



# Logic, Algebra and Truth Degrees

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## Proceedings

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# On an implication-free reduct of $MV_n$ chains

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## Abstract

Let  $\mathbf{L}_{n+1}$  be the MV-chain on the  $n+1$  elements set  $\mathbf{L}_{n+1} = \{0, 1/n, 2/n, \dots, (n-1)/n, 1\}$  in the algebraic language  $\{\rightarrow, \neg\}$  [3]. As usual, further operations on  $\mathbf{L}_{n+1}$  are definable by the following stipulations:  $1 = x \rightarrow x$ ,  $0 = \neg 1$ ,  $x \oplus y = \neg x \rightarrow y$ ,  $x \odot y = \neg(\neg x \oplus \neg y)$ ,  $x \wedge y = x \odot (x \rightarrow y)$ ,  $x \vee y = \neg(\neg x \wedge \neg y)$ . Moreover, we will pay special attention to the also definable unary operator  $*x = x \odot x$ .

In fact, the aim of this paper is to study the  $\{*, \neg, \vee\}$ -reducts of the MV-chains  $\mathbf{L}_{n+1}$ , that will be denoted as  $\mathbf{L}_{n+1}^*$ , i.e. the algebra on  $\mathbf{L}_{n+1}$  obtained by replacing the implication operator  $\rightarrow$  by the unary operation  $*$  which represents the square operator  $*x = x \odot x$  and which has been recently used in [4] to provide, among other things, an alternative axiomatization for the four-valued matrix logic  $J_4 = \langle \mathbf{L}_4, \{1/3, 2/3, 1\} \rangle$ . In this contribution we make a step further in studying the expressive power of the  $*$  operation, in particular we will focus on the question for which natural numbers  $n$  the structures  $\mathbf{L}_{n+1}$  and  $\mathbf{L}_{n+1}^*$  are term-equivalent. In other words, for which  $n$  the Łukasiewicz implication  $\rightarrow$  is definable in  $\mathbf{L}_{n+1}^*$ , or equivalently, for which  $n$   $\mathbf{L}_{n+1}^*$  is in fact an MV-algebra. We also show that, in any case, the matrix logics  $\langle \mathbf{L}_{n+1}^*, F \rangle$ , where  $F$  is an order filter, are algebraizable. What we present here is a work in progress.

## Term-equivalence between $\mathbf{L}_{n+1}$ and $\mathbf{L}_{n+1}^*$

Let  $X$  be a subset of  $\mathbf{L}_{n+1}$ . We denote by  $\langle X \rangle^*$  the subalgebra of  $\mathbf{L}_{n+1}^*$  generated by  $X$  (in the reduced language  $\{*, \neg, \vee\}$ ). For  $n \geq 1$  define recursively  $(*)^n x$  as follows:  $(*)^1 x = *x$ , and  $(*)^{i+1} x = *((*)^i x)$ , for  $i \geq 1$ .

A nice feature of the  $\mathbf{L}_{n+1}^*$  algebras is that we can always define terms characterising the principal order filters  $F_a = \{b \in \mathbf{L}_{n+1} \mid a \leq b\}$ , for every  $a \in \mathbf{L}_{n+1}$ .

**Proposition 1.** *For each  $a \in \mathbf{L}_{n+1}$ , the unary operation  $\Delta_a$  defined as*

$$\Delta_a(x) = \begin{cases} 1 & \text{if } x \in F_a \\ 0 & \text{otherwise.} \end{cases}$$

*is definable in  $\mathbf{L}_{n+1}^*$ . As a consequence, for every  $a \in \mathbf{L}_{n+1}$ , the operation  $\chi_a$  that corresponds to the characteristic function of  $a$  (i.e.  $\chi_a(x) = 1$  if  $x = a$  and  $\chi_a(x) = 0$  otherwise) is definable as well.*

*Proof.* The case  $a = 1$  corresponds to the Monteiro-Baaz Delta operator and, as is well-known, it can be defined as  $\Delta_1(x) = (*)^n x$ . For  $a = 0$  define  $\Delta_a(x) = \Delta_1(x) \vee \neg \Delta_1(x)$ ; then  $\Delta_a(x) = 1$  for every  $x$ . Now, assume  $0 < a = i/n < 1$ . It is not difficult to show that one can always find a sequence of terms (operations)  $t_1(x), \dots, t_m(x)$  over  $\{*, \neg\}$  such that  $t_1(t_2(\dots(t_m(x))\dots)) = 1$

if  $x \in F_a$  while  $t_1(t_2(\dots(t_m(x))\dots)) < 1$  otherwise. Then  $\Delta_a(x) = \Delta_1(t_1(t_2(\dots(t_m(x))\dots)))$  for every  $x$ .

As for the operations  $\chi_a$ , define  $\chi_1 = \Delta_1$ ,  $\chi_0 = \neg\Delta_{1/n}$ , and if  $0 < a < 1$ , then define  $\chi_a = \Delta_a \wedge \neg\Delta_{a-1/n}$ .  $\square$

It is now almost immediate to check that the following implication-like operation is definable in every  $\mathbf{L}_{n+1}^*$ :  $x \Rightarrow y = 1$  if  $x \leq y$  and 0 otherwise. Indeed,  $\Rightarrow$  can be defined as

$$x \Rightarrow y = \bigvee_{0 \leq i \leq j \leq n} (\chi_{i/n}(x) \wedge \chi_{j/n}(y)).$$

Actually, one can also define Gödel implication on  $\mathbf{L}_{n+1}^*$  by putting  $x \Rightarrow_G y = (x \Rightarrow y) \vee y$ .

On the other hand, it readily follows from Proposition 1 that all the  $\mathbf{L}_{n+1}^*$  algebras are simple. Indeed, if  $a > b \in \mathbf{L}_{n+1}$  would be congruent, then  $\Delta_a(a) = 1$  and  $\Delta_a(b) = 0$  should be so. Recall that an algebra is called *strictly simple* if it is simple and does not contain proper subalgebras. It is clear then that in the case of  $\mathbf{L}_{n+1}$  and  $\mathbf{L}_{n+1}^*$  algebras, they are strictly simple if  $\{0, 1\}$  is their only proper subalgebra.

**Remark 2.** It is well-known that  $\mathbf{L}_{n+1}$  is strictly simple iff  $n$  is prime. Note that, for every  $n$ , if  $\mathbf{B} = (B, \neg, \rightarrow)$  is an MV-subalgebra of  $\mathbf{L}_{n+1}$ , then  $\mathbf{B}^* = (B, \vee, \neg, *)$  is a subalgebra of  $\mathbf{L}_{n+1}^*$  as well. Thus, if  $\mathbf{L}_{n+1}$  is not strictly simple, then  $\mathbf{L}_{n+1}^*$  is not strictly simple as well. Therefore, if  $n$  is not prime,  $\mathbf{L}_{n+1}^*$  is not strictly simple. However, in contrast with the case of  $\mathbf{L}_{n+1}$ ,  $n$  being prime is not a sufficient condition for  $\mathbf{L}_{n+1}^*$  being strictly simple. In Lemma 7 below we will provide some examples of prime  $n$  for which  $\mathbf{L}_{n+1}^*$  is not strictly simple, in view of Theorem 6.

**Lemma 3.**  $\mathbf{L}_{n+1}^*$  is strictly simple iff  $\langle (n-1)/n \rangle^* = \mathbf{L}_{n+1}^*$ .

*Proof.* The ‘only if’ direction is trivial. In order to prove the converse, assume that  $\langle a_1 \rangle^* = \mathbf{L}_{n+1}^*$  for  $a_1 = (n-1)/n$ . For  $i \geq 1$  let  $a_{i+1} = t_i(a_i)$  such that  $t_i(x) = *x$  if  $x > 1/2$ , and  $t_i(x) = \neg x$  otherwise. Since  $\mathbf{L}_{n+1}^*$  is finite, there is  $1 \leq i < j$  such that  $a_j = a_i$  and so  $A_1 := \{a_i \mid i \geq 1\} = \{a_i \mid 1 \leq i \leq k\}$  for some  $k$  such that  $a_i \neq a_j$  if  $1 \leq i, j \leq k$ . Let  $A = A_1 \cup A_2 \cup \{0, 1\}$  where  $A_2 = \{\neg a \mid a \in A_1\}$ . Since  $*1 = 1$  and  $*x = 0$  if  $x \leq 1/2$ ,  $A$  is the domain of a subalgebra  $\mathbf{A}$  of  $\mathbf{L}_{n+1}^*$  over  $\{*, \neg, \vee\}$  such that  $a_1 \in A$ , hence  $\langle a_1 \rangle^* \subseteq \mathbf{A}$ . But  $\mathbf{A} \subseteq \langle a_1 \rangle^*$ , by construction. Therefore  $\mathbf{A} = \langle a_1 \rangle^* = \mathbf{L}_{n+1}^*$ .

**Fact:** Under the current hypothesis (namely,  $\langle a_1 \rangle^* = \mathbf{L}_{n+1}^*$ ): if  $n$  is even then  $n = 2$  or  $n = 4$ . Indeed, suppose that  $\langle a_1 \rangle^* = \mathbf{L}_{n+1}^*$  and  $n$  is even. If  $n = 2$  or  $n = 4$  then clearly  $\mathbf{L}_{n+1}^*$  is strictly simple. Now, assume  $n > 4$ . Observe that: (1) for any  $a \in \mathbf{L}_{n+1}^* \setminus \{0, 1\}$ ,  $*a = i/n$  such that  $i$  is even; and (2) if  $i < n$  is even then  $\neg(i/n) = (n-i)/n$  such that  $n-i$  is even. That being so, if  $i/n \in (A_1 \cup A_2) \setminus \{a_1, \neg a_1\}$  (recall the process described above) then  $i$  is even. But then, for instance,  $3/n \notin \mathbf{A} = \langle a_1 \rangle^* = \mathbf{L}_{n+1}^*$ , a contradiction. This proves the Fact.

From the **Fact**, assume now that  $n$  is odd, and let  $a = ((n+1)/2)/n$  and  $b = ((n-1)/2)/n$ . Since  $\neg a = b$ ,  $\neg b = a$  and  $a, b \in A$  then, by construction of  $A$ , there is  $1 \leq i \leq k$  such that either  $a = a_i$  or  $b = a_i$ . If  $a = a_i$  then  $a_{i+1} = *a = 1/n$  and so  $a_{i+2} = \neg a_{i+1} = \neg 1/n = (n-1)/n = a_1$ . Analogously it can be proven that, if  $b = a_i$  then  $a_1 = a_j$  for some  $j > i$ . This shows that  $A_1 = \{a_1, \dots, a_k\}$  is such that  $a_{k+1} = a_1$  (hence  $a_k = 1/n$ ). Now, let  $c \in \mathbf{L}_{n+1}^* \setminus \{0, 1\}$  such that  $c \neq a_1$ . If  $c \in A_1$  then the process of generation of  $A$  from  $c$  will produce the same set  $A_1$  and so  $\mathbf{A} = \mathbf{L}_{n+1}^*$ , showing that  $\langle c \rangle = \mathbf{L}_{n+1}^*$ . Otherwise, if  $c \in A_2$  then  $\neg c \in A_1$  and, by the same argument as above, it follows that  $\langle c \rangle = \mathbf{L}_{n+1}^*$ . This shows that  $\mathbf{L}_{n+1}^*$  is strictly simple.  $\square$

**Lemma 4.** If  $\mathbf{L}_{n+1}$  is term-equivalent to  $\mathbf{L}_{n+1}^*$  then  $\mathbf{L}_{n+1}^*$  is strictly simple.

*Proof.* If  $\mathbf{L}_{n+1}$  is term-equivalent to  $\mathbf{L}_{n+1}^*$  then  $\odot$  is definable in  $\mathbf{L}_{n+1}^*$ , and hence  $\langle (n-1)/n \rangle^* = \mathbf{L}_{n+1}^*$ . Indeed, we can obtain  $(n-i-1)/n = ((n-1)/n) \odot ((n-i)/n)$  for  $i = 1, \dots, n-1$ , and  $1 = -0$ . By Lemma 3 it follows that  $\mathbf{L}_{n+1}^*$  is strictly simple.  $\square$

**Corollary 5.** *If  $\mathbf{L}_{n+1}$  is term-equivalent to  $\mathbf{L}_{n+1}^*$  then  $n$  is prime.*

*Proof.* If  $\mathbf{L}_{n+1}$  is term-equivalent to  $\mathbf{L}_{n+1}^*$  then  $\mathbf{L}_{n+1}^*$  is strictly simple, by Lemma 4. By Remark 2 it follows that  $n$  must be prime.  $\square$

**Theorem 6.**  *$\mathbf{L}_{n+1}$  is term-equivalent to  $\mathbf{L}_{n+1}^*$  iff  $\mathbf{L}_{n+1}^*$  is strictly simple.*

*Proof.* The ‘only if’ part is Lemma 4. For the ‘if’ part, since  $\mathbf{L}_{n+1}^*$  is strictly simple then, for each  $a, b \in \mathbf{L}_{n+1}$  where  $a \notin \{0, 1\}$  there is a definable term  $\mathbf{t}_{a,b}(x)$  such that  $\mathbf{t}_{a,b}(a) = b$ . Otherwise, if for some  $a \notin \{0, 1\}$  and  $b \in \mathbf{L}_{n+1}$  there is no such term then  $\mathbf{A} = \langle a \rangle^*$  would be a proper subalgebra of  $\mathbf{L}_{n+1}^*$  (since  $b \notin \mathbf{A}$ ) different from  $\{0, 1\}$ , a contradiction. By Proposition 1 the operations  $\chi_a(x)$  are definable for each  $a \in \mathbf{L}_{n+1}$ , then in  $\mathbf{L}_{n+1}^*$  we can define Łukasiewicz implication  $\rightarrow$  as follows:

$$x \rightarrow y = (x \Rightarrow y) \vee \left( \bigvee_{n>i>j\geq 0} \chi_{i/n}(x) \wedge \chi_{j/n}(y) \wedge \mathbf{t}_{i/n, a_{ij}}(x) \right) \vee \left( \bigvee_{n>j\geq 0} \chi_1(x) \wedge \chi_{j/n}(y) \wedge y \right)$$

where  $a_{ij} = 1 - i/n + j/n$ .  $\square$

We have seen that  $n$  being prime is a necessary condition for  $\mathbf{L}_{n+1}$  and  $\mathbf{L}_{n+1}^*$  being term-equivalent. But this is not a sufficient condition: in fact, there are prime numbers  $n$  for which  $\mathbf{L}_{n+1}$  and  $\mathbf{L}_{n+1}^*$  are not term-equivalent.

**Lemma 7.** *If  $n$  is a prime Fermat number greater than 5 then  $\mathbf{L}_{n+1}$  and  $\mathbf{L}_{n+1}^*$  are not term-equivalent.*

*Proof.* Recall that a Fermat number is of the form  $2^{2^k} + 1$ , with  $k$  being a natural number. We are going to prove that if  $n$  is a prime Fermat number and  $a_1 = (n-1)/n$ , then  $\langle a_1 \rangle^*$  is a proper subalgebra of  $\mathbf{L}_{n+1}^*$  (recall Theorem 6 and Lemma 3). Thus, let  $n > 5$  be a prime Fermat number, that is, a prime number of the form  $n = 2^m + 1$  with  $m = 2^k$  and  $k > 1$ . The  $(m-1)$ -times iterations of  $*$  applied to  $a_1$  produce  $((n+1)/2)/n$ , that is:  $(*)^{m-1}(a_1) = ((n+1)/2)/n$ . Since  $*(((n+1)/2)/n) = 1/n$ , the constructive procedure for generating the algebra  $\langle a_1 \rangle^*$  described in the proof of Lemma 3 shows that  $\langle a_1 \rangle^* = \mathbf{A}$  has  $2m+2$  elements:  $m$  elements in  $A_1$ , plus  $m$  elements in  $A_2$  corresponding to their negations, plus 0 and 1. Since  $2m+2 < 2^m+1 = n$  as  $n > 5$ ,  $\langle a_1 \rangle^*$  is properly contained in  $\mathbf{L}_{n+1}$ , and it is different from  $\{0, 1\}$ .  $\square$

The first Fermat prime number greater than 5 is  $n = 17$ . It is easy to see that

$$\langle 16/17 \rangle^* = \{0, 1/17, 2/17, 4/17, 8/17, 9/17, 13/17, 15/17, 16/17, 1\}.$$

Actually, we do not have a full characterisation of those prime numbers  $n$  for which  $\mathbf{L}_{n+1}$  and  $\mathbf{L}_{n+1}^*$  are term-equivalent. But computational results show that for prime numbers until 8000, about 60% of the cases yield term-equivalence.

### Algebraizability of $\langle \mathbf{L}_{n+1}^*, F_{i/n} \rangle$

Given the algebra  $\mathbf{L}_{n+1}^*$ , it is possible to consider, for every  $1 \leq i \leq n$ , the matrix logic  $\mathbf{L}_{i,n+1}^* = \langle \mathbf{L}_{n+1}^*, F_{i/n} \rangle$ . In this section we will show that all the  $\mathbf{L}_{i,n+1}^*$  are algebraizable in the sense of Blok-Pigozzi [1], and the quasivarieties associated to  $\mathbf{L}_{i,n+1}^*$  and  $\mathbf{L}_{j,n+1}^*$  are the same, for every  $i, j$ .

Observe that the operation  $x \approx y = 1$  if  $x = y$  and  $x \approx y = 0$  otherwise is definable in  $\mathbf{L}_{n+1}^*$ . Indeed, it can be defined as  $x \approx y = (x \Rightarrow y) \wedge (y \Rightarrow x)$ . Also observe that  $x \approx y = \Delta_1((x \Rightarrow_G y) \wedge (y \Rightarrow_G x))$  as well.

In order to prove the main result of this section, we state the following:

**Lemma 8.** *For every  $n$ , the logic  $L_{n+1}^* := \mathbf{L}_{n,n+1}^* = \langle \mathbf{L}_{n+1}^*, \{1\} \rangle$  is algebraizable.*

*Proof.* It is immediate to see that the set of formulas  $\Delta(p, q) = \{p \approx q\}$  and the set of pairs of formulas  $E(p, q) = \{\langle p, \Delta_0(p) \rangle\}$  satisfy the requirements of algebraizability.  $\square$

Blok and Pigozzi [2] introduce the following notion of equivalent deductive systems. Two propositional deductive systems  $S_1$  and  $S_2$  in the same language are *equivalent* if there are translations  $\tau_i : S_i \rightarrow S_j$  for  $i \neq j$  such that:  $\Gamma \vdash_{S_i} \varphi$  iff  $\tau_i(\Gamma) \vdash_{S_j} \tau_i(\varphi)$ , and  $\varphi \dashv\vdash_{S_i} \tau_j(\tau_i(\varphi))$ . From very general results in [2] it follows that two equivalent logic systems are indistinguishable from the point of view of algebra, namely: if one of the systems is algebraizable then the other will be also algebraizable w.r.t. the same quasivariety. This will be applied to  $\mathbf{L}_{i,n+1}^*$ .

**Lemma 9.** *The logics  $L_{n+1}^*$  and  $L_{i,n+1}^*$  are equivalent, for every  $n$  and for every  $1 \leq i \leq n-1$ .*

*Proof.* It is enough to consider the translation mappings  $\tau_1 : \mathbf{L}_{n+1}^* \rightarrow \mathbf{L}_{i,n+1}^*$ ,  $\tau_1(\varphi) = \Delta_1(\varphi)$ , and  $\tau_{i,2} : \mathbf{L}_{i,n+1}^* \rightarrow \mathbf{L}_{n+1}^*$ ,  $\tau_{i,2}(\varphi) = \Delta_{i/n}(\varphi)$ .  $\square$

Finally, as a direct consequence of Lemma 8, Lemma 9 and the observations above, we can prove the following result.

**Theorem 10.** *For every  $n$  and for every  $1 \leq i \leq n$ , the logic  $L_{i,n+1}^*$  is algebraizable.*

As an immediate consequence of Theorem 10, for each logic  $\mathbf{L}_{i,n+1}^*$  there is a quasivariety  $\mathcal{Q}(i, n)$  which is its equivalent algebraic semantics. Moreover, by Lemma 9 and by Blok and Pigozzi's results,  $\mathcal{Q}(i, n)$  and  $\mathcal{Q}(j, n)$  coincide, for every  $i, j$ . The question of axiomatising  $\mathcal{Q}(i, n)$  is left for future work.

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