42.6% in the Impressionist style. In 13.5% of the misclassifications ℓ-SHE$^{\sqcap(\mathbb{Q})}$ warns that there is evidence to believe that a painting belongs to the Post-Impressionist style. An example of an outlier in the Post-Impressionist style is *Sorrowing Old Man* ($vincent\_van\_gogh\_41$ in the Painting-91-BIP dataset) by van Gogh, where $B(vincent\_van\_gogh\_41) = 0.17$, $I(vincent\_van\_gogh\_41) = 0.87$ and $PI(vincent\_van\_gogh\_41) = 0.00$.

Results obtained by each ℓ-SHE for both datasets are presented in Table 10. Note first that ℓ-SHE$^{G(\mathbb{Q})}$ gets the lowest general accuracy, 48.2%. Besides, the ℓ-SHE$^{\text{RPL}}$ version is again the proposal with highest general accuracy, but ℓ-SHE$^{\sqcap(\mathbb{Q})}$ gets 57.0% of general accuracy. Hence similar accuracies to other were obtained for both proposals, ℓ-SHE$^{\text{RPL}}$ and ℓ-SHE$^{\sqcap(\mathbb{Q})}$. However, accuracies for the Impressionist and the Post-Impressionist styles are higher for ℓ-SHE$^{\text{RPL}}$, and the ℓ-SHE$^{\sqcap(\mathbb{Q})}$ version shows an accuracy for the Impressionism style close to a random classifier. Therefore from the data analysis it might be concluded that ℓ-SHE$^{\text{RPL}}$ is the most accurate classifier.

Table 10 Percentages of accuracy obtained in the QArt-Dataset and the Painting-91-BIP datasets for each ℓ-SHE version.

Dataset	Art style painting	ℓ-SHE version		
		ℓ-SHE$^{\text{RPL}}$	ℓ-SHE$^{G(\mathbb{Q})}$	ℓ-SHE$^{\sqcap(\mathbb{Q})}$
Painting-91-BIP	Baroque	86.5	100.0	96.0
	Impressionism	40.2	18.3	34.2
	Post-Impressionism	53.9	26.4	40.7
	General accuracy	60.2	48.2	57.0
QArt-Dataset	Baroque	90	100.0	93.3
	Impressionism	70	56.7	60
	Post-Impressionism	60	40	53.3
	General accuracy	73.3	65.6	68.9

9 Conclusions, and Future Work

The art style classification algorithm ℓ-SHE has been presented and analysed considering the three different versions defined, which are determined by the three logics RPL, $G(\mathbb{Q})$ and $\sqcap(\mathbb{Q})$. The accuracy acquired in the QArt-Dataset with the three logics (Table 10) is similar to other works that use qualitative colour descriptors [13]. Regarding Painting-91-BIP dataset, the results obtained (Table 10) are similar, but a bit the two classifiers built in [15]. However, contrary to those classifiers based on machine learning methods, the ℓ-SHE classification provides explanations of right classifications, and also of some of the outliers by giving a second option. Hence each classification method has both advantages and disadvantages. In this way, comparing in detail different approaches for art style classification is future work.

On the other hand, all the ℓ-SHE versions show a low accuracy for Impressionist style in the Painting-91-BIP dataset. This flaw in the classification might be explained in terms of art genres: individual portraits are scant in Renoir's paintings from the QArt-dataset, whereas this is the main type of painting in Renoir's paintings from Painting-91-BIP dataset. This is an important aspect to consider for future work.

In both datasets the ℓ-SHE$^{\mathrm{RPL}}$ version gets the highest general accuracy among the ℓ-SHE approaches. Indeed, the general accuracy for the QArt-Dataset obtained by the ℓ-SHE$^{\mathrm{RPL}}$ version is 73.3%, whereas the general accuracies for the QArt-Dataset obtained by the ℓ-SHE$^{G(\mathbb{Q})}$ and the ℓ-SHE$^{\sqcap(\mathbb{Q})}$ are 65.6% and 68.9%, respectively (see Table 10). In addition, the general accuracy for the Painting-91-BIP obtained by the ℓ-SHE$^{\mathrm{RPL}}$ version is 60.2%, and the general accuracies for the Painting-91-BIP dataset obtained by the ℓ-SHE$^{G(\mathbb{Q})}$ and the ℓ-SHE$^{\sqcap(\mathbb{Q})}$ are 48.2% and 57%, respectively (see Table 10). Thus the ℓ-SHE$^{\mathrm{RPL}}$ seems to be the best approach on both datasets.

Other future work includes introducing other logical formalisms and other aggregation methods to represent the different art styles and the use of reasoning mechanisms to draw conclusions about the relationship between new and classical styles. For this purpose it would be important to add new art styles to the dataset. Moreover, we expect to show the ℓ-SHE outcomes to art experts in order to get feedback from them and use it to improve the ℓ-SHE algorithm.

Moreover, adding art-genre information, studying the complexity of ℓ-SHE algorithm, and comparing it to machine learning methods are future work. Also, it would be relevant to explore a SMT-based (Statistical Machine Translation) approach in future extensions of the ℓ-SHE algorithm. Finally, we will study the possibility of enriching our algorithm with abduction procedures to improve the accuracy.

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