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ClusDM: A Multiple Criteria Decision Making Method for Heterogeneous Data Sets

Aïda Valls Mateu

Foreword by Vicenç Torra

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Pel meu estimat Toni

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Foreword

This book describes ClusDM, a system for dealing with multicriteria decision making problems. In particular, ClusDM permits either to construct a ranking between existing alternatives or to select the best ones. The main characteristic of the system with respect to the user perspective is that it does not restrict him/her to work in the numerical scale when evaluating the alternative criteria. Instead, it permits the user to use at the same time criteria evaluated in numerical and ordinal scales. Additionally, and from the operational point of view, the system uses a novel approach. The computation of the ranking between alternatives does not satisfy the condition of independence of irrelevant alternatives. This condition, often included in decision systems for technical (and computational) reasons, is here removed. In this way, the final evaluation (or the final ranking position) of an alternative is affected/influence by the others. Clustering techniques are used for establishing these kinds of influences.

By the way, the development of a system is not complete until its formal properties are studied and the results of some applications are considered. Both issues are described in the book in their corresponding chapters. In addition, in relation to applications and, more specifically, to help users to evaluate the results of ClusDM, a set of quality measures were proposed (one measure for each of the stages that define the system). A detailed description of one application showing all these elements at work is also included in the text. This description includes a comparison of the ClusDM results with the corresponding human decisions.

Bellaterra, March 2003

Vicenç Torra
Researcher of the IIIA-CSIC

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Abstract

This thesis presents a new methodology for decision making. In particular, we have studied the problems that consider more than one criterion, which is known as Multiple Criteria Decision Making (MCDM) or Multiple Criteria Decision Aid (MCDA). The difference relies on the fact of imitating the behaviour of the decision maker (i.e. develop a method that makes decisions) or giving to the decision maker some additional information that allows him to understand the mechanism of solving decisions (i.e. the decision maker can learn from the use of the method). Our proposal fits better in the MCDA approach, but has also similarities with the MCDM perspective. On one hand, the method we have designed is independent enough to not require a deep understanding of the process by the decision maker. On the other hand, we have carefully studied the process and the method is able to extract knowledge about the decision problem, which is given to the user to let him know any special characteristics of the data analysed.

ClusDM is a new method to solve multicriteria decision problems. It is able to find a ranking of alternatives or to select the best ones. Some extensions to the classical numerical approach have been studied, such as, fuzzy or ordinal values. However, we have noticed that they require having a common scale for all criteria. This thesis faces the problem of managing different types of criteria at the same time. ClusDM follows the utility approach, which considers two steps to sort a decision problem out: the aggregation and the ranking. We have included some additional steps in order to improve the process: (i) the explanation phase and (ii) the quality measurement phase. With these additional stages, ClusDM is able to build a qualitative preference ranking, which can be easily understood by the decision maker. As well as, some quality measures have been defined in order to give an idea of the trustworthiness of the ranking.

Chapter 1.

Introduction

In this introductory chapter, we will briefly situate the research area where we have been working: multicriteria decision aid. We will also present the goals formulated at the beginning of this thesis and we will summarise our main contributions to the field. Finally, we will describe the structure of this document.

1.1 What is multicriteria decision aid

The study of decision problems has a long history, and in the last decades has been one of the major research fields in decision sciences. People have to make lots of decisions during their life. Moreover, some of these decisions are directly related to the main worries of humankind, such as survival, security or perpetuation [Yu,1990]. The mathematical modelisation of these decision making problems started in the 19th century with economists and applied mathematicians like Pareto, VonNeumann, Morgenstern and many more. The first approaches considered monocriterion decision problems. In 1951 two research teams introduced the multicriteria problem: Koopmans [Koopmans,1951] and Kuhn & Tucker [Kuhn&Tucker,1951]. That is, the problem of finding the best alternative (or a ranking of all of them) considering multiple conflicting criteria or goals.

It was in late 60s that multicriteria decision making research experimented an explosive growth. In 1972 the first international conference in Multiple Criteria Decision Making was done. From then on, this is an active area of research as can be observed in the many conferences organised every year and the specialized journals (e.g. *Journal of Multi-Criteria Decision Analysis*). Some associations gather the researchers in the field (e.g. the *International Society on Multiple Criteria Decision Makin*g [MCDA,2002] or the *European Working Group on MCDA* [EWGMCDA,2002]).

Although we have been talking about Multiple Criteria Decision Making (MCDM), this is not the only denomination that can be found. In Europe, it is most common the use of MCDA, which stands for Multiple Criteria Decision Aid. This differentiation is not only in the name but also in the underneath philosophy of how to help decision makers to make better decisions (called the French School). MCDA researchers devote

their main efforts to develop methods to help the user to understand the preference model behind his/her decisions. This approach includes for instance: graphical tools to represent the data or interactive methods to build a model of the preferences. On the other hand, the American School of MCDM takes a more descriptive approach. The goal is to build a model of the behaviour of the decision makers and let them to apply the model to solve new problems.

The method that we will explain in this thesis fits better with the MCDA approach. As it will be introduced in the next section, we have developed a methodology for decision making but having into account that the decision maker must understand the solution and the degree of confidence he should attribute to it, as well as, he must also be aware of how the solution has been obtained in order to let him to modify the elements that take part in the process.

1.1.1 Motivation and goals

The main difficulty in MCDA[1] problems lies in the fact that usually there is no objective or optimal solution for all the criteria. Thus, some trade-off must be done among the different points of view to determine an acceptable solution. Therefore, it is not an easy problem at all, which explains the large amount of publications in the area in the last decades.

Although MCDA problems have been studied in the Operational Research area for a long time, recently there is an increasing interest in including Artificial Intelligence techniques to the classical numerical methods [Bana e Costa,1990]. Sometimes, the knowledge available about the alternatives cannot be expressed numerically (or it is difficult to use numbers instead of other types of values) [Wang,2001]. For instance, assume that we need to have into consideration the height of each person in a team; if we do not have any tool to measure the height, it is difficult to give a numerical value for each person. On the contrary, it is very natural to say: “tall”, “short”, “very tall”, etc. Therefore, different approaches to the use of non-numerical values in MCDA have been developed (see Chapter 2).

However, very few methods consider the possibility to have matrices with heterogeneous criteria (different types and/or different scales). This limitation to a common scale for all criteria forces the data suppliers to use values that could be different to the ones they would normally use. Other approaches let the user to provide heterogeneous data, which is automatically translated into a unified scale before their processing [Herrera&Herrera-Viedma,2000]. In this case, the transformation obtained does not contain all the information that the person has initially provided. For this reason, sometimes it is argued that is better to allow only a unique scale for providing the data. We agree with the authors (e.g. [Delgado et al., 1998]) who argue that, in spite of the increase of the ambiguity, uncertainty or contradiction in the data, the more information we have the better understanding of the alternatives.

So, after detecting the problem of heterogeneous criteria, we became interested in studying this situation and **we concentrated our efforts in developing a different**

[1] In this section, we will use the acronym MCDA but what is explained is also valid for MCDM.

approach that is able to handle different types of criteria without making an explicit translation into a common domain a priori.

In the MCDA field, three kinds of problems are distinguished [Vincke,1992]: *choice problems, ranking problems* and *sorting problems*. The goal of the decision maker in each type of problem is different: in choice problems the aim is to find the best alternative, in ranking problems we want to know the *goodness* of all alternatives, which is usually presented as a ranking from the best to the worst, and in sorting problems we want to know which alternatives belong to each class of a predefined set of classes.

As a natural extension of our previous research in unsupervised learning methods, particularly in clustering, **we have focused on *choice* and *ranking* problems** instead of the *sorting* one, which is classically solved with a supervised approach [Zopounidis,2002]. As it will be seen, this unsupervised approach has been considered during all the stages of the decision process. In spite of losing some power, we have intended to not require the user to know technical details about the methods in order to provide an easy tool that does not need a learning stage.

The necessity to give a qualitatively described result has been argued by different authors. The rationale behind this belief is that human decision makers understand better a linguistic statement characterising the selected alternative (or ranking of alternatives) than a numerical result or even a membership function. In [Bana e Costa, 1990] different components of the ideal solution are identified: not only the position of each alternative in the ranking is important, but also the intensity of preference of each one and the degree of truth of the result. The reliability of the system can change depending on the degree of consensus of the different criteria. Thus, if the decision maker's confidence in the system makes him follow the recommendation without doubt and the alternative chosen is not good enough, the result will be disastrous, especially in critical situations. The decision maker will also welcome the addition of other information about the reasons of having obtained a bad or good result. **We have devoted special attention to the definition of quality measures and linguistic descriptions of the decision making process**.

In particular, we have studied a new semantics for qualitative criteria based on the concept of antonym. The key idea is that we can infer the meaning of a term knowing the terms that express an opposite value [de Soto&Trillas, 1999]. This relation among the qualitative terms can be represented with a negation function. In [Torra,1996] an extended negation function is proposed. In [Valls&Torra,1999c] we studied in detail the use of the negation functions and their induced semantics, observing that it is adequate to capture more information than with ordinal qualitative criteria. Nowadays, the fuzzy representation of the semantics of qualitative terms is the most widespread, however, from the expert's point of view, it is easier to give the information needed to build a negation function than the information required to build a fuzzy set. Therefore, **we have focused on the use of the negation based semantics during the decision analysis and, especially, in the description of the ranking of alternatives.**

In summary, the goal of this doctoral dissertation is ***the development of a multi-criteria decision method for choice and ranking problems that allows different types of values and domains without making an explicit translation into a common domain. In addition, some quality measures and linguistic explanations will be part of the result.***

1.2 Contributions

The main contributions of this research work in the MCDA field are the following:

- We have designed a new methodology for multi-criteria decision problems with heterogeneous data, called ClusDM. In particular, we have developed a system that considers three different types of values: numerical, qualitative and Boolean. Moreover, each qualitative criterion can have its own set of available linguistic terms.

- We have studied the use of a new kind of qualitative description, based on negation functions. We have used this simple representation of the semantics for the linguistic terms during all the stages of the process. At the end, the ranking of alternatives (or the selected one) is explained to the decision maker using this type of values.

- We have developed a method to adapt one of the preference vocabularies provided by the experts to describe the overall preferences of the alternatives, that is, to explain the ranking obtained. The use of terms that are already known for the user can make things easier to him. In particular, we provide a way of selecting one of the vocabularies depending on the similarities among the meaning of its terms and the meaning of the groups of alternatives we want to characterise. Moreover, we have developed some algorithms to select the most appropriate terms among the ones in the vocabulary, to produce new terms and to adapt the meaning of these terms according to the characteristics of the result.

- We have identified different key points in any MCDA process where the quality of the partial results generated should be evaluated. In particular, we have defined different quality measures for the different stages of our method. With these measures we can give an overall value of the trustworthiness of the final result. This kind of information is very useful for the decision maker in order to pay more or less attention to the recommendations of the system.

- We have developed a methodology that is able to detect conflicting elements. In particular, the decision maker is notified about alternatives that have received opposite preference evaluations for different criteria and about criteria that do not agree with the majority. With this additional information, the user is able to modify the data set, for instance, dropping alternatives that may not have been considered, including additional criteria or modifying their weights.

1.3 Structure of the Thesis

This document is divided into 8 chapters. The first one has given an introduction to the problem we have studied and the goals and contributions of the thesis.

Chapter 2 presents a brief survey of the MCDA research field. From a general view, we go through the classical approaches to multicriteria decisions until focusing on uncertainty in utility-based models using ordinal qualitative criteria.

Chapters 3, 4 and 5 explain in detail the methodology we have developed, called ClusDM (Clustering for Decision Making). Four phases can be distinguished. The first one, the aggregation phase is explained in Chapter 2; the second, the ranking phase, is detailed in Chapter 3; finally, Chapter 4 includes the explanation and quality measurement phases.

ClusDM can be seen as an aggregation operator for qualitative preference criteria. The properties of this new decision operator are defined and proved in Chapter 6. We have studied the usual properties required to this type of operators. We begin with the basic properties: symmetry, idempotence and monotonicity. Then, we continue with more elaborate properties. We will see that ClusDM does not satisfy some of these properties.

The methodology we propose has been implemented using Lisp and Java. The Lisp code is included in a system called *Radames*, which performs aggregation of numerical data, qualitative data, heterogeneous data, data matrices and trees. In particular, the ClusDM methodology is the one used in aggregation of qualitative and heterogeneous data sets. The Java code is integrated in an agent developed in Jade, called *ClusDMA*. Chapter 7 gives some details about these systems and explains the results obtained in three different application domains.

Finally, an overview of the thesis can be found in Chapter 8. This chapter also gives some interesting directions to continue this work.

Chapter 2.

Review of Multicriteria Decision Aid techniques

In this chapter we explain the different approaches for solving a Multiple Criteria Decision Aid problem. We begin with the definition of the problem and the concepts we will deal with during this dissertation. Then, the main characteristics of the most used approaches are given: Multi-attribute Utility Theory and Outranking Relations. Then, the Rough Sets approach is described due to the similarities with our research. The chapter finishes reviewing the field of multicriteria decision in the case of imprecision and uncertainty.

2.1 Formalisation

In this section, we formalize the multi-criteria decision problem. We define the concepts and nomenclature used in this document. This section is included because there is not a common standard for the denomination and nomenclature of the elements that participate in this kind of decision-making frameworks.

2.1.1 Concepts

These definitions are adapted from the ones provided in [Roy,2000]. In the rest of the document we will follow the notation introduced here.

Actor: Any individual, group of individuals or entity which can play a role, directly or not, in the decision process.

Decision-maker: Actor for whom the decision-aid tools are developed and implemented.

Analyst: Actor who is responsible for the decision-aid process.

Action: A generic term used to designate the object of the decision. In practice, the term action may be replaced by such terms as scenario, operation, investment or solution, depending on the situation. We will follow the notation: $A = a_1, a_2, \ldots, a_m$. This is, m different actions in the set of possible actions A.

Alternative: Action that can be implemented independently of the other actions. This term can be used instead of Action when this independence condition is fulfilled.

Potential Action: Action which could be implemented or which is interesting for the analysis during the decision process.

Point of view: A class of effects or attributes which share the same goal or the same type of concerns, thought pertinent by at least one of the actors for evaluating and comparing actions.

Scale: Set of elements, S, (called "degrees") ranked according to a complete order, reflecting the preferences of the decision-maker for a particular point of view.
Different scales can be considered according to the allowed operators on the set of elements. Some of the most common scales are: numerical, ordinal and categorical.

Vocabulary: Set of elements (or degrees) expressed using linguistic terms. We will refer to the vocabulary of a particular qualitative criterion as T.

Criterion: Application g from the set of actions to a scale, such that it appears meaningful to compare two actions a_1 and a_2 according to a particular point of view, on the sole basis of $g(a_1)$ and $g(a_2)$. We will follow $C = \{c_1, c_2, \ldots, c_p\}$ to denote the criteria, being g_j the function attached to c_j. Also, with v_{ij} we denote the value of $g_j(a_i)$. That is $v_{ij}=g_j(a_i)$.

Threshold: Value that is used to take into account the imprecision on the result of certain comparisons. It permits to establish the equivalence between two alternatives evaluated different in a given scale. For example, we can define an Indifference threshold, a Preference threshold or a Dispersion threshold.

Weight: Value that indicates the relative importance of one criterion in a particular decision process. It models the different roles that an

actor would like the different criteria to play in the elaboration and argumentation of comprehensible preferences.
The concrete interpretation of this concept depends on the method, as it will be illustrated later.

Ideal Point: Point in the criterion space that has the maximum value for each dimension.

Nadir Point: Point in the criterion space that has the minimum value for each dimension.

Preference relation: Binary relation that expresses how much an action is preferred over another one. Several scales can be used for expressing these preferences. A preference relation is an application R: $A \times A \rightarrow S$.

Preferential Independence: A subset of criteria C is preferentially independent of C^c (the complement of C) iff any conditional preference among elements of C, holding all elements of C^c fixed, remain the same regardless of the levels at which C^c are held.
A classical example of non preferential independence is the following one: we have two criteria g_1 and g_2, being g_1= {red wine, white wine}, g_2= {meat, fish}. Most people prefer red wine to white wine with meat, but white wine to red wine with fish. Therefore, preference in g_1 depends on g_2.

Cluster: Considering alternatives as points in a p-dimensional space, clusters may be described as continuous regions of this space containing a relatively high density of points, separated from other such regions by regions containing a relatively low density of points [Everitt,1977]. Clusters described in this way are sometimes referred to as *natural clusters*. Other definitions can be found in the literature, however, the advantage of considering clusters in this way is that it does not restrict the shape of clusters as rigidly as do other proposed definitions. For example, definitions suggesting that objects within a cluster should be closer to each other than to objects in other clusters restrict one to the consideration of spherical shapes.

2.1.2 Multiple criteria decision problem

Having a defined set A of actions and a consistent[2] family C of criteria on A, a multiple criteria decision problem is the one that, with respect to G, either aims to find:

a) a subset of A that contains the best actions,
b) an assignment of the actions into predefined categories, or
c) a rank of the actions in A from best to worst.

Each of these objectives defines a different multicriteria decision problem, called: (a) choice problem, (b) classification or sorting problem (depending on if the categories are preferentially ordered or not) and (c) ranking problem.

The main difficulty lies in the fact that it is an ill-defined mathematical problem because there is no objective or optimal solution for all the criteria. Thus, some trade-off must be done among the different points of view to determine an acceptable solution for the decision problem.

2.1.3 MCDM versus MCDA

Multicriteria decision making (or multiple criteria decision making, MCDM) can be understood a part of the more general area of research: Multicriteria decision aid (MCDA). MCDA develops tools to help decision-makers in solving a decision problem with several points of view that have to be taken into account. This is not an easy task because often these points of view are contradictory, consequently, it is not always possible to find a unique solution that is the best for all the points of view.

MCDA intends to give tools that allow the decision-maker to capture, analyse and understand these points of view, in order to be able to find the way in which the decision process must be handled. This is called a *constructivist* approach.

Multicriteria decision making (MCDM) has a more *descriptive* approach. In MCDM it is supposed that there exists "*something*" that will allow the decision-maker to determine which are the best alternatives. This is done using a utility function - if it can be discovered and described in mathematical terms - or using mechanisms based on comparisons among the different alternatives or options. Thus, the main goal is to observe the behaviour of decision-makers and try to help them to understand the mechanisms intrinsic into the decision process, as well as, to be aware of all the factors that influence the result.

MCDM is mainly developed in United States of America (known as American School), while the constructivist approach of MCDA is the one adopted by most of the European researchers (French School) [Roy&Vanderpooten,1996].

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[2] A set of criteria that is **exhaustive** ($g_j(a) = g_j(b),\ \forall j \Rightarrow$ no preference between a and b), **cohesive** ($g_j(a) = g_j(b),\ \forall j \neq k$ and a preferred to b for $g_k \Rightarrow a$ preferred to b) and **nonredun-dant** (leaving out one criterion leads to the violation of one of the previous requirements) is said to be **consistent**.

Inside the MCDA research area we can distinguish: multiple objective decision making (MODM) and multiple attribute decision making (MADM). The former deals with problems where the decision space is continuous. MODM has been widely studied with mathematical programming methods, which have a well-formulated theoretical frame in which this optimisation problem can be studied making different assumptions on the variables as well as on functions that define the model and constraints. More information on MODM can be found in [Hwang&Masud, 1979], [Slowinski&Teghem, 1990], [Lai&Hwang, 1996] and [Ehrgott&Gandibleux, 2002]. Recently, evolutionary algorithms have been applied to MODM and they seem more appropriate to deal with problems with multiple solutions than conventional optimisation techniques [Fonseca&Fleming, 1995]. The second type, MADM, concentrates on problems with discrete decision spaces, in which the alternatives have been predetermined in advance. In the literature, it is usual to use MCDM or MCDA to refer only to the second class of problems, MADM, which is the one we are working on.

Since the beginning of the MCDA research field, many different methods have been proposed. Each method has its own characteristics and there are many ways one can classify them. For example, we can separate methods with a *single* decision maker and methods with a *group* of decision makers. The methods involving more than one decision maker are included in the research field of Group Decision Making and Negotiation (an introduction to the field can be found in [DeSanctis&Gallupe,1987] or [Jelassi et. al.,1990]). Another classification distinguishes *deterministic*, *stochastic* and *fuzzy* methods. The deterministic approach considers that the decision making problem (i.e. the alternatives, criteria, etc.) are perfectly described before applying the decision method. The stochastic or probabilistic case corresponds to a type of modelling in which the criteria are viewed as random variables. Finally, fuzzy methods consider different types of uncertainty and imprecision in some of the elements of the decision making problem. In section 2.5, we will give some details about uncertainty in MADM.

MADM or MCDM methods are also classified into two distinct families: *Aggregation* approaches (based on the Multi-Attribute Utility Theory) and *Order-focussed* approaches (based on Outranking relations). In the following sections we will explain the main ideas of these approaches for the deterministic case. Last section is devoted to the review of methods that allow imprecision and uncertainty, because our work is focused on the case of single decision maker MADM problems with uncertainty in the values of the criteria.

2.2 Multiattribute utility theory

Multiple Attribute Utility Theory (MAUT) has its bases in the philosophical doctrine of the Anglo-Saxon culture called Utilitarism. It was introduced in Economy by Von-Neumann and Morsgenstern to model the behaviour of economic agents. In the 60's these concepts were introduced to the decision making field (two interesting references are [Fishburn,1970] and [Keeney&Raiffa,1976]). MAUT is based on the idea that any decision-maker attempts unconsciously to maximise some function that aggregates the utility of each different criterion.

$$U = U(c_1, c_2, ..., c_p) \qquad \text{Eq. 2.1}$$

In MAUT, data is usually provided through a decision matrix, with alternatives as rows and criteria as columns (see Table 1). The values in this decision matrix can be provided by a single expert (i.e. an actor) or by different ones.

	c_1	c_2	...	c_p
a_1	v_{11}	v_{12}	...	v_{1N}
a_2				
...				
a_m				

Table 1. Decision matrix

Different models exist according to different expressions for function U in Eq. 2.1. The simplest model considered in MAUT is the additive one. Here, U is an additive combination of utility of the criteria. This is, the function U is expressed as:

$$U(a) = \sum_{j=1}^{n} U_j(c_j(a))$$

where U_j (the utility function of criterion c_j) is a strictly increasing function that returns values in a common scale, in order to allow the criteria to be compared and added without problems with different units of measurement. Moreover, additional conditions must be fulfilled to use this model [Vincke,1992]: each criterion must be a preference relation that induces a complete preorder, and any subset of criteria must be preferentially independent.

It has to be noted that in the additive model, other combination functions than the addition can be used to combine the utility function U_j. In particular, U can be defined as the arithmetic mean or the weighted mean of the U_j.

Apart from the use of additive utility functions, it is also possible to use other utility functions, such as the multiplicative utility one. The multiplicative model enables the

consideration of the interactions among the different criteria. This model is expressed as:

$$U(a) = \prod_{j=1}^{n} U_j\left(c_j(a)\right)$$

where, as before, U_j is the utility function for criterion c_j.

A key issue in utility-based approaches is the determination of the marginal utility functions, U_j. These functions transform the scale of the corresponding criterion into numerical utility values. The construction of these functions is a difficult issue. The usual case is to build them from the information provided by some domain experts. In this case, the process of eliciting the parameters of the model is usually done through an interactive interrogation procedure (known as *Direct Methods*). However, this is a non non-easy and time-consuming process. For this reason, research on alternative methods not requiring the intensive participation of the experts have also been considered in the literature (*Indirect Methods*), where the utility functions are estimated on the basis of the global judgements made by the decision-maker on the alternatives. [Fishburn,1967] and [Vincke,1992] describe several methods for function estimation.

Once the U_j are known, the MAUT methods consider two steps to be followed [Chen&Klein, 1997], [Henig&Buchanan, 1996]:

- *Aggregation (rating)*: a global value for each alternative is computed, *U(a)*, which gives a general idea of the utility of the alternative considering all the criteria at the same time;
- *Ranking or sorting*: the utility values obtained in the first step are used to find the best alternative, to rank them or to classify the alternative into some predefined groups.

When possible, different measures of interpersonal agreement or individual consistency are applied in order to give more information to the decision maker about the characteristics of the decision problem.

Another model based on the MAUT principles is the Analytic Hierarchy Process (AHP) developed by Saaty in 1980 [Saaty,1980]. In this model, the MCDM problem is decomposed into a system of hierarchies from which a $m \times p$ matrix is built. The matrix is constructed by using the relative importance of the alternatives in terms of each criterion separately. Each row of this matrix is the principal vector of an $p \times p$ reciprocal matrix determined by pairwise comparisons of the impact of the m alternatives on the i-th criterion.

A comparison of these three models (the additive, the multiplicative and the AHP) can be found in [Triantaphyllou,2000].

2.3 Outranking methods

The outranking approach was introduced in the 60s by Roy based on his work on real-world applications. The intention was to overcome some of the difficulties of the aggregation approaches of those days, such as the use of qualitative criteria.

This approach focuses the attention to the fact that in MCDA problems one tries to establish preference orderings of alternatives ([Roy,1991], [Perny&Roy,1992]). As each criterion usually leads to different ranking of the alternatives, the problem is to find a consensued ranking. The outranking methods perform pairwise comparisons of alternatives to determine the preferability of each alternative over the other ones for each particular criterion. Then, a concordance relation is established by aggregating the relative preferences. Moreover, a discordance relation is also established, which is used to determine veto values against the dominance of one alternative over the others. Finally the aggregation of the concordance relation yields the final dominance relation.

The basis of these methods is the definition of an outranking relation *S*. By definition, *S* is a binary relation: *a'Sa* holds if we can find sufficiently strong reasons for considering the following statement as being true in the decision maker's model of preferences:

" *a*' is at least as good as *a* "

The reasons for validating this assertion have to be found in the criterion space. Two conditions must be fulfilled in order to accept that *a'Sa* holds:

1st. A *concordance* condition: a majority of criteria must support *a'Sa* (classical majority principle)

2nd. A non *discordance* condition: among the non concordant criteria, none of them strongly refutes *a'Sa* (respect of minorities principle)

There are different ways of implementing these conditions and different levels of requirement. Let us explain them in more detail.

Concordance is measured in two steps. Firstly, we measure the contribution of each criterion, c_j, to the outranking relation *a'Sa*. We define the partial concordance of one criterion so that it follows these two conditions: concordance is 1 when the j[th] criterion fully supports *a'Sa* and concordance is 0 when the criterion does not support *a'Sa* at all.

$$concordance_j(a',a)=\begin{cases} 1 & \text{if } c_j(a') \geq c_j(a)-q_j \\ 0 & \text{if } c_j(a') \leq c_j(a)-p_j \\ \dfrac{p_j-\left(c_j(a)-c_j(a')\right)}{p_j-q_j} & \text{if } c_j(a)-p_j \leq c_j(a') \leq c_j(a)-q_j \end{cases}$$

where p_j is the preference threshold and q_j is the indifference threshold of the j[th] criterion. These thresholds define 5 different intervals in the domain of preference of the criterion, as it is shown in Figure 1: P_j means "strict preference", Q_j is "weak preference" and I_j corresponds to "indifference".

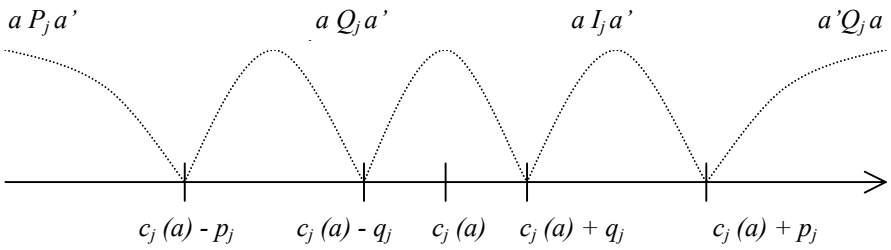

Figure 1. Thresholds in a criterion

Secondly, the overall concordance value is obtained using the partial concordances. We can use the weights associated to each criterion, w_j, to adjust the influence of each of them.

$$Concordance(a',a) = \sum_{j=1}^{p} w_j \cdot concordance_j(a',a)$$

With respect to the discordance condition, outranking methods use the discordance measurement to introduce the opportunity of the non concordant criteria to express their strong opposition, a *veto*, denoted v_j.

If $c_j(a') < c_j(a) - v_j$, for some criterion c_j, then *a'Sa* is rejected.

In these methodologies, the decision process has three steps:

α: isolate the smallest subset of alternatives $A_0 \subset A$ liable to justify the elimination of all actions belonging to $A \setminus A_0$;

β: assign each alternative to an appropriate pre-defined category according to what we want it to become afterwards;

γ: build a partial (or complete) pre-order on the subset A_0 of those among the alternatives of A.

Different methods implement this process using different algorithms. Some of the most well-known outranking models are ELECTRE, PROMETHEE, MAPPAC and PRAGMA (see e.g. [Bana e Costa,1990]).

2.4 Another approach: the Rough Sets Theory

Despite that the two major models used in MCDA are the ones based on Utility functions and Outranking relations, there are other approaches that face up the problem from other perspectives. In this section, we will give some details about the Rough Sets approach, because there are some similarities between their ideas and goals and the ones of the methodology we propose, ClusDM.

The rough sets theory was formulated by Pawlak [Pawlak,1982] to deal with inconsistency and vague description of objects. The theory is based on the concept of indiscernibility relation, which induces a partition of the objects into blocks of indiscernible (i.e. indistinguishable) objects, called elementary sets. Being X the universe of discourse, any subset Y of X can be expressed in terms of these blocks either precisely or approximately. In the second case, the subset may be represented by two sets called the *lower* and *upper approximations* of Y. A rough set is then defined using these approximation sets.

The lower and upper approximation sets are built from a data matrix of examples. In decision making, an example is formed by a description of an alternative in terms of different criteria and the final decision value given to the alternative by the decision maker after solving the problem. That is, if we use the concepts of machine learning, the rough sets approach is a *supervised method*, because we require the knowledge of some solved problems in order to build a model to solve new ones. In fact, the rough sets methodology was introduced as a method to infer decision rules from a set of examples.

An interesting characteristic of the rough set approach is that it is possible to deal with heterogeneous data sets without having to use a unified domain. The rules are generated from the analysis of the elements in the lower, upper and boundary approximations of the different solutions. That is, the values of the elements in these sets (in spite of the type and domain) define the conditions of the rules for the different conclusions (i.e. decision results).

Until now, we have introduced the classical rough set approach using an indiscernibility relation. However, there are some generalisations of the method to deal with fuzzy sets, fuzzy indiscernibility relations [Dubois&Prade,1990], or to substitute indiscernibility by a weaker binary similarity relation or even a fuzzy similarity relation [Greco et. al.,1998]. Another generalisation refers to the treatment of missing values. The classical approach requires a complete data matrix, while other works have relaxed this condition to allow the presence of missing values [Greco et. al.,2000].

The application of rough sets to multiple criteria decision making began in the 90's [Slowinski,1993]. The original rough set approach is not able, however, to deal with preference-ordered criteria and decision classes. Moreover, the rough sets theory is devoted to classification problems, while MCDA also deals with choice, ranking and sorting situations. In fact, initially it was only used in MCDA classification applications [Pawlak,1997].

In [Greco et. al.,2001] there is a good explanation of how rough sets theory can be adapted to deal with the particular characteristics of sorting, choice and ranking decisions. The main modification is the substitution of the indiscernibility relation by a dominance relation. Indiscernibility is not able to deal with ordinal properties. In

particular, it can not detect some inconsistencies. For example, alternative a_1 is better than a_2 with respect to all the considered criteria, but the decision maker considers than a_1 is worse than a_2 as a solution to the decision problem. In order to detect this inconsistency, the rough approximation must handle the ordinal properties of the criteria. This can be naturally achieved with dominance relations[3]. In the case of multicriteria choice and ranking problems, other extensions are needed because the data matrices used in the classical rough sets theory do not allow the representation of preferences between alternatives. Here, Greco et al. propose to operate on pairwise comparison tables, where rows represent pairs of alternatives for which multicriteria evaluations and the global preference relation are known.

Therefore, we can see that the rough set theory is also useful in MCDA. Moreover, some of its characteristics are also the goals that we will face up in this thesis: the integration of heterogeneous criteria, the possibility of having missing values and the explanation of the result in a language that is easy for the decision maker.

The main differences between the rough set approach and the one presented in this thesis are: (i) the type of information required to the decision maker and (ii) the type of result obtained. The rough set case needs to have a set of solved decision examples in order to build the rules that explain how to make decisions in the future. In our proposal, we directly deal with the data of an unsolved decision making problem and find the solution for this particular case. Using AI terminology: rough sets is a supervised method while ClusDM is unsupervised. Regarding the type of result, rough sets build a set of decision rules that can help the decision maker to solve future problems, however, the new problems must be similar to the ones used to build the model in order to be able to apply the same rules. In our case, the method is applied to a particular decision case. The solution is also expressed in a language that is familiar to the decision maker, together with additional information that can help him to understand the problem (e.g. alternatives with conflicting values or criteria that do not agree with the majority).

[3] *x dominates y* if *x* is at least as good as *y* for all criteria.

2.5 MCDM with imprecision and uncertainty

Traditionally, MCDA was concerned about decisions under certainty. That is, the parameters and values needed are known with certainty. Later on, the MCDA community became aware of the necessity to develop methods that were able to handle uncertain information. Here, we have to distinguish the study of decisions under risk from the study of decisions under uncertainty, understood as vagueness.

In decision making under risk, the lack of information is about the occurrence of the "state". These problems are usually handled with stochastic programming or Bayesian analysis, because the probability of occurrence of the states is known. On the other hand, we have situations with components that are intrinsically vague, called uncertain. In particular, the management of uncertainty is important in: (1) the evaluation of the alternatives with respect to the criteria, that is, v_{ij} are not known with certainty, and (2) assessing the relative importance of criteria (i.e. the weights w_j).

In this work we are not concerned with risky situations, but rather with the uncertainties as vagueness. This uncertainty arises due to different situations:

- unquantifiable information: some properties can not easily be described using numbers, then linguistic terms are usually used. For example, the comfort of a car can be evaluated with terms as good, fair, poor, etc. This type of criteria is called qualitative.

- incomplete information: obtaining a precise numerical value for some measurements is sometimes a difficult task, because the measurement equipment is not precise enough, such as the velocity of a car.

- non obtainable information: when the methodology involved in a measurement is complex and time consuming approximations of the value are used.

- partial ignorance: the experts that provide the data do not always know all the details of all criteria for all alternatives. This natural ignorance about some criteria or alternatives introduces imprecision in the global process.

The research that attempts to model imprecision into decision analysis is done basically with probability theory or fuzzy set theory [Lai&Hwang,1996]. Probability theory is claimed not to capture the human behaviour, because it models the imprecision considering random or stochastic processes in a statistical way, as if the lack of precision was a matter of randomness. On the contrary, fuzzy set theory [Zadeh,1968] models imprecision in a more human-like way, taking into account the subjectivity of the expert rather than employing only objective probability measures [Zadeh,1978].

In [Bana e Costa, 1990] an introduction to the problem of uncertainty in decision making is done. The basic ideas and problems of the management of uncertainty in the outranking utility approaches are explained. In this section, we will concentrate on the utility approach. More information about fuzzy preference relations and its aggregation

operators can be found in [Orlovski,1978], [Kackprzyk&Fedrizzi,1990], [Fodor&Roubens,1994], [Chiclana et. al.,1998] and [Zapico,2000].

As it has been said in the definition of an MCDA problem, the decision maker faced one of 3 type of goals: (i) choice, (ii) classification or sorting and (iii) ranking. For each case, different methodologies are required. [Zopounidis,2002] makes a good review of methods for the classification and sorting tasks. In the rest of the chapter, we will focus on the ranking and choice problems.

When the utility approach is taken, uncertainty is usually understood as fuzzy evaluations of the alternatives with regard to the criteria. A survey of the fuzzy MCDM methods is done in [Ribeiro,1996]. In Table 2 there is a summary of different approaches to deal with fuzziness. In the column named 'Phase' it is distinguished whether the method deals with the aggregation stage (I) or the ranking phase (II). In the next columns, the nature of both the criteria, the weights is indicated and the solution obtained.

Regarding the criteria, we must distinguish two approaches to fuzziness. We may consider a criterion c_j as a fuzzy set, so that the values v_{ij} indicate the membership degree of the alternative a_i to this fuzzy criterion. For example, the comfort of a car can be evaluated using fuzzy set, and each car has a degree of comfort expressed in the interval [0,1]. On the other hand, we may consider the possible values of a criterion c_j as being uncertain, that is, v_{ij} are linguistic terms. For example, the comfort of a car can have a domain with the values "good", "not-bad" and "uncomfortable". In this case, each linguistic term is a fuzzy set.

Aggregation rule	Phase	Criteria	Weights	Solution	Authors
OWA operators	I + II	fuzzy	crisp	crisp	Yager [Yager,1988]
Evidential logic rule	I	fuzzy	crisp	crisp	Baldwin [Baldwin,1994]
Choquet integral	I	fuzzy	fuzzy	crisp	Choquet [Choquet,1968]
Sugeno integral	I	fuzzy	fuzzy	crisp	Sugeno [Sugeno,1974]
Hierarchical aggregation	I	crisp	crisp	fuzzy	Laarhoven& Pedrycz [Laarhoven& Pedrycz,1983]
Max min	I + II	fuzzy	crisp	crisp	Bellman & Zadeh [BellmanZadeh,70]
Max min	I + II	fuzzy	crisp or fuzzy	crisp	Yager [Yager,1978; Yager,1981]
Weighted average (WA)	I + II	fuzzy	fuzzy	fuzzy	Baas&Kwakernaak [Baas&Kwakernaak ,1977]
WA. Extension principle + α-cuts + intervals	I	fuzzy	fuzzy	fuzzy	Dong, Shah & Wong [Dong&Shah,1985; Dong&Wong,1987]
WA. Approximate extension principle		fuzzy	fuzzy	fuzzy	Dubois & Prade [Dubois&Prade,80]
WA. Extension principle		fuzzy	fuzzy	fuzzy	Schmucker [Dong,1985]
Weighted average	I	fuzzy	fuzzy	crisp	Tseng & Klein [Tseng&Klein,92]

Table 2. Aggregation operators used MCDM (adapted from [Ribeiro,1996])

Before coming into details of the methods, it is important to notice that the *complexity* of the ranking phase depends on the type of the result of the rating phase. That is, if the result of the aggregation is a crisp value, the ranking is straightforward (just select the alternative with the highest value), on the other hand, for other types of results this process can be difficult. For instance, when the result is a fuzzy set, a ranking method to order them must be used. However, there is not a unified methodology for ordering fuzzy sets (see [Klir&Yuan,1995] for details).

The first four methods in Table 2 consider the first approach, so v_{ij} are membership degrees (i.e. crisp numbers), while the rest of the methods use the second approximation, in which the values v_{ij} are linguistic terms with a fuzzy set that gives us its semantics.

Yager uses the OWA operator [Yager,1988], which is an aggregation operator that averages the values giving different weights to the values rather than weights to the criteria. Baldwin proposes to do a simple weighted average and then use a linguistic filter to obtain the level of satisfaction of the criteria. The filters are fuzzy sets as "most", "all", "few", etc. Choquet and Sugeno define fuzzy integrals to make the consensus of the fuzzy values; the weights are given by fuzzy measures of the form: m: $\wp(C) \rightarrow [0,1]$, which define the importance of any subset of C. Several characterizations of Choquet integrals are available, see e.g. [Narukawa&Murofushi, 2002]. In addition, T-conorm fuzzy integral [Murofushi&Sugeno, 1991] generalize Choquet and Sugeno integrals.

The method proposed by Laarhoven and Pedrycz in 1983 is a variation of Saaty's method AHP for dealing with uncertainty. Saaty used the classification trees to deal with intermediate values like "about three". Laarhoven and Pedrycz fuzzify the crisp values obtained from pairwise comparisons, as in Saaty's approach, and use the approximate algorithms of Dubois and Prade to perform the algebraic operations on the fuzzy numbers.

Bellman and Zadeh gave, in 1970, a max-min approach to the aggregation process. The final result is a fuzzy set whose membership function is the degree to which an alternative is a solution. This membership is obtained from the following aggregation function:

$$\mu_D(a_i) = min\left(w_1 * \mu_{c1}(a_i), w_2 * \mu_{c2}(a_i), ..., w_n * \mu_{cn}(a_i)\right)$$

$$\text{with } \sum_j w_j = 1$$

The ranking phase will choose the alternative a_i with the maximum membership to the decision fulfilment. That is the reason why the process is called Max-Min.

Yager assumed the Bellman and Zadeh's max-min principle, but the importance of the criteria is represented as exponential scalars. This is based on the idea of linguistic hedges of Zadeh [Zadeh,1983], which are assigned according to linguistic variables (e.g. μ^2 corresponds to "very"). Formally,

$$\mu_D(a_i) = min\left(\mu_{c1}(a_i)^{\alpha 1}, \mu_{c2}(a_i)^{\alpha 2}, ..., \mu_{cn}(a_i)^{\alpha n}\right) \text{ for } \alpha > 0$$

The rest of the approaches (Baas and Kwakernaak, Dong et al., Dubois and Prade, Schmucker, and Tseng and Klein) propose different methods to compute a weighted average. They deal with fuzzy values and fuzzy weights, so the arithmetic operations needed to calculate the average must be defined for fuzzy numbers. Baas and Kwakernaak formalise the problem as a continuous differentiable function, whose largest maximum is found through the calculus of derivatives. These derivatives are used to calculate the weighted average. Dong, Shah and Wong calculate the weighted average for some α-cuts of the fuzzy sets, using interval operations and the extension principle. From the results obtained for each α-cut the final fuzzy set is built. Dubois and Prade proposed an approach based on the L-R approximation (triangular fuzzy sets are represented with three numbers: *l*, *m* and *u*, corresponding to the lower, medium and upper bounds, respectively; accordingly, the arithmetic operations are redefined

using the tuples (*l,m,u*)). Schmucker discretizes the fuzzy numbers into a finite set of points, then calculates their discrete weighted average, and finally approximates the resulting fuzzy set. However, this method has problems since not all algebraic operations result in convex fuzzy numbers. Tseng and Klein gave, in 1992, an approach based on the idea of transforming the fuzzy linguistic values into numeric values by means of a defuzzification process (they use the centre of the area covered by the fuzzy number). When crisp numbers have been obtained the aggregation process belongs to the numerical case.

Another approach is to use the order among the linguistic values in a fuzzy criterion, instead of using fuzzy sets or probability theory. Methods that use this approach are given in Table 3.

In the previous techniques (the ones in Table 2), when the original values are linguistic labels in a certain set (each corresponding to a fuzzy set), the fuzzy set obtained may not correspond to any of the linguistic terms in the original term set. Thus, a linguistic approximation process is needed to find the most suitable linguistic term [Herrera&Herrera-Viedma,1997]. This process consists of finding a label whose meaning is the same or the closest (according to some metric) to the meaning of the membership function obtained after the aggregation. In order to avoid this problem, the methods in Table 3 combine the values by direct computation on labels.

Aggregation rule	Phase	Criteria	Weights	Solution	Authors
Plurality rule	I	ordinal linguistic	crisp	set of linguistic labels	axiomatization in [Roberts,1991]
Median	I	ordinal linguistic	crisp	linguistic label	median based operators in [Domingo&Torra,02c]
LOWA / LWD, LWC, LWA	I + II	ordinal linguistic	crisp/ ordinal linguistic	linguistic label	Herrera et al. [Herrera&Herrera-Viedma,1997]
WM ordinal OWA	I + II	ordinal linguistic	crisp	linguistic label	Yager [Yager,1998]
Sugeno integral	I+II	ordinal linguistic	ordinal linguistic	ordinal linguistic	Sugeno [Sugeno,1974; Marichal&Roubens,99]
QWM,QOWA, QWOWA, QChoquet Integral	I + II	ordinal linguistic	ordinal linguistic	linguistic label	Godo & Torra [Godo&Torra,2000; Godo&Torra,2001]
2-tuple WA 2-tuple OWA	I	ordinal linguistic	crisp	linguistic label	Martínez&Herrera [Herrera&Martínez,00b]
antonym-based aggregation	I	ordinal linguistic	-	linguistic label	Torra [Torra,2001]

Table 3. Aggregation operators for ordinal linguistic values

The first approach is known as Plurality rule or Plurality function, and corresponds to the selection of the most frequent label. In fact, the definition does not return a single label but a set of labels that appear more often.

The LOWA operator is a Linguistic version of the OWA operator [Yager,1988]. This method assumes an implicit numerical scale underlying the ordinal linguistic one. Then, if all the criteria take values in an ordinal qualitative scale $L=\{l_l,...,l_r\}$, the Linguistic OWA of the linguistic values of an alternative a_i, with respect to a weighting vector W is recursively defined as:

$$C^m(W, v_i) = C^2\left((w_1, 1-w_1), \left(a_{i,\sigma(j)}, C^{m-1}(W', v_i')\right)\right) \text{ for } m > 2$$

with $w_i \in [0,1]$ and $\sum_j w_j = 1$.

Where $v_i' = (v_{i,\sigma(2)}, ..., v_{i,\sigma(n)})$ and $W' = (w_2/(1-w_1), ..., w_n/(1-w_1))$, and $C^2((w_1, w_2), (v_{i,1}, v_{i,2})) = l_k$, where

$$k = min\left(r, height\left(v_{i,\sigma(2)}\right) + round\left(w_1 \cdot \left(height\left(v_{i,\sigma(1)}\right) - height\left(v_{i,\sigma(2)}\right)\right)\right)\right).$$

In these expressions, σ is a permutation of v_i such that $v_{i,\,\sigma(j)} >= v_{i,\,\sigma(j+1)}$, and $height(v_{i,j})$ returns the position of the label within the scale L.

This method carries out an implicit conversion of the labels into the natural numbers corresponding to their position.

The LWD (linguistic weighted disjunction), LWC (linguistic weighted conjunction) and LWA (linguistic weighted averaging) are operators that consider linguistic weights for each criterion. The formulation follows Yager's Min and Max operators based on T-conorms and T-norms, respectively (see [Herrera&Herrera-Viedma,1997] for more details).

Yager's operators [Yager,1998] for qualitative values are based on the idea of the median, that is, the result is the value which is in the median position among the other values to aggregate. The Weighted Median (WM) corresponds somehow to a weighted average, and the Ordinal OWA operator replaces the classical arithmetic weighted mean by the weighted median in the OWA definition. However, as both operators are based on the median, they force the result to be one of the values that are combined, which is not always desirable.

In [Marichal&Roubens,1999] the use of the Sugeno integral as an aggregation operator for multiple criteria is analysed. It is proved that this measure has some desirable properties of aggregation operators (it is a fuzzy measure, is continuous, is idempotent in the first n arguments and is comparison meaningful for ordinal scales).

In [Godo&Torra,2000], a set of qualitative weighted mean-like operators are defined. Their main characteristic is that it is not necessary to use any kind of numerical interpretation of the qualitative (i.e.linguistic) values. They re-define all the arithmetic operations needed to apply some numerical aggregation operators (WM, OWA and WOWA) to handle linguistic terms in an ordered domain. In [Godo&Torra,2001] the extension of the Choquet integral to ordinal values is done.

The Choquet measure allows the user to express the interactions between the sources, which cannot be done with the WM, OWA and WOWA operators.

A different approach is the one presented by Martínez and Herrera. A different representation of ordinal linguistic vocabularies is given. In this approach, the semantics is implicit, that is, it is encoded in the aggregation operators. They define a 2-tuple as pair (s_i, α_i), where s_i is a linguistic label and α_i is a number in [-0.5, 0.5], which indicates the distance to the closest label. Some functions to translate 2-tuples into numerical values and viceversa are given. With this functions, some classical numerical operators are redefined for the case of 2-tuples.

In 1996 another way of giving semantics to a qualitative vocabulary was defined in [Torra,1996]. It is based on the concept of antonyms: we can infer the meaning of a term if we know the terms that express an opposite value. In [Torra,2001] an operator to aggregate data described with different vocabularies is explained. It is based on building a unified vocabulary and putting the original values to this common one. In this thesis, we will explain a new methodology that uses this concept of antonyms but avoiding the necessity to work with a common vocabulary.

Chapter 3.

ClusDM (Clustering for Decision Making)

This chapter explains the new multi-criteria decision aid methodology we propose, called *ClusDM*, which stands for *Clus*tering for *D*ecision *M*aking. Its name comes from the use of clustering algorithms to solve the decision-making problem, as it will be explained in this chapter.

This methodology has been designed for dealing with heterogeneous data sets because there is a lack of MCDA tools for this type of problems. One of the key points of this method is that it can deal with different types of variables during all the stages of the decision-making analysis. As it has been explained in the previous chapter, the existing approaches perform a transformation of the original data into a common domain. In our method, we are always dealing directly with the data provided by the experts, in order to avoid the modification of the information available in those data.

Although we will explain our method as a ranking decision tool, it can also be used to solve selection decision problems. In fact, a selection problem can be seen as a subtype of ranking problems in which we are only interested in distinguishing the group of best alternatives.

In this chapter we will explain part of the ClusDM methodology. Before starting this explanation, section 3.1 is devoted to describe the scales we use. Then, in section 3.2 we give an outline of the ClusDM methodology, giving some details of the four stages of the process: Aggregation, Ranking, Explanation and Quality measurement. Section 3.3 is devoted to the explanation in detail of the aggregation stage. The rest of the stages will be explained in the following chapters.

3.1 Considerations on the scales in ClusDM

It has been reviewed in Chapter 2 that the evaluation of alternatives in relation to a given criterion can be done in many different scales. The most common MCDA methods deal with a single common scale. ClusDM is a general methodology that is able to handle heterogeneous criteria. In our design and implementation of the methodology, we have considered the following ones:

- quantitative or numerical scale
- ordered qualitative or ordinal scale (i.e preference values)
- non-ordered qualitative scale (i.e. nominal or categorical values)
- Boolean scale (i.e. binary values)

Although we have restricted ourselves to these types of values, we would like to note that any other type of value that has a distance function defined in its domain could also be used.

To operate on the values of these scales, in particular to compute similarities between pairs of values, some assumptions are needed on the semantics of the values. In the case of quantitative, categorical and Boolean scales, the definition of distances or similarities has been widely studied (we will review some possibilities in section 3.3.2). For the case of ordered qualitative values, we can find in the literature several approaches to the definition of the underlying semantics of the scale, which is the basis for the similarity and aggregation operations [Torra,2001].

Explicit semantics: A mapping exists that translates each linguistic term in a numerical or fuzzy value. Operations on the linguistic values are defined on terms of the corresponding operations in the numerical or fuzzy scale.

Implicit semantics: Operations are defined assuming an implicit mapping function from the original scale into a numerical one. The typical case is to replace each term by its position in its domain.

Operations restricted on the ordinal scale: New operations in a given scale are only defined in terms of operations axiomatically defined in that scale. Allowed operations are maximum, minimum, t-norm, t-conorm and operations defined from them.

Working on any of these settings present advantages and disadvantages:

- In the case of explicit semantics, operations are well defined and sound. However, the experts are required to supply additional information, in particular, they must provide a mapping for each scale.
- Implicit semantics provide easy to use operations but, instead, semantics is coded - and fixed- in the operators. Counterintuitive results can be obtained if the application does not follow the assumptions considered.
- Operators restricted on the ordinal scale also lead to sound results. Nevertheless, some of the basic operations are difficult to be defined by non-experienced users, as their meaning is sometimes difficult to grasp. This is the case of defining ordinal t-norms and t-conorms.

ClusDM uses a negation-based semantics. This can be seen as an alternative to the explicit semantics approach as it builds an explicit mapping from the set of linguistic terms into the unit interval. This mapping is inferred from a negation on the set of terms. This approach avoids the use of operators with coded semantics. Now the user is only required to supply a negation function instead of a complete explicit mapping from terms to numbers. This approach is easier for the experts because the negation of a term can be interpreted as its antonym, following [de Soto&Trillas,1999].

In the rest of this section we describe the negation functions we consider and how the semantics is inferred.

Negation based semantics for linguistic terms

Negation is a well-known operation in multi-valued logics that is defined over a set of ordered linguistic labels (i.e. terms) T=$\{t_0, ..., t_n\}$ (with $t_0<...<t_n$). It is axiomatically defined as a function from T to T, that satisfies the following conditions:

N1) $\text{if } t_i < t_j \text{ then } N(t_i) > N(t_j)$ for all t_i, t_j in T

N2) $N(N(t_i)) = t_i$ for all t_i in T

In fact, when these conditions hold, the set of ordered linguistic terms T completely determines the negation function. This is so because for each set of ordered linguistic terms T=$\{t_0, ..., t_n\}$ there exists only one negation function that satisfies N1 and N2 [Agustí et al.,1991]. This negation function is defined by:

$$N(t_i) = t_{n-i} \quad \text{for all } t_i \text{ in T}$$

According to this last result, when conditions N1 and N2 are required, the negation function assumes vocabularies where each term in the pair $< t_i, t_{n-i} >$ is equally informative. Although in decision making, equal informativeness is sometimes not adequate, it is not always possible for the expert to define an interval or a fuzzy set for each term because that would require a degree of accuracy that the expert cannot always supply. To allow non-equal informativeness without requiring experts to supply detailed information on the semantics of the terms, [Torra,1996] introduced a new class of negation functions over linguistic terms. With this approach an expert can provide additional information about the meaning of the terms in a more natural way. These new negation functions are defined from T to $\wp$(T) (i.e., parts of T) weakening conditions N1 and N2.

Definition 1. [Torra,1996] A function *Neg* from T to $\wp$(T) is a negation function if it satisfies:

C0) *Neg* is not empty and convex

C1) $\text{if } t_i < t_j \text{ then } Neg(t_i) \geq Neg(t_j)$ for all $t_i, t_j \in$ T

C2) $\text{if } t_i \in Neg(t_j) \text{ then } t_j \in Neg(t_i)$

In this definition C1 and C2 are generalisations, respectively, of N1 and N2. In fact C2 is a generalisation of N3 (given below) that is equivalent to N2.

N3) if $t_i = Neg(t_j)$ then $t_j = Neg(t_i)$

C0 is a technical condition. It means that for all t_i in T, $Neg(t_i)$ is not empty ($Neg(t_i) \neq \emptyset$) and convex (a subset X of T is convex if and only if for all t_x, t_y, t_z in T such that $t_x < t_y < t_z$ and $t_x, t_z \in X$ then $t_y \in X$). In other words, C0 establishes that $Neg(t_i)$ is a non-empty interval of terms in T.

Now, let us turn into the semantics. For a vocabulary T, the semantics of a term is understood as a subset of the unit interval. Let $I(t_i)$ be the subset attached to term t_i; in this case the set $P = \{I(t_0),...,I(t_n)\}$ corresponds to the semantics of all terms in T. It is assumed that the sets recover the unit interval and that the intersection of any two sets is empty or punctual (if they are contiguous). That is, $\cup_{I \in P} I = [0,1]$ and $I(t_i) \cap I(t_j) = \varnothing$.

However, not all partitions in the unit interval are adequate as semantics for a set of linguistic labels. In fact, the relations among labels that a negation function establishes should also be true in the intervals in *P* when the negation in the unit interval is considered. In particular, the consistency of *P* in relation to the most usual negation function N(x)=1-x was mathematically defined. Informally, when consistency is required, the following two conditions hold: (i) the negation of all the elements of the interval attached to t_i belongs to the intervals attached to the negation of t_i; (ii) if $Neg(t_i) = \{t_{i0},...,t_{ik}\}$, then neither the term t_{i0} nor the term t_{ik} are "superfluous" in relation to the negation function. This latter condition means that there exists at least one element of the interval attached to t_i such that its negation belongs to $I(t_0)$ (respectively to $I(t_{ik})$). Given a negation function, there are several consistent semantics. In particular, the following one (which is the one we are going to use) is consistent with N(x)=1-x:

Definition 2. [Torra,1996] Let *Neg* be a negation function from T to $\wp$(T), according to Definition 1; we define P_{Neg} as the set $P_{Neg} = \{[m_0, M_0],...,[m_n, M_n]\}$ where

$$I(t_i) = [m_i, M_i] = \left[\frac{\sum_{t<t_i} |Neg(t)|}{\sum_{t \in T} |Neg(t)|}, \frac{\sum_{t \leq t_i} |Neg(t)|}{\sum_{t \in T} |Neg(t)|} \right] \qquad \text{Eq. 3.1}$$

where |X| stands for the cardinality of the set X.

It is important to note that that the classical semantics is obtained when the negation function is restricted to satisfy $|Neg(t_i)| = 1$. In that case, $I(t_i) = [i/(n+1), (i+1)/(n+1)]$,

which corresponds to having all the intervals with the same measure (i.e., the same precision). According to that, this approach extends the classical negation functions for multi-valued logics and relates them with the usual implicit semantics (note that the central point of the interval $I(t_i)$, $(i+1/2)/(n+1)$, is proportional to the position of the term t_i normalized in [0,1]: i/n.

3.2 The ClusDM methodology

In this section we will introduce a methodology for multi-criteria decision aid, which follows the utility-based model. As it has been explained in section 2.2, these multi-criteria decision methods distinguish two different stages: (1) the aggregation of alternatives and (2) their ranking. Our methodology follows the same strategy but we have included two additional stages: (3) an explanation stage to give semantics to the ranking obtained, and (4) an evaluation stage to measure the quality of the result. With these new stages we want ClusDM to be a useful decision aid more than a simple decision making procedure. That is, our goal is to give recommendations to the user rather than make an automatic decision.

Therefore, the ClusDM methodology distinguishes the following steps:

> STAGE 1. *Aggregation or Rating Phase*: The values of each alternative are analysed in order to find another evaluation for the alternative that allows us to compare it with the others and decide which one is the best.
>
> STAGE 2. *Ranking Phase*: The alternatives are compared and ranked on the basis of the value given in the aggregation phase.
>
> STAGE 3. *Explanation Phase*: In addition to the list of ordered alternatives, a qualitative term is attached to each alternative, in order to give some semantics to their relative position in the ranking in comparison to the positions of the ideal and nadir alternatives. So, the alternatives near the ideal will be denoted as "*optimum*" or "*very_good*" ones, the ones near the nadir will be the "*very_bad*" options. The others will receive a term according to their values.
>
> STAGE 4. *Quality Measurement Phase*: some quality measures are given, which can be useful for the decision maker in order to decide the reliability of the ranking.

In Figure 2 we can see a schema of the flow of data. We begin with a data matrix with m alternatives and p criteria. At the end, we have a qualified set of alternatives (each alternative has a linguistic term t_i that describes the appropriateness to be selected as a solution for the decision problem) and a report with additional information.

During the analysis of the decision matrix, the method extracts useful information for the decision-maker. All the details about this data and the way it is obtained will be included to this final report. The ClusDM methodology has been designed having in

mind that the user will be reluctant to make a machine-based decision. He needs some guarantee of the quality of the ranking given by the system. ClusDM pretends to be a useful aid for decision makers supplying them all the useful knowledge that can be extracted form the data during the aggregation, ranking and explanation stages.

As it has been said in the introduction of this chapter, section 3.3 reviews the aggregation stage. The ranking phase is described in chapter 4 and the last two ones are explained in chapter 5.

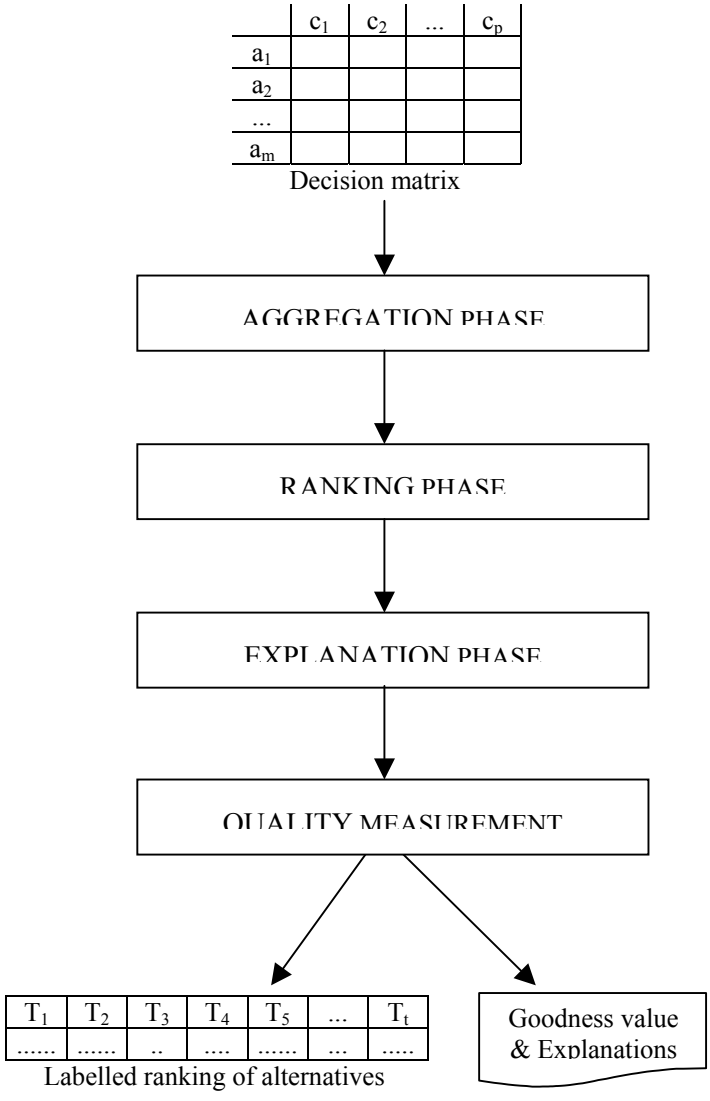

Figure 2. Stages of the ClusDM process

3.3 Aggregation

The first stage of the multicriteria decision process consists of aggregating the different values given to each alternative, and obtain a new one that synthesises the information provided by the individual criteria. When working with homogeneous values, the result of the aggregation stage is a new value of the same nature than the original ones. For example, the Weighted Average operator is usually applied to a set of numerical values, producing a new numerical value. However, when the criteria are heterogeneous, it is not obvious which should be the type of values of the result. This is so because not all the scales can give the same accuracy when describing the alternatives.

We have implemented a system, called Radames, which allows the aggregation of many different data representation structures (e.g. data matrices, trees, vectors). The case studied in this thesis concerns the aggregation of vectors describing an alternative. In particular, we work with a data matrix whose rows are vectors with qualitative or heterogeneous values. For the rest of cases (e.g. numerical or Boolean data), the most well-known aggregation operators have been studied and implemented [Valls, 1997].

For qualitative or heterogeneous value we propose the use of the ClusDM methodology to obtain a new qualitative criterion. That is, ClusDM can be seen as a MCDA methodology or as an aggregation or fusion operator.

In ClusDM, the result of the first stage is a qualitative non-ordered vocabulary, although after the ranking and explanation stages it will become an ordered preference qualitative criterion. The selection of a qualitative preference scale is based on the comparison of the different scales we are considering: numerical, qualitative (preferences or categories) and Booleans. The most informative type is the numerical one, and the least informative is the Boolean one. Qualitative values are in the middle, the greater the cardinality of their domains; the more differences can be stressed. In fact, sometimes Boolean can be considered as a qualitative variable with two values in the domain.

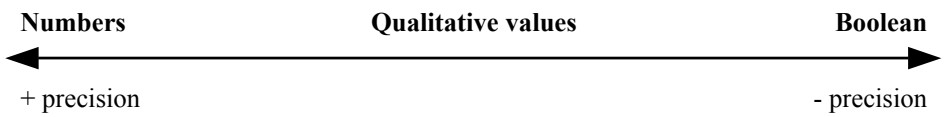

Figure 3. Precision of the different types of values

The transformation of one scale into another has two different effects (see Figure 3). On one hand, the translation of numbers into terms (or Booleans) implies a reduction of information because different numbers will be transformed into the same term. On the other hand, transforming qualitative values into numerical ones implies substituting a term by a number. The subsequent operations with this number will treat

it as a precise value, which is introducing error because the number is only an interpretation of a term that is actually covering an interval of values.

Considering that changes from one type of representation to another produces a loss of some kind of information, we decided to take a position in the middle. Thus, the result of ClusDM will be a qualitative term describing each alternative.

After studying qualitative domains, we have seen that the linguistic terms of a vocabulary define a partition on the set of alternatives, because the alternatives that take the same value are indistinguishable, according to the expert. Therefore, we can formulate our aggregation goal as: to obtain a new partition of the set of alternatives having into account all the information provided by the criteria (i.e. experts). Each cluster in this partition will correspond to a new linguistic value in the domain of the new social (i.e. agreed) criterion [Valls, 2000a].

To obtain a partition (i.e. a non-overlapping set of clusters) we can use clustering methods. During the clustering process the objects form groups according to their similarity, which is measured comparing the values of the alternatives for the different criteria. To find these groups or clusters, each object is compared to the others.

We have studied the application of clustering to qualitative and heterogeneous data sets. In the next section, there is a brief overview of clustering techniques, making special emphasis on the ones that are more appropriate to be used as an aggregation operator. Section 3.3.2 explains how to obtain the aggregation of the alternatives in the decision matrix by means of a clustering tool called *Sedàs*.

Although we will concentrate on our clustering system *Sedàs*, any other clustering technique could be applied. In any case, it is important to note that this aggregation method does not hold the condition of irrelevant alternatives[4] [Arrow,1963], because (using clustering) it is not possible to obtain the consensus value of an alternative without taking into account the rest.

3.3.1 Review of Clustering methods

Clustering methods are traditional techniques to obtain a partition of a set of objects [Everitt,1977], [Jain&Dubes,1988]. A clustering process has two phases (Figure 4):

(a) The construction of a *similarity* matrix that contains the pairwise measures of proximity between the alternatives. Several similarity or dissimilarity functions can be used. Each one has different properties, and it is not possible to determine which is the best for a particular set of data. In [Anderberg,1973] and [Baulieu,1989] there is a review of some of these measures and their interrelationships.

(b) The construction of a set of clusters, in which similar objects belong to the same cluster. Many different methods have been developed [Jain&Dubes,1988]. Up to now, it is impossible to define a way to choose neither the best method, nor the best for a particular problem. These methods can are divided into two families:

[4] This condition is usually satisfied by the aggregation methods in MAUT.

- *Hierarchical Agglomerative clustering methods*: clusters are embedded forming a tree. The root is the most general cluster, which contains all the objects (i.e. alternatives), and the leaves are the most specific groups, that contain a unique alternative.

- *Partitioning clustering methods*: clusters are mutually exclusive. They are generated optimising a 'clustering criterion'.

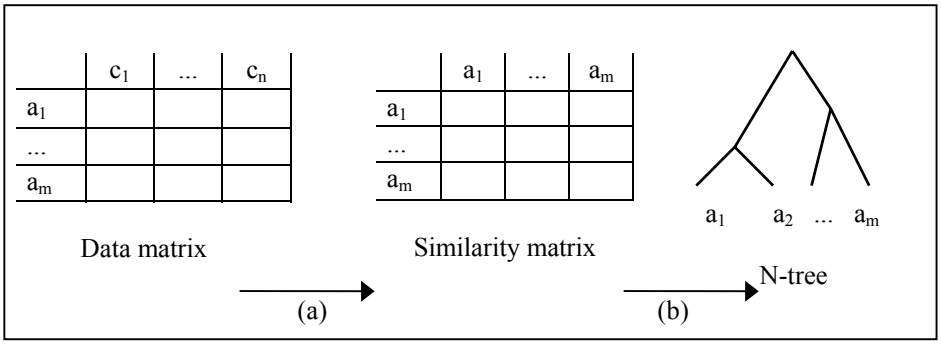

Figure 4. Clustering process

We will follow the hierarchical agglomerative approach. That is, once the similarity relation is defined for each pair of alternatives in the data matrix, the clustering will proceed to build a tree. A tree is a nested sequence of partitions over the set of alternatives. Formally,

Definition 3. [Gordon, 1987] A tree over a set of alternatives A is defined as a set τ of subsets of A that satisfies the following conditions:

1. $A \in \tau$
2. $\varnothing \notin \tau$
3. $\{a_i\} \in \tau$ for all $a_i \in A$
4. $M \cap N \in \{\varnothing, M, N\}$ for all $M, N \in \tau$

With this conditions we can have binary or n-ary trees, although usually clustering trees are forced to be binary (each node has only two children). The use of binary trees is justified in terms of the facility with which these structures are obtained and treated. However, binary trees are not as much close to the knowledge they represent as n-trees.

The clustering process, besides of returning the set of nodes of the clustering tree, assigns to each node a cohesion value, h_α, of the cluster it represents. This value corresponds to a measure of similarity of the last union (i.e. when all the subclusters have been gathered to form the cluster that the node represents). Therefore, for any pair of alternatives (a_i, a_j) that belongs to the cluster α, the following condition is fulfilled: $d(a_i, a_j) \leq h_\alpha$, where d is the dissimilarity function (i.e. the opposite of the similarity) used to compare the alternatives during the clustering process.

As it will be seen in the next section, we have focused on the study of a particular subset of clustering methods known as SAHN [Sneath&Sokal,1973]: Sequential, Agglomerative, Hierarchical and Non-overlapping methods. The clustering algorithm for these methods can be summarised as follows:

```
STEP 0. Construction of the initial similarity matrix

STEP 1. Selection of the alternatives (i.e. objects) that
        are more similar. Those alternatives will form
        the new cluster

STEP 2. Modification of the similarity matrix as follows:
     2.1. Elimination of the alternatives that belong to
          the new cluster
     2.2. Insertion of the new cluster in the similarity
          matrix
     2.3. Calculation of the similarity between the new
          cluster and the rest of objects (using the
          clustering criterion)

STEP 3. Repeat steps 1-2 until we have a single cluster
```

At step 1, the method can gather only two objects (in this way we build a binary tree) or gather all those alternatives with maximum similarity (so we obtain a n-tree). With respect to the clustering criterion that appears in step 2.3, it is used to recalculate the similarity matrix when a new cluster has been created. There are different approaches, such as the Single Linkage, the Ward's method, the Centroid Clustering analysis, etc. (see [Everitt,1977] for more details). Some of them will be reviewed in the next section.

As it has been said, the result of the clustering process is a tree. Trees are generally pictured using dendrograms (see Figure 5). A monotonic dendrogram is the graphical representation of an ultrametric (i.e. cophenetic) matrix. More formally, a dendrogram is defined as a rooted terminally-labeled weighted tree in which all terminal nodes are equally distant from the root [Lapointe&Legendre, 1991]. The weights of this tree are given by the heights h_α, which correspond to the cohesion values of the clusters α. So, for a tree τ with $M,N \in \tau$ (two internal nodes), the following property is fulfilled: if $M \cap N \neq \varnothing$, $h_M \leq h_N \leftrightarrow M \subset N$.

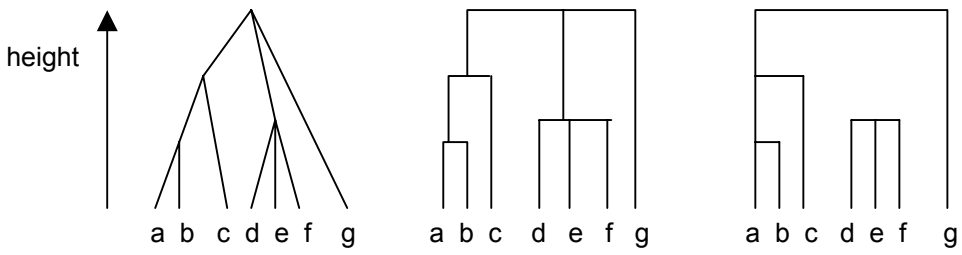

Figure 5. Three different formats for representing dendrograms

Alternative characterisations of a dendrogram can be found in the literature. Gordon [Gordon, 1987] states that a necessary and sufficient condition for a monotonic dendrogram is that the set h_{ij} satisfies the ultrametric condition:

$$h_{ij} \leq \max(h_{ik}, h_{jk}) \text{ for all } a_i, a_j, a_k \in A$$

where h_{ij} is the height of the internal smallest cluster to which both alternatives a_i and a_j belong.

Nevertheless, some of the trees generated by the clustering criteria do not fulfil this ultrametric condition. So, they are not monotonous. They are said to present *inversions* or *reversals*. For example, in
Figure 6 we can see that clusters α=(g,h) and β=(i,j) merge at a level lower that the level at which α was created.

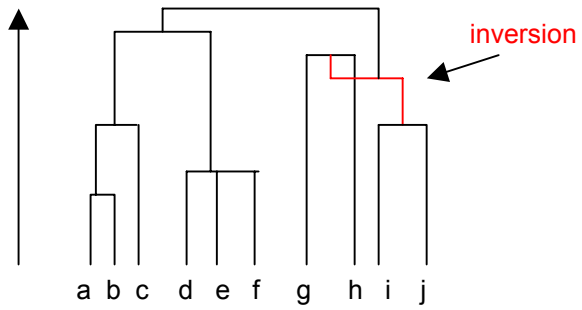

Figure 6. Dendrogram of a non-monotonous tree

Non-monotonous trees may cause problems when the tree is cut in order to obtain a partition of the set of alternatives.

Partitions are obtained making a horizontal cut of the tree at a particular height. The height at which the tree is cut determines the abstraction level achieved. Increasing the cut level we obtain a smaller number of bigger (more general) clusters.

3.3.2 Our generic clustering system: *Sedàs*

We have implemented a generic SAHN clustering system, called *Sedàs* [Valls et. al., 1997]. All the scales mentioned in section 3.1 are allowed in *Sedàs*: numerical, ordered qualitative preferences, categorical and Boolean. However, any other scale with a subtraction function defined in its domain can be included in the system. *Sedàs* has been incorporated to the *Radames* system, in order to be used as an aggregation operator.

The interface allows the user to choose from a list of similarity functions and a list of clustering techniques the most adequate to each particular data set. The system includes, among others, the following classic weighted dissimilarity functions. Being v_{ij}

the value of the i-th criterion of alternative a_j, and v_{ik} the value of the i-th criterion of alternative a_k, we can calculate the dissimilarity $d(a_j,a_k)$ using:

- Distance based on Differences

$$\frac{\sum_{i=1}^{p}\left(v_{ij}-v_{ik}\right)}{p} \qquad \text{Eq. 3.2}$$

- Manhattan Distance

$$\sum_{i=1}^{p}\left|v_{ij}-v_{ik}\right| \qquad \text{Eq. 3.3}$$

- Mean Character Difference (M.C.D.)

$$\frac{\sum_{i=1}^{p}\left|v_{ij}-v_{ik}\right|}{p} \qquad \text{Eq. 3.4}$$

- Taxonomic or Euclidean Distance

$$\sqrt{\frac{\sum_{i=1}^{p}\left(v_{ij}-v_{ik}\right)^2}{p}} \qquad \text{Eq. 3.5}$$

- Minkowski Distance

$$\sqrt[r]{\sum_{i=1}^{p}\left|v_{ij}-v_{ik}\right|^r} \qquad \text{Eq. 3.6}$$

- Pearson Correlation Coefficient

$$\frac{\sum_{i=1}^{p}\left(v_{ij}-\overline{v_i}\right)\left(v_{ik}-\overline{v_k}\right)}{\sqrt{\sum_{i=1}^{p}\left(v_{ij}-\overline{v_i}\right)^2\cdot\sum_{i=1}^{p}\left(v_{ik}-\overline{v_k}\right)^2}} \qquad \text{Eq. 3.7}$$

This dissimilarity functions have been generalised to be applied to numerical, ordered qualitative, categorical and Boolean data [Valls et. al., 1997]. For a numerical criterion with range [a,b], we put the values into the unit interval [0,1] before applying the dissimilarity function. Ordered qualitative values are translated into numbers in [0,1] using their negation-based semantics. The difference v_{ij} - v_{ik} for categorical values takes only two possible values: 0 if they are different or 1 if they are equal. Finally, the Boolean values are treated as categorical ones. This functions can also be adapted to consider different weights for the different variables (i.e. criteria, attributes) [Gibert&Cortés, 1997].

If the decision matrix has missing values (that is there are some unknown values), the system is able to calculate the similarity among the pairs of objects. If v_{ij} or v_{ik} are unknown, *Sedàs* can operate in two modes: a) the rest of values of this criterion are used to calculate the average value, which is used instead of the unknown value; b) this criterion is ignored in the comparison of the two alternatives, a_j and a_k, so p is decreased in 1 unit because we are dealing with less criteria.

Using the data in the similarity matrix, *Sedàs* executes the algorithm explained in the previous section. In step 2.3, a clustering criterion is needed to compare the new-created cluster with the rest of elements of the similarity matrix. To determine the similarity of this new element with respect to the others, many methods have been defined. Some of the most known approaches are available in our system, such as:

- Single Linkage or Nearest Neighbour

  This criterion considers that the dissimilarity value between a new cluster α and an object[5] o_k is equal to the minimum distance between the objects in the cluster and the object outside o_k.

$$d(\alpha, o_k) = \min_{o_i \in \alpha} d(o_i, o_k)$$

  Graphically,

[5] An object can be a single alternative or a cluster generated in a previous step.

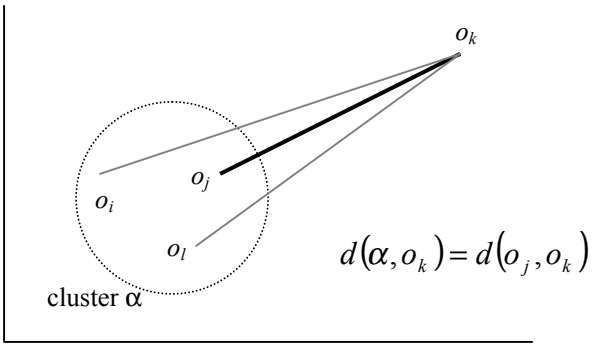

Figure 7. Single Linkage

- Complete Linkage or Furthest Neighbour

This criterion assumes a similar behaviour than the Single Linkage, however, it considers that the dissimilarity value between a new cluster α and an object o_k is equal to the maximum distance between the objects in the new cluster and the object outside it, o_k.

$$d(\alpha, o_k) = \max_{o_i \in \alpha} d(o_i, o_k)$$

Graphically,

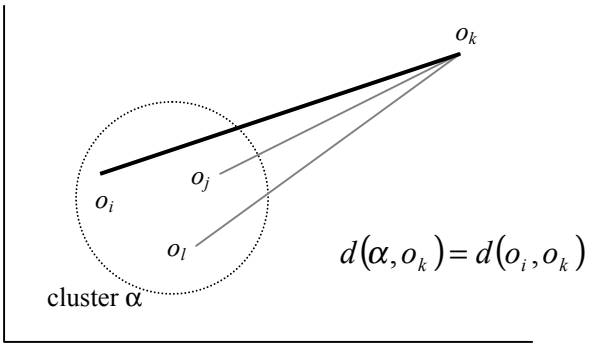

Figure 8. Complete Linkage

- Arithmetic Average

A measure in between of the two previous ones is the one known as Arithmetic Average criterion. It takes as a dissimilarity value between a new cluster α and an object o_k, the average distance between the objects in the cluster and the object outside o_k.

$$d(\alpha, o_k) = \frac{\sum_{o_i \in \alpha} d(o_i, o_k)}{|\alpha|}$$

- Centroid Clustering

This approach is based on the calculation of the prototype of each cluster. Let us denote as o_α the prototype of the cluster α. This prototype or centroid is defined as follows: $o_\alpha = (\overline{c_1}, \overline{c_2}, ..., \overline{c_p})$, where $\overline{c_i}$ is the average value of the criterion c_i considering the alternatives that belong to α. Using this prototype or centroid, the distance between the new cluster and an outside object, o_k, is defined.

$$d(\alpha, o_k) = d(o_\alpha, o_k)$$

This averaging function needed to calculate the prototype of the cluster depends on the type of scale. In Table 4 we can see some examples of averaging functions for the scales we are dealing with:

Scale	**Functions**
Numerical	Arithmetic average, Weighted Arithmetic average, OWA
Categorical	Max-min, Voting Techniques, Averages (translating terms into numbers)
Boolean	Voting

Table 4. Some averaging functions to build prototypes

In the case of qualitative domains with a negation function, we propose the translation of the values into numbers and the application of a numerical averaging operator. We recommend the use of the Weighted Arithmetic average or the OWA operator, depending on the kind of weights we are interested to apply.

- Median Cluster Analysis

This criterion established that the dissimilarity between a cluster α (formed by the union of objects o_i and o_j) and the object o_k (which does not belong to α) is the length of the bisectrix of the angle corresponding to o_k, considering a triangle formed by these three objects. This is illustrated in Figure 9.

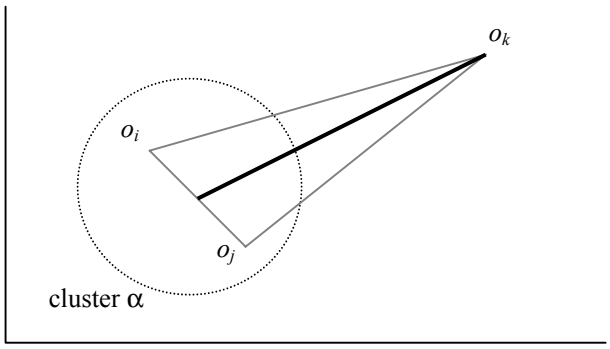

Figure 9. Median Cluster Analysis

$$d([o_i, o_j], o_k) = \sqrt{\frac{1}{2}d^2(o_i, o_k) + \frac{1}{2}d^2(o_j, o_k) - \frac{1}{2}d^2(o_i, o_j)}$$

For n-trees, this definition can be generalized as follows:

$$d(\alpha, o_k) = \sqrt{\frac{1}{2}\max_{o_i \in \alpha} d^2(o_i, o_k) + \frac{1}{2}\min_{o_i \in \alpha} d^2(o_i, o_k) - \frac{1}{2}d^2(a, b)}$$

$$\text{where } a = \max_{o_i \in \alpha} d^2(o_i, o_k) \text{ and } b = \min_{o_i \in \alpha} d^2(o_i, o_k)$$

That is, we build a triangle using two of the objects that belong to the cluster: the one that is nearest to o_k and the one that is furthest with respect to o_k.

Using these clustering criteria, *Sedàs* is able to generate n-ary trees. We decided to discard the binary approach in order to avoid the arbitrary choice of two elements to be joined when there are several with the same similarity. Moreover, with this method we eliminate the chaining of clusters that have the same distance between them.

Not all these clustering criteria produce monotonous trees. In particular, the Centroid and the Median Cluster Analysis methods may generate trees with inversions. So, when *Sedàs* generates a partition P from the tree, it checks that the clusters in the partition are mutually exclusive, that is, $M \cap N = \varnothing$ for all $M, N \in P$. For instance, the partition induced in Figure 10, P={(a,b,c),(d,e,f),(g),(h),(g,h,i,j),(i,j)}, is not correct because does not hold this condition.

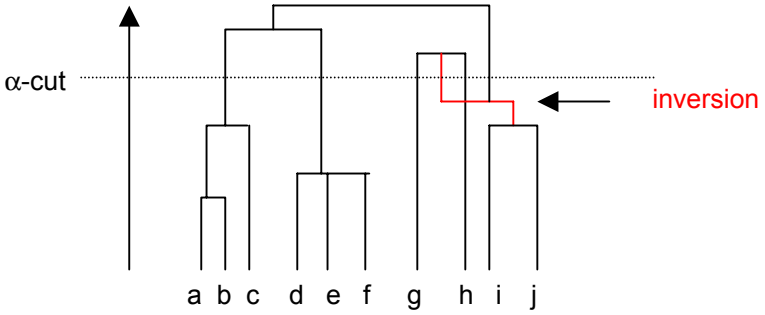

Figure 10. Making a cut in a tree with inversions

3.3.3 Using *Sedàs* as an aggregation operator

We have studied and compared the trees obtained using different similarity functions and clustering criteria [Valls et. al., 1997]. The main conclusion reached is that clustering criterion has less influence on the structure of the tree generated than the similarity function. In Table 5 we can see the comparison of different trees obtained from the same data matrix with several clustering criteria and similarity functions. The table give the distance between pairs of trees. We have used the distance defined in [Newmann,1986] and [Barthélémy&McMorris,1986]:

$$d_\tau(\tau,\tau') = |\tau \cup \tau'| - |\tau \cap \tau'|$$

Looking at the distances between trees, we can see that the distance is highly related to the similarity function used rather than to the classification method. This is reflected by means of small distances between trees obtained with the same similarity function (Differences or Mean Character Difference), and greater distances when different similarity functions are considered. We can see, for example, that when we choose the similarity function Differences (Dif), the trees obtained by means of the Arithmetic Average (Dif_a) and the Median procedure (Dif_m) have a distance of 8. On the other hand, when Arithmetic Average is considered with several similarity functions we have d_τ(Dif_a, MCD_a)=13. Notice that the distances in the upper right frame are greater than the others in the same column/row.

	Dif_a	Dif_m	Dif_s	MCD_a	MCD_m	MDC_s
Dif_a	0	8	9	13	17	13
Dif_m		0	13	17	19	17
Dif_s			0	12	16	12
MCD_a				0	6	0
MCD_m					0	6
MCD_s						0

Symbols glossary
Dif: Differences Distance
MCD: Mean Character Difference
a: Arithmetic average
m: Median procedure
s: Single Linkage

Table 5. Distances between trees

Assuming that the selection of the clustering criterion does not causes great differences in the structure of a tree if the similarity between the elements is well established, we recommend the use of the ***Centroid Clustering*** criterion for aggregating the values of the alternatives. The rationale for this decision is that this method is based on the concept of prototype. The prototype is the pattern of the cluster, and it is used to determine the relation from one cluster to the other clusters and objects analysed. As it will be seen in the next chapters, the following stages of the ClusDM methodology are also based on the prototype of the clusters in the partition obtained after the cutting of the tree. For this reason, we consider that it is appropriate that the aggregation stage also works with prototypes.

After fixing the clustering criterion to the use of the Centroid Clustering, we studied the most usual similarity functions:

- the Differences distance may compensate a negative difference in one criterion with a positive difference in another one. This is an important drawback since two different objects can be considered as equal if the differences compensate each other;
- the Manhattan distance is based on a city made of blocks, so the distance between two opposite corners of a building is the length of the two streets you have to walk to arrive to the other side;
- the Taxonomic distance considers that if you have to cross a square from one corner to the opposite one, you can walk through the square. So, the distance between these two opposite corners is the length of the line that crosses them;
- the Minkowski distance is a generalization of the Taxonomic distance that considers more than 2 criteria, but the properties are the same;
- the Pearson Correlation Coefficient is based on the lineal relations between alternatives. It measures the correlation between two alternatives comparing their values to the average for each criterion. Some dimensional properties on the data set are required for applying this distance [Sneath&Sokal, 1973].

Having into account that the goal of our methodology is to be able to deal with heterogeneous data sets. As it has been said in chapter 1, it is particularly interesting the case of having qualitative preference criteria with different vocabularies. For this reason, we recommend the use of the Manhattan distance. The basis of this similarity function is more appropriate to the characteristics of a qualitative domain because

when we compare two linguistic terms, we will use a numerical translation of this terms, however, the number represents an interval (like one face of a building block) instead of a single point (like the corner of a square).

Although in this point of the explanation we are suggesting the use of the Manhattan distance together with the Clustering criterion, we must remember that the ClusDM methodology is more general, and these are only some parameters that can be changed.

In our system, *Sedàs*, these parameters are required to build the n-tree. As it has been said, to obtain a partition this tree is cut at an appropriate level. In our case, this level is determined by the number of clusters we want to obtain. Remember, that each of these clusters must receive a different term in the vocabulary of the new preference criterion. So, the number of clusters is proportional to the length of the vocabulary. In general, 7 it is said to be the ideal number of terms that a person is able to handle [Miller,1956], however, this number might not be adequate in some cases.

We propose to use the lengths of the vocabularies of the criteria provided by the experts to have an idea of the number of clusters we are looking for. Using this criterion, *Sedàs* takes a number of clusters as close as possible to the number of linguistic terms used in the criteria. If there is no qualitative criterion, then a good approximation is to take $max(1,log_2 d)$, where d is the number of different values considering all alternatives. This value is based on the proposal of [Dougherty et. al.,1995]: they define the best number as the maximum of *1* and $2*log_{10} d$. However, this approximation gives a number of labels too small, which implies losing too much information. After making different tests, we recommend the use of logarithm base 2.

Despite of being interested in a partition, it is also useful to know the complete tree of clusters, which is giving us the relation among the alternatives at different levels. Looking into the subclusters of a particular cluster we can obtain a more precise clustering of the alternatives, which allows us to distinguish different categories inside a cluster. On the other hand, if we look at higher clusters in the tree, we can see the similarities among the clusters of our partition.

Finally, once the alternatives have been aggregated in clusters, *Sedàs* automatically assigns a symbolic name to each cluster. This partition and the prototype of each of its clusters are the inputs of the following stage: Ranking.

Chapter 4.

Ranking stage

The ranking of the alternatives is applied after the aggregation of the values in the decision matrix. In general, the ranking procedure depends on the type of result provided by the previous stage. In our case, the aggregation produces a set of clusters and each cluster can be represented by a prototype alternative, which is built according to the values of the alternatives that belong to the cluster, as it has been explained in chapter 3.

Therefore, the goal of this stage is to determine automatically the preference among the clusters, that is, their ranking. In this way, at the end of the process, the class at the first position of the ranking will contain the most preferred alternatives (according to the new overall criterion). To obtain these preferences on the clusters, their prototypes will be used.

The study of different ranking techniques have brought us to distinguish two different situations:

CASE A: All the criteria in the decision matrix are expressing preferences over the alternatives. That is, each criterion is giving an order of the alternatives according to some preference opinion or property.

CASE B: The criteria are expressing different views of the data, which can be preferences or just descriptive properties (e.g. educational degree, job, and age).

The first case is the one that is usually studied in MCDA research [Vincke,1992]. Nevertheless, sometimes there are descriptive properties that should also be taken into account in the decision making process.

In the following sections we will explain the ranking methodology used in the two different cases. A formal definition of the method is done at the beginning of the section, to continue with the explanation of how to apply each method to the ranking of clusters.

4.1 Ranking using Principal Components Analysis

The ranking in CASE A is done using the multivariate statistical method called *Principal Components Analysis* (PCA). To obtain a good ranking with PCA, criteria are required to be correlated with each other. This situation happens when the criteria are the opinions of different experts about the alternatives. Although the experts may have different points of view, if it is possible to define "the best ranking" for the set of alternatives, and experts really know the decision problem, there is supposed to be a high degree of correlation.

The method of *Principal Components* [Pearson, 1901] obtains linear transformations of a set of correlated variables such that the new variables are not correlated. This is a useful technique for statistical analysis of multivariate data, in particular, to describe the multivariate structure of the data.

Although the Principal Components Analysis is usually a descriptive tool, it can be also used for other purposes. For example, PCA can be applied to obtain a ranking of observations [Zhu, 1998].

In this section, we will explain in detail the mathematical basis of a Principal Components Analysis. We will see some properties that are interesting for its use as a ranking tool. Furthermore, we will define some measures and procedures to interpret the results. Finally, we will detail how PCA must be applied to the ranking phase in a multicriteria decision problem.

4.1.1 How to perform a Principal Components Analysis

Considering that we have a data matrix, X, where the alternatives are defined in a certain basis, the PCA will make a change in the basis, so that, the new space is defined by orthogonal axes. However, PCA is not applied directly to the matrix X [Jackson,1991]. We use a $p \times p$ symmetric, non-singular matrix, M.

Principal Components are generated one by one. To find the first principal component we look for a linear combination of the variables that has maximum sample variance. Then, the second vector will be obtained with the same goal subject to the fact of being orthogonal to the first vector, and so on. The solution to this maximisation problem is based on the fact that the matrix M can be reduced to a diagonal matrix L by premultiplying and postmultiplying it by a particular orthonormal matrix U. This diagonalisation is possible because M is a $p \times p$ symmetric, non-singular matrix.

$$U'MU = L$$

With this diagonalisation we obtain p values, $l_1, l_2, ..., l_p$, which are called the characteristic roots or eigenvalues of M. The columns of U, $u_1, u_2, ..., u_p$, are called the characteristic vectors or eigenvectors of M. Geometrically, the values of the characteristic vectors are the direction cosines of the new axes related to the old.

Having the set of data, X, described by p variables, $x_1, x_2, \ldots, x_p$, we can obtain the eigenvectors corresponding to this data and produce new p uncorrelated variables, $z_1, z_2, \ldots, z_p$. The transformed variables are called the *principal components* of x.

The new values of the alternatives are called z-scores, and are obtained with this transformation:

$$z = U' x^* \qquad \text{Eq. 4.1}$$

where x^* is $p \times 1$ vector that has the values of an alternative after some scaling.

4.1.2 Types of Principal Components Analysis

The matrix M, from which the principal components are obtained, is defined as described in Eq.4.2.

$$M = \frac{Y'Y}{n} \qquad \text{Eq. 4.2}$$

Different types of principal components analysis exist according to the definition of variable Y in terms of X. Here we underline the three different possibilities [Jackson,1991].

- **Product matrix**
  The first approach consists in taking $Y = X$, that is, perform the analysis from the raw data. However, there are not many inferential procedures that can be applied in this case.

- **Covariance matrix**
  The second approach consists in centring the data, so that $Y = X - \overline{X}$. In this case, we scale the data to be distances from the mean (which is actually a translation of the points). Thus we transform the variables such that all of them have mean equal to 0, which makes them more comparable. It is important to notice that, in this case, the matrix M obtained is the covariance matrix of X.
  In the calculation of the covariances the mean is subtracted from the data, so it is not necessary to do it in advance. Then, we obtain the principal components using Eq.4.1, where x^* will be the result of subtracting the mean from the data values.

$$z = U'\left[x - \bar{x}\right] \qquad \text{Eq. 4.3}$$

where x is a $p \times 1$ vector that has the values of an alternative on the original variables, and $\bar{x}$ is also a $p \times 1$ vector that has the mean of each variable.

The covariance matrix is denoted S and it is calculated as follows:

$$S = \begin{bmatrix} s_1^2 & s_{12}^2 & \cdots & s_{1p}^2 \\ s_{21}^2 & s_2^2 & \cdots & s_{2p}^2 \\ \cdots & \cdots & \cdots & \cdots \\ s_{p1}^2 & s_{p2}^2 & \cdots & s_p^2 \end{bmatrix} \quad \text{Eq. 4.4}$$

where s_i^2 is the variance of x_i, and the covariance of (x_i, x_j) is calculated as follows:

$$s_{ij} = \frac{n\sum x_{ik}x_{kj} - \sum x_{ik}\sum x_{jk}}{n(n-1)}.$$

PCA based on the covariance matrix is widely applied because the inferential procedures are better developed for this kind of matrix than for any other situation [Jackson,1991]. However, there are some situations in which the covariance matrix should not be used: (i) when the original variables are expressed in different units or (ii) when the variances are different (even though the variables are in the same units). The use of a covariance matrix in these two situations will give undue weight to certain variables (i.e. those that have a large range of values or a large variance).

- **Correlation matrix**

To avoid the weighting of certain variables, we can work with variables with a common deviation equal to 1. This is obtained by centring and standardising the variables. So, the matrix M is, in this case, the correlation matrix of X.

The correlation matrix, denoted by R, is computed as follows:

$$R = D^{-1}SD^{-1} \quad \text{Eq. 4.5}$$

where D is the diagonal matrix of standard deviations of the original variables:

$$D = \begin{bmatrix} s_1 & 0 & \dots & 0 \\ 0 & s_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & s_p \end{bmatrix} \qquad \text{Eq. 4.6}$$

The use of correlation matrices is also very common and it is usually the default option in some computer packages (e.g. Minitab). Inferential procedures for this type of matrices are also well defined.

In this case, the z-scores are obtained using Eq.4.3 but using standardised values for x^*. That is, we have to subtract the mean to the data and divide it by the standard deviation. Then, we must multiply it by the eigenvectors.

$$z = U' D^{-1} \left[x - \bar{x} \right] \qquad \text{Eq. 4.7}$$

As it has been previously said, the results obtained with each type of scaling are different. For example, the eigenvectors, *U*, and the z-scores, *z*, are different. In fact, there is no one-to-one correspondence between the principal components obtained from a correlation matrix and those obtained from a covariance matrix.

Other types of vectors can be derived from the characteristic vectors (U-vectors) obtained either with the covariance or the correlation matrix. We are interested in the V-vectors, which properties will be described in the next section. The transformation of the characteristic vectors is done in order to obtain principal components in other scales, in which other properties are fulfilled.

V-vectors are the ones obtained with the following transformation:

$$V = UL^{1/2} \qquad \text{Eq. 4.8}$$

$$\text{i.e.} \quad v_i = u_i \sqrt{l_i} \qquad \text{Eq. 4.9}$$

$$\text{i.e.} \quad v_{ij} = u_{ij} \sqrt{l_i} \qquad \text{Eq. 4.10}$$

Giving weights to the variables:

To give different importance to each variable, we must adjust the matrix used in the PCA (either the correlation or the covariance matrix) using a diagonal matrix with the weights of each variable. Then, the matrix used for the multivariate analysis will be:

$$X \cdot \begin{bmatrix} weight_{c1} & 0 & ... & 0 \\ 0 & weight_{c2} & ... & 0 \\ ... & ... & ... & ... \\ 0 & 0 & ... & weight_{cp} \end{bmatrix} \quad \text{Eq. 4.11}$$

4.1.3 Properties

Let us describe the properties of the results obtained in the two most popular PCA approaches: the covariance matrix and the correlation matrix.

PCA based on covariances:

The U-vectors are orthonormal; that is, they are orthogonal and have unit length. Therefore, they are scaled to unity (i.e. the coefficients of these vectors will be in the range [-1,1]). Using these vectors we produce principal components that are uncorrelated and have variances equal to the corresponding eigenvalues. The contribution of each variable to the formation of the i-th principal component is given by the magnitude of the coefficients of u_i, with the algebraic sign indicating the direction of the effect [Dillon&Goldstein, 1984].

V-vectors are also orthogonal but they are scaled to their roots. In this case, the principal components will be in the same units as the original variables. The variances will be equal to the squares of the eigenvalues.

Interpretation of principal components is often facilitated by computing the component loadings, which give the correlation of each variable and the respective component. So, the loading for the j-th variable on the i-th principal component is:

$$\frac{u_{ij}\sqrt{l_i}}{\sqrt{S_{jj}}} \quad \text{Eq. 4.12}$$

Note that the numerator is actually $v_{ij.}$

PCA based on correlations:

The properties of U-vectors are the same as the ones explained for the case of the covariance matrix. Therefore, the interpretation of their coefficients is the same.

With regard to the V-vectors, in this case, they hold important property: their coefficients show the correlations between the principal components and the original variables, because the variances of the standardised variables are all equal to 1. Thus, if the coefficient v_{ij} is equal to 1 it means that the *i*-th principal component and the *j*-th

variable are positively correlated, and if v_{ij} is equal to -1 they are negatively correlated. However, we lose the property of obtaining z-scores in the domain of the original variables.

4.1.4 Stopping rule:

The Principal Component Analysis allows us to reduce the multidimensionality of the data, and represent the information of the initial data set in a *k*-space smaller than the original (with *p* variables), that is, $k<<p$. In the *k*-space the data is easily interpretable. However, the determination of which should be the value *k* is not straightforward. The larger *k* is, the better the fit of the PCA model; the smaller *k* is, the simpler the model will be.

There are different stopping criteria (see [Jackson, 1991]). They are based in the fact that the characteristic roots, $l_1, l_2, ..., l_p$, are decreasingly ordered, that is, $l_1 > l_2 > ... > l_p$. That means that the first characteristic vector is the one that accounts for a higher proportion of variability. These stopping criteria range from methods that evoke formal significance tests to less formal approaches involving heuristic graphical arguments.

For the covariance input, the stopping criteria are usually related to the statistical significance of the eigenvalues. However, for the correlation matrix, these statistical testing procedures no longer apply.

An alternative approach consists of more ad hoc criteria. For example, the cumulative percentage of the variance extracted by successive components, or the Jolliffe's criterion (called Broken Stick), which consists of selecting the *k* vectors, u_j, such that $l_j > g_j$, where g_j is:

$$g_j = \frac{1}{p}\left(\sum_{i=j}^{p}(1/i)\right) \qquad \text{Eq. 4.13}$$

An adaptation of this formula to the case of having variables with uniform distributions is:

$$g'_j = \frac{(n-j+1)(p+j+1)}{\sum_{i=1}^{p}(n-i+1)(p-i+1)} \qquad \text{Eq. 4.14}$$

For the case of the correlation matrix, this variance approach lacks clear meaning, because the standardisation of the data produces a dimensionless standard score space, where the sum of the eigenvalues is equal to the number of variables, *p*. The most frequently used extraction approach in this case is the selection of the components whose eigenvalues are greater than one. The rationale for this criterion is that any component should account for more “variance” than any single variable (remember that variances are equal to 1 because data have been centred and standardised).

4.1.5 Interpretation of the results

A Principal Components Analysis is usually performed for descriptive purposes. In this framework, it is useful to know the global variance of the data we are studying. There is a direct relation between the sum of the original variances and the sum of the characteristic roots obtained with the PCA.

$$Tr(L) = l_1 + l_2 + l_3 + \ldots l_p \qquad \text{Eq. 4.15}$$

In the case of doing the PCA with the correlation matrix, it holds that $Tr(L) = p$ because the variables have been previously standardised.

The value $Tr(L)$ is used to calculate the proportion of the total "variance" attributable to the i-th component, which is $l_i/Tr(L)$.

Another measure that is interesting is the contribution of each observation, j, to the formation of a particular component, i, denoted $CTR_i(j)$. With this information, we can detect observations that if they were removed from the analysis, the result would be the same. These observations have low contribution values.

$$CTR_i(j) = \frac{z_i^2(j)}{l_i} \qquad \text{Eq. 4.16}$$

We can also measure the cosine of the angle between an alternative j and the component i, which gives us an idea of the quality of the representation of the alternative if it is projected into the i-th component.

$$\cos_i^2(j) = \frac{z_i^2(j)}{d^2(j,G)} \qquad \text{Eq. 4.17}$$

being d the Euclidean distance between the observation j and the centre of gravity (which is 0 if the data is standardised).

Graphically,

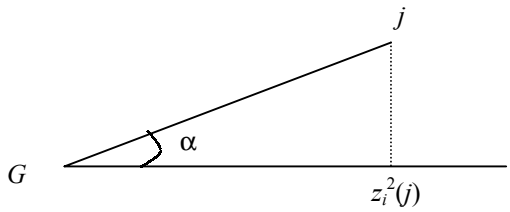

Figure 11. Measuring the quality of representation of alternative j

The measure $\cos_i^2(j)$ is, actually, the square of the cosine of the angle α in Figure 11. If we denote as A the distance between *j* and *G*, and B is the distance between $z_i^2(j)$ and *G*, we can see that when Eq.4.17 is equal to 0, A and B are perpendicular, and if Eq.4.17 is equal to 1 then A=B, so *j* is the same as $z_i^2(j)$, which means that there is no loss of information in the change between one space and the other.

We can define a measure of the quality of the representation of a particular observation *j* in a *k*-space (formed with the *k* first components). The maximum value of *QLT* (quality) is 1, which means that the observation is completely representable with the *k* components.

$$QLT_k(j) = \sum_{i=1}^{k} \cos_i^2(j) \qquad \text{Eq. 4.18}$$

On the other hand, it is very interesting to know the meaning of the new space defined by the eigenvectors obtained in the PCA in terms of the initial variables. It is possible to make a dual analysis with X^T, that is, transposing the data matrix, with which we consider the variables as rows (as observations) and the individuals as columns. Then, using the PCA, we obtain an *m*-space where we can represent the variables in terms of a set of uncorrelated axis (that represent uncorrelated observations). An important property is that this *m*-space is related to the *p*-space obtained with matrix *X*. With this relation, we can use the *p*-space to represent the variables without having to perform the second analysis.

Once we have the variables represented together with the observations, we can use the measures Eq.4.16, Eq.4.17, and Eq.4.18 to infer the meaning of the principal components. In [Volle,1985] there are some guidelines about the process to follow for the interpretation of the new axes, in the case of using the correlation matrix. Note that if we calculate the projection of the variable x_j into the i-th component, $z_i(x_j)$, we can write the contribution and cosine in terms of the V-vectors, because $z_i(x_j) = v_i(x_j)$.

$$CTR_i(x_j) = \frac{v_i^2(x_j)}{l_i} = u_i^2(x_j) \qquad \text{Eq. 4.19}$$

We can see that the contribution of a variable to the i-th component is given by the square of the U-vector obtained when performing the PCA of *X*. The sign of u_i says if it has contributed positively of negatively.

On the other hand, with respect to Eq. 4.17, the distance of each variable to the centre of gravity is 1 (because the data has been standardised). So, the cosine is equal v_i and also it is equal to the correlation between the variable and that component. If $\cos^2_i(x_j)$ is near to 1, x_j can explain the meaning of the axis, because it is really well represented by this axis. In addition, if v_i is near to 1, x_j is positively correlated with the component (and if v_i is -1, it is negatively correlated).

$$CORR_i(x_j) = v_i^2(x_j) \qquad \text{Eq. 4.20}$$

Finally, there are some measures for the global correlation of the initial variables. One of them is the calculation of the determinant of the covariance or the correlation matrix. In the case of the correlation matrix, R, the determinant is sometimes referred to as the "scatter coefficient" [Jackson, 1991]. This coefficient is bounded between 0 (all of the variables are perfectly correlated) and p (all of the variables are uncorrelated).

$$|R| = l_1 \cdot l_2 \cdot \ldots \cdot l_p \qquad \text{Eq. 4.21}$$

Another measure is the addition of the individual correlation of each variable to the first component, but having into account the sign of their direction (positive if it has the same direction than the component, and negative otherwise). If all the variables are positively correlated, the sum is equal to the first eigenvalue l_i, so the percentage of correlation is l_i/p.

4.1.6 Application of the PCA to rank order

The principal components found with a PCA can be used to rank the observations [Slottje et. al., 1991]. In the simplest case, we have a set of highly correlated variables and the stopping criterion selects only one component to represent the data. Then, the projections of the observations in this component, z_1, completely define an order among them.

In the case of needing more than one component to represent the information of our set of data, we can combine the components considering the proportion of variance explained by each one. In [Zhu, 1998] the position of each alternative a_j is given by:

$$POS_j = \sum_{i=1}^{k} \frac{l_i}{p} |z_{ij}| \qquad \text{Eq. 4.22}$$

In this expression, all the values of the observations in the original variables must be positive. If this is not the case, some adjustments must be introduced to Eq.4.22 (see [Zhu,1998]).

We propose to use the Principal Components Analysis to rank the alternatives only if one component is enough to represent our data. If more than one component are needed, the interpretation of the result is far more complicated to automatize. In addition, the measure that qualify the goodness of a ranking obtained with the PCA can only be applied for the case of having the projection of the alternatives in one component (this will be explained in more detail in chapter 5). Therefore, when the first component is not enough to represent the data and perform the ranking, we will use an

alternative procedure based on the similarity to an ideal alternative, which is explained in section 4.2.

Now, we are going to see in detail the process that must be followed to obtain the rank order of the partition of alternatives that we have got in the clustering phase. We want to mention here, that usually the PCA is used as a descriptive tool for an statistical expert that knows how to interpret the results in each of the different steps of the process. However, we want to include PCA in a decision-making method that can be implemented and executed automatically to obtain the ranking of the alternatives without the help of any expert in PCA. For this reason, we have studied in depth this statistical procedure and have selected some measures that can provide a useful knowledge to the decision maker without having to know the insights of this statistical method [Valls&Torra,2002].

First of all, we have to decide which type of PCA to use. As we have seen, there are different ways of performing a PCA depending on the kind of matrix from which we obtain the eigenvectors and eigenvalues. We propose to use the correlation matrix because it will allow us to have variables with different variances. Remember that, in our decision-making framework the variables are the criteria[6], which can have different types of values and different domains.

In the moment of having to perform the ranking, we have the following information available: a data matrix with the alternatives described according to a set of criteria, the grouping of this alternatives into similarity classes and, finally, the prototype of each class (in terms of the same criteria). With the prototypes we can build another matrix, *B*, of the form:

	Criterion 1	...	*Criterion p*
Prototype Class A			
...			
Prototype Class G			

Table 6. Prototypes matrix, denoted by *B*

Then, we have two data matrices that can be used to obtain the first principal component: the original data matrix, *X*, and the prototypes matrix, *B*. In principle, PCA could be performed in each of the two matrices. However, the second one has a very short number of objects (between 4 and 9, which are the usual cardinalities of linguistic vocabularies). This is not good for PCA, which is a technique to be used when the number of variables (i.e. criteria) is smaller than the number of alternatives (i.e. classes or objects). Moreover, the values in the matrix of prototypes have not been provided by the experts, they are the result of some computation over the original values, which can introduce error in the interpretation of the result. So, although the objects that we want to rank are the ones in matrix *B*, we should not perform the PCA directly with these data. The PCA will be done in the original data matrix, and then, the prototypes of the classes will be introduced in the new space in order to be ranked.

[6] In the data matrix we can have criteria given by a single expert or by different experts.

We can distinguish 5 steps in the process of applying the Principal Components Analysis to our data. These steps must be followed sequentially. At the end, we will have a ranking of the classes and some values that will be used to measure the goodness of the result, and to infer the relations among the variables (i.e. preference criteria).

STEP 1 – Apply the Principal Components Analysis to the data matrix. Obtain the eigenvalues, l_i, eigenvectors, u_i and V-vectors, v_i.

STEP 2 – Check if the first component is enough to perform the ranking. To decide whether it is enough or not, we must apply a stopping criteria (section 4.1.4) and see if the number of selected components is one or greater. As we are working with correlation matrices, we propose to use the criteria that selects those vectors that account for more than a 1% of variance, that is, l_i>1.

If we need more than one principal component to represent our data, we will execute step 4 (to obtain some additional information) and end.

STEP 3 – Use the first V-vector to know the meaning of the first component. A value near zero means that the variable has no influence in the interpretation of the component, while the higher the absolute value of the variable, the more the component is saying the same than the variable. We can apply Eq.4.20 to calculate the relation between each variable and the first axis and find the variables with higher correlation.

Once, we have got the variables that can explain the meaning of the axis, we need to know if they are positively or negatively correlated, this can be found looking directly into the V-values of the first axis, v_1. The sign indicates the direction of the variable in relation to the component. This is particularly interesting because we must determine which is the direction of the first component in order to know which are the best alternatives. In our case, all the variables are expressing preferences, where the higher the value, the more preferred the alternative is. Thus, the sign of coefficients of v_1 should be the same if all the criteria agree in giving the same kind of preference (good or bad) to the same alternatives. When a criteria is saying the contrary than the others, its sign will be the opposite of the others. In case of having a set of positively correlated variables of similar dimension to the set of negatively correlated variables, we will stop the MCDA process because the direction of the first component cannot be established.

STEP 4 – Calculate the contribution of each variable to the formation of the first principal component (Eq.4.19). If a variable did not contributed to the formation of the first axis, it means that this variable does not give any useful information for the determination of the axis to be used in the ranking.

When a variable highly contributes to the second principal component and not to the first one, we can say that this variable is in contradiction (it is perpendicular) to our social axis, which is the first one.

If a variable does not contribute to any axis, it means that it can be eliminated from the analysis and the result would not be significantly different.

STEP 5 – Find the z-scores of the prototypes in the first principal component, z_1, using (Eq.4.3), where x^* are the columns of the prototypes matrix. Before, these values have been centred and standardised.

The z-scores tell us the position of each class into a line, which defines a total order among them. The direction of the director vector of this line determines which is the best and worse position. This direction has been found in step 3. Thus, the ranking of the classes we were looking for is already set.

If the process finishes successfully, in step 5 we have obtained the z-scores in the first principal component, z_1. However, the values of z_1 do not belong to a predefined real interval. To be used in the following stages of the MCDM process, we need to know the position of the clusters in the [0,1] interval. To perform this scaling for a given prototype, j, we use Eq.4.23.

$$z_{01}(j) = \frac{z_1(j)}{z_1(a_{ideal}) - z_1(a_{nadir})} \qquad \text{Eq. 4.23}$$

The a_{ideal} is a fictitious alternative that takes the best possible value for each criterion. If this alternative existed, it will be the most preferred by the decision maker. On the other hand, the a_{nadir} is a fictitious alternative with the worst possible value for each criterion.

4.2 Ranking based on the similarity to the Ideal alternative

The second procedure, denoted as CASE B in the description of the ranking phases, corresponds to the situation in which criteria are not correlated enough. For this case, we propose the application of another ranking technique based on similarity functions. Due to the distinct opinions of the experts (or criteria suppliers) or the incomparable meaning of the criteria, we will need a separable measure, which compares the objects criterion by criterion.

We assume that for each criterion there is a single value of its domain, v_{ij}, which is the best. That is, if alternatives were only described with this criterion, the ones with value v_{ij} will be selected by the decision maker. With the values v_{ij} we build an ideal alternative, denoted a_{ideal}, which is the one that has the best value for each criterion. This ideal alternative is the same one considered in the previous section to locate alternatives in the [0,1] interval.

The ranking is based on the comparison of prototypes with respect to the ideal alternative. The alternatives that belong to the class whose prototype is nearer to a_{ideal} are the best ones. To compare them we must use a similarity measure, like the ones used during the clustering process.

With this approach, the position in $\Re$ of a cluster is given by:

$$z(j) = similarity\left(prototype_j, a_{ideal}\right) \qquad \text{Eq. 4.24}$$

where the lower the z, the better the cluster is.

A similar approach is the one known as TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), developed by Yoon and Hwang [Hwang&Yoon,1981]. TOPSIS is based on the concept that the selected alternative should have the shortest distance from the ideal solutions and the farthest distance from the negative-ideal (nadir) solution. Therefore, they define a measure of the relative closeness to the ideal as:

$$C_{i^*} = \frac{\sqrt{\sum_{j=1}^{p}\left(v_{ij} - v_{j-}\right)^2}}{\sqrt{\sum_{j=1}^{p}\left(v_{ij} - v_{j^*}\right)^2} + \sqrt{\sum_{j=1}^{p}\left(v_{ij} - v_{j-}\right)^2}}$$

That is, they calculate the Euclidean distance between the alternative a_i and the ideal, defined as $a_{ideal} = (v_{1^*}, v_{2^*}, ..., v_{p^*})$, and the Euclidean distance between the alternative a_i and the nadir one, $a_{nadir} = (v_{1-}, v_{2-}, ..., v_{p-})$. Then the ranking of the alternatives in found according to the preference rank order of C_{i^*}.

Using the TOPSIS approach, if we have two alternatives with same similarity to the ideal, the one that is furthest from the nadir is the one considered as best than the other

one. If we represent it in a two-dimensional space (Figure 12), we can see, that the alternative more distant to the nadir is the one that has a greater difference in the values given by the two criteria (*a* is considered as 0.5 for one criterion and 0.8 for the other). Their corresponding closeness preference values according to TOPSIS will be: $C_{a^*} = 0.64$ and $C_{b^*} = 0.62$. So, the best one is *a*.

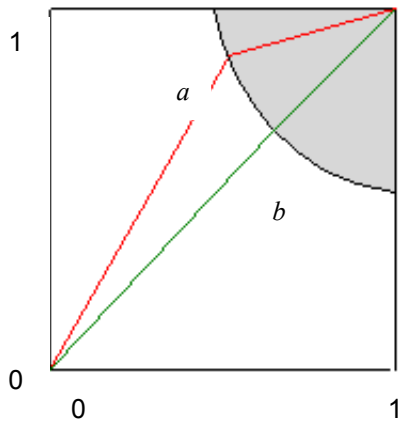

Figure 12. Ranking of alternatives with TOPSIS

However, this approach does not have into account the agreement between the criteria. Under our point of view, alternative *a* is as good as *b* with respect to the goal of achieving the values of the ideal solution. The difference between them is related to the knowledge we have about their goodness. For this reason, we propose to consider them as equal and give extra knowledge to the decision maker about the trustworthiness of their position in the preference ranking. As it will be explained in more detail in the next chapter, our confidence on *b* is greater than on *a*, because the two criteria give the same value to *b*, whereas our knowledge about *a* is that it can be as good as 0.8 indicates, or it can be not so good as 0.5 says. For this reason, the ranking method we propose only compares the prototypes with the ideal alternative.

Moreover, after studying the properties and behaviour of different similarity measures to rank the clusters, we propose the use of the Manhattan distance if we have qualitative criteria in our decision matrix. The Manhattan distance (Eq.3.3) is appropriate when working with numbers that represent linguistic terms, as it has been argued in section 3.3.3, where it has been recommended to be used in the aggregation process.

If no qualitative criterion is considered, we recommend to apply the same measure used in the first stage, so that the same conditions apply during all the process (this is, to avoid different similarity functions in the same process because each similarity function has different properties).

4.2.1 Application of the similarity-based ranking

As said, this ranking procedure will be used in case of having non-correlated preference criteria or descriptive criteria with a non-ordered domain. The information provided by the aggregation stage is the same than in the PCA ranking: a data matrix with the alternatives described according to a set of criteria, the grouping of this alternatives into similarity classes and, finally, the prototype of each class in terms of the same criteria.

To find the ranking, we start with the prototypes of the clusters. For each prototype we measure the similarity (or distance) to the *ideal* alternative. The result will indicate a degree of preference of a particular cluster.

Repeating this distance measurement for all the prototypes we obtain a numerical degree of preference of all clusters (we denote by $z(j)$ the numerical value of the j-th cluster Eq. 4.24). Using these values we can determine an order among the clusters.

Now, we have got a rough approximation of the position of the clusters in a numerical interval [a,b]. As we have explained in section 4.1.6, the values that the following stages require must be in the [0,1] interval. For this reason we must apply the same transformation function that was indicated for the PCA method, Eq. 4.23, which is reproduced here:

$$z_{01}^{*}(j) = \frac{z(j)}{z(a_{ideal}) - z(a_{nadir})}$$

In this case, $z(a_{ideal})$ will be 0 because the distance between the ideal solution and itself is 0. Moreover, the values we obtain will be ordered from best to worse, that is, the alternative with a lower z_{01}^{*} will be the best one, whereas in the PCA ranking the ordering was the opposite. For this reason the following transformation is applied to the z_{01}^{*} values.

$$z_{01} = 1 - z_{01}^{*} \qquad \text{Eq. 4.25}$$

After these calculations, the result of the ranking stage for case B is the same than case A: we have obtained a totally ordered set of clusters. This leads to an ordered partition of the alternatives. This ordered partition defines a new qualitative ordered criterion.

Chapter 5.

Explanation and Quality stages

The outcome of the ranking stage is an ordered set of clusters, where each cluster is defined in terms of several alternatives. This cluster has also associated a value in the [0,1] interval corresponding to a rough approximation of its position on the "social axis". In this section we describe how to associate a linguistic term to each cluster (and, therefore, to each alternative). The linguistic terms will replace the numerical rough approximations computed in the previous stage. To complete the process and obtain a new qualitative preference criterion, we must establish not only the vocabulary but also the negation-based semantics of this criterion.

In the first part of this chapter, the complete methodology to build the new qualitative criterion is explained. Several algorithms have been developed in order to deal with all the special situations and obtain a good vocabulary with an appropriate semantics. This is very important because these are the tools that we give to the user to understand the result of the decision making process.

The second part of the chapter is devoted to the evaluation of the goodness of this new criterion, which we have called: the quality measurement stage. This goodness is calculated from the information provided at the different stages of the process: the aggregation through clustering, the ranking (with the Principal Components Analysis or with the Similarity calculation) and the vocabulary building. Many different factors are analysed and included in a final qualitative measure of the trustworthiness of the resulting criterion. However, we also recommend having into account not only the final qualification but also the partial quality measures of each stage.

5.1 Giving semantics to the ordered set of clusters

The main goal of this phase of the process is to give meaning to the ordered qualitative domain of the new-created criterion. At this stage, the values of this domain are terms artificially generated in the first stage. We want to change these terms by others that have a meaning easily understandable for the decision maker.

We propose a new method to select the most appropriate linguistic terms to describe each cluster of alternatives. With these terms we build the vocabulary and semantics of the new overall criterion.

The vocabulary can be obtained from the ones used by the different preference criteria in the data matrix, or it can be given by the user. Once we have the set of possible terms to be used, we apply a new assignation procedure to select the best term for each cluster. During this process, we can split up some terms to obtain others with a finer semantics, that is, to generate more precise terms. The new linguistic labels are obtained using linguistic hedges.

When the selection of the terms to be used has been done, the new vocabulary has been established. The next step consists of giving the semantics to these terms that is, building the negation function over this vocabulary.

5.1.1 The vocabulary of the result

To determine which is the most appropriate set of terms to be used in the new criterion, we distinguish two different situations:

CASE C: The decision maker provides a vocabulary to be used in this stage. This vocabulary must consist of a finite ordered set of terms and a negation function over these terms.

CASE D: No vocabulary is given by the decision maker. Then, the system has to choose one of the vocabularies of the criteria provided by the experts when they have filled the decision matrix.

We believe that the less parameters the user has to define when running a decision support system, the more encouraged to use it he will be. The large amount of information required to the decision maker may be a counterpart for its use in daily situations. For this reason, we will only consider CASE C when there is no possibility to describe the result with the vocabularies of the original criteria. For example, in Table 7 we have that the three criteria are not appropriate for expressing a preference ranking over the alternatives. Thus, the user should provide a vocabulary like the one in the last row.

	lowest value . . . *largest value*
Weight	lean, thin, normal, corpulent, fat, overweighted
Distance	same_place, close, near, far, remote, outlying
Waiting time	very_short, short, acceptable, long, very_long
Preference	terrible,bad,not-recommendable,acceptable, recommendable,good,very_good,ideal

Table 7. Qualitative vocabularies of the criteria vs. preference vocabulary for the raking

We can see that the vocabularies in Table 7 are ordered sets of terms, but the higher value does not necessary mean that it is the desired value. For example, concerning the weight, we may prefer a corpulent person than a fat or a normal one.

In CASE D or when some of the vocabularies of the criteria are already expressing preferences over the alternatives, we can use their values to qualify the clusters of alternatives without having to ask to the decision maker. In this case, we have the problem of choosing a vocabulary among the possible ones. We have defined a distance measure between ordered qualitative vocabularies, d_v, based on the fact that each vocabulary is a set of bounded closed non-overlapping intervals in [0,1].

First, we define a *centre function* as a function that assigns to each value x_i in [0,1] another value in [0,1] that is the value of the central point of the interval $(m,M]$ to which x_i belongs to. This centre function is a left continuous step function.

Having two vocabularies, V_A and V_B, we denote A and B their corresponding centre functions, such that, for any $x \in [0,1]$,

$$A : x \to a_x$$
$$B : x \to b_x$$

where a_x is the central point of the interval of A to which x belongs, and b_x is the central point of the interval of B to which x belongs.

Then, we define a measure of similarity between vocabularies as follows:

$$d_v(V_A, V_B) = d_v(A, B) = \left[\int_0^1 d^2(a_x, b_x) dx \right]^{1/2} \qquad \text{Eq. 5.1}$$

where $d^2(a_x, b_x) = (a_x - b_x)^2$.

It can be easily seen that $d(a_x, b_x) = \sqrt{(a_x - b_x)^2}$ is the Euclidean distance between two points.

Theorem: $d_v(V_A, V_B)$ is a distance.

Proof.

(1) Positivity.

According to the definition of $d_v(V_A, V_B)$, the result cannot be negative, $d_v(V_A, V_B) \geq 0$.

Let's proof that if $d_v(V_A, V_B) = \left[\int_0^1 d^2(a_x, b_x)dx\right]^{1/2} = 0$ then $V_A = V_B$

We will show that when $d_v(V_A, V_B) = 0$, for any $x \in (0,1]$, $d^2(a_x, b_x) = 0$, which means that a_x and b_x are always equal ($V_A = V_B$).

Let us suppose that there exists $x' \in (0,1]$, such that $d^2(a_{x'}, b_{x'}) = (a_{x'} - b_{x'})^2 \neq 0$, as A and B are left-continuous step functions, for any $x' \in (0,1)$, there exists an $x'' \in (0,1)$, $x'' < x'$ such that $(a_x - b_x)^2 = (a_{x'} - b_{x'})^2$ for any $x \in [x'', x']$. So,

$$\int_0^1 d^2(a_x, b_x)dx \geq \int_{x''}^{x'} d^2(a_x, b_x)dx = \int_{x''}^{x'} (a_x - b_x)^2 dx = \int_{x''}^{x'} (a_{x'} - b_{x'})^2 dx = (a_{x'} - b_{x'})^2 (x'' - x')$$

as $(a_{x'} - b_{x'})^2 \neq 0$ and $(x'' - x') > 0$, we have that the previous expression is positive, i.e., $\int_0^1 d^2(a_x, b_x)dx \geq (a_{x'} - b_{x'})^2 (x'' - x') > 0$, which contradicts the original assumption $d_v(V_A, V_B) = \left[\int_0^1 d^2(a_x, b_x)dx\right]^{1/2} = 0$.

So, it is not possible to find any $x' \in (0,1]$ such that $d^2(a_{x'}, b_{x'}) = (a_{x'} - b_{x'})^2 \neq 0$. Therefore, $a_{x'} = b_{x'}, \forall x' \in [0,1]$, i.e. $V_A = V_B$

(2) Symmetry.

For any V_A, V_B,

$$d_v(V_A, V_B) = d_v(A, B) = \left[\int_0^1 d^2(a_x, b_x)dx\right]^{1/2} = \left[\int_0^1 d^2(b_x, a_x)dx\right]^{1/2} = d_v(B, A) = d_v(V_B, V_A)$$

since $d^2(a_x, b_x) = (a_x - b_x)^2$ is symmetric.

(3) Triangle inequality.

We want to show that $d_v(V_A, V_B) \leq d_v(V_A, V_C) + d_v(V_C, V_B)$.

We know that $d(a_x,b_x) \le d(a_x,c_x)+d(c_x,b_x)\ \ \forall x \in [0,1]$, because it is a distance.

From this inequality we can also have, $d^2(a_x,b_x) \le (d(a_x,c_x)+d(c_x,b_x))^2$ or $d^2(a_x,b_x) \le d^2(a_x,c_x)+d^2(c_x,b_x)+2d(a_x,c_x)\cdot d(c_x,b_x)$.

So, if we introduce the bounded integral in each operand, $\int_0^1 d^2(a_x,b_x)dx \le \int_0^1 d^2(a_x,c_x)dx + \int_0^1 d^2(c_x,b_x)+2\int_0^1 d(a_x,c_x)\cdot d(c_x,b_x)dx$, the inequality is also true.

Since $\int_0^1 d(a_x,c_x)\cdot d(c_x,b_x)dx \le \left(\int_0^1 d^2(a_x,c_x)dx \cdot \int_0^1 d^2(c_x,b_x)dx\right)^{1/2}$, we have that $\int_0^1 d^2(a_x,b_x)dx \le \int_0^1 d^2(a_x,c_x)dx + \int_0^1 d^2(c_x,b_x)+2\left(\int_0^1 d^2(a_x,c_x)dx \cdot \int_0^1 d^2(c_x,b_x)dx\right)^{1/2}$ or $\int_0^1 d^2(a_x,b_x)dx \le \left[\left(\int_0^1 d^2(a_x,c_x)dx\right)^{1/2} + \left(\int_0^1 d^2(c_x,b_x)\right)^{1/2}\right]^2$ which is exactly the triangle inequality property:

$$\left[\int_0^1 d^2(a_x,b_x)\right]^{1/2} \le \left[\int_0^1 d^2(a_x,c_x)\right]^{1/2} + \left[\int_0^1 d^2(c_x,b_x)\right]^{1/2}$$

□

This distance measure take values in [0,0.25], being 0 the value indicating that two vocabularies are identical, and being 0.25 the maximum distance value for two different criteria. This maximum is obtained when the intervals of the two negation-based vocabularies are completely different.

Proof.

The difference between the centers of two overlapping intervals reaches its limit when these intervals are large and are positioned far from one to each other. The maximum length of the intervals is achieved having the minimum number of terms. That is, having a vocabulary with only 1 term, and the other one with 2 terms (Figure 13).

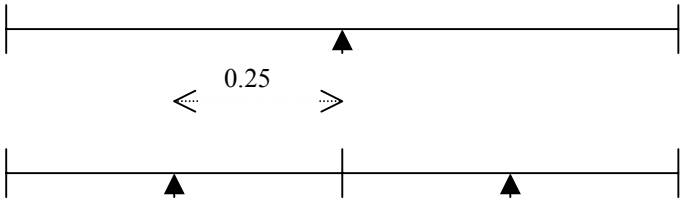

Figure 13. Maximum distance between overlapping negation-based intervals

In this situation, the maximum difference of the centers is 0.25 for all the points in the domain [0,1]. Therefore, for all x in [0,1], we have

$$d^2(a_x, b_x) = 0.25^2 = 0.0625$$

which can be substituted to d_v to obtain the maximum distance value:

$$d_v(V_A, V_B) = \left[\int_0^1 0.0625 dx\right]^{1/2} = 0.25$$

□

We apply the distance d_v to measure the similarity between each vocabulary given by the experts and the result of the ranking phase, which is a set of ordered names of clusters.

For each vocabulary of the criteria we have a negation function that allow us to obtain the interval $(m,M]$ corresponding to each term (using Eq.3.1). Obtaining the centre of this interval (i.e. a_x) is straightforward. Moreover, for each cluster we know the position of the prototype in the interval [0,1]. Being b_x the centres of the intervals, it is possible to know the boundaries of the intervals. Therefore, we have all the information needed to calculate the integral in expression Eq. 5.1.

The criterion whose vocabulary is the most similar to the set of clusters is selected to be used to explain the meaning of those clusters.

5.1.2 Assigning the most appropriate term to each cluster

Once we have the final vocabulary selected (or provided by the user), we have to assign a term of this vocabulary to each class. This term will describe the suitability of the cluster for the decision problem. Moreover, we can only use each term once, because if more than one cluster receives the same term, they will be indistinguishable.

We have a method to solve this selection problem. Some intuitive assumptions have been considered:

- no cluster with a position, z_{0l}, lower than 0.5 will receive a positive term
- no cluster with a position, z_{0l}, higher than 0.5 will receive a negative term
- if a cluster is near the centre, 0.5, it will receive the neutral term
- the neutral term, if exists, will have a negation equal to itself

With this requirements, we have developed the following procedure that divides the vocabulary into three parts: positive terms (those with a preference higher to 0.5), negative terms (those with a preference lower than 0.5) and the neutral term (the one whose negation is itself, and its value is 0.5). For knowing the position see the semantics induced by the negation function (Eq.3.1).

According to the negation function it is possible that the selected vocabulary does not have any neutral term. In this case we will have the vocabulary divided into two sets, instead of three.

The procedure has 6 steps:

1. Find the cluster with corresponding z-value equal to $0.5 \pm \xi$, which will be denoted $C_{neutral}$
2. If it exists then assign to it the neutral term, $T_{neutral}$ (if the vocabulary does not have a neutral term, it will be provided by the user). For further calculations, consider that $C_{neutral}$ is positioned in z_{01}=0.5.
3. Divide the clusters into two groups:
   Positive Clusters (positioned between 0.5 and 1) and
   Negative Clusters (positioned between 0 and 0.5)
4. Divide the vocabulary into two groups:
   Positive Terms (following $T_{neutral}$) and
   Negative Terms (preceding $T_{neutral}$)
5. If the granularity of any group is smaller than the number of clusters of the corresponding group, apply the algorithms `Making_new_labels` and `Make_names` until we have the same number of terms than clusters.
6. Apply the algorithm `Explain_result` to the 2 groups independently.

Two additional algorithms have been defined in order to sort out two particular steps of this assignation process [Valls&Torra, 2000b]. Firstly, we will see the algorithm to assign terms than are able to explain the result (i.e. the alternatives according to the clusters). The inputs to the algorithm are the set of ordered clusters and the set of ordered terms to be used to qualify the clusters.

```
Algorithm Explain_result is
 k := number of clusters to be explained
 if k=number of terms then
     Assign these k terms to the k clusters
 else
     Take the best cluster of the set (C_best)
     While k>0 do
        Take all those terms in the vocabulary that have at least
         k-1 worse terms [t_b..t_a]. Moreover, t_a should not be
         better than any previously assigned label.
```

```
        If  similarity(C_best, Ideal) belongs to one of the
         intervals of the terms in [t_a..t_b]  then
            C_best takes the term corresponding to this interval
        else
            if similarity(C_best, Ideal)>I(t_a) then
                     C_best takes t_a (the best possible label)
            elsif similarity(C_best, Ideal)<I(t_b) then
                     C_best takes t_b (the worst possible label)
            end if
        end if
        k := k-1;
        If   k = number of terms that follow the assigned term
         then
            Assign these k labels to the k remaining clusters
            k := 0;
        else
         Take the cluster that follows C_best in the ranking, and
          call it C_best
        end if
      end while
  end if
end algorithm.
```

This method pretends to give the most appropriate term to each cluster maintaining always the ranking among them. However, we suppose that the decision maker is particularly interested in knowing which are the best alternatives, because he is trying to make a good decision. Thus, we start the process with the selection of the most suitable term for the first cluster in the ranking, provided that we leave enough terms for the rest of clusters.

This algorithm needs a set of terms equal or larger than the set of clusters. If the vocabulary selected does not have enough terms, we have designed an method to create new terms using the ones that we have in the vocabulary. The key idea is to split some terms up and use a qualifier to distinguish the two new parts. So, the problem is reduced to the selection of the labels most adequate to be split.

As we have some information (given by the negation function) about the meaning of the labels in a vocabulary, we can use it to guide the process. A label that has more than one label in its negation indicates that there are slight differences between some of the alternatives assigned to it, in some sense, there is a gradation in

the meaning of the label, and each degree corresponds with a label in the negation. Under this interpretation, this label is a candidate to be split up.

```
algorithm Making_new_labels is

  repeat
    {t_left, t_right}:= split the most suitable label, t_k
       T := remove t_k from T
       T := add t_left and t_right to T
  until we have the desired number of terms

end algorithm.
```

We assume that the labels in a vocabulary cover all the possible values in [0,1]. Each label t_i corresponds to a fixed interval $[m_i, M_i]$, as in Figure 14.

The splitting method begins by looking for the possible cut points. This is done with the help of the negation function, which is used to calculate the numeric intervals of each label. Then, these values are projected into the opposite part of the domain [0,1] to find out which labels have more specific meanings.

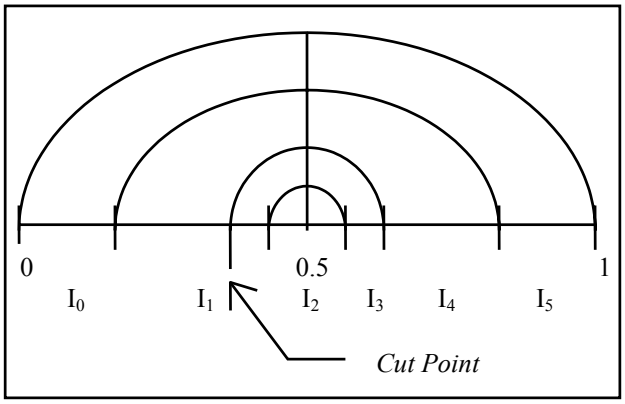

Figure 14. The negation procedure for generating new terms

Once we have got the cut points, we apply each one of these cuts to the vocabulary separately. Thus, we obtain a new possible vocabulary for each cut point. Then, each new vocabulary is compared to the ordered names of the clusters (the result of the second stage) using the distance we have defined, d_v. The vocabulary that is closer to the partition is chosen, and two new labels are obtained from the one we have split. If we already do not have enough labels, we repeat the process of applying the cut points but now they are applied to this new vocabulary.

However, it is possible to have some situations where the negation cannot produce the number of new terms required [Valls&Torra, 1999a]. For example, when the negation function is the classical one, we cannot obtain any new term because all have the same dimension. Then, if the clusters obtained are concentrated on one side of the vocabulary (if they are mainly good or bad), we will have a lack of terms.

In this particular case, where the negation-based semantics cannot help, the solution proposed consists of identifying the term that has a larger number of clusters to explain, and split it up. This process can be repeated until we have produced the desired number of terms.

When a term is selected to be split, t_i, we have to divide its corresponding interval $[m_i, M_i]$ and obtain $[m_i, c_i]$ and $[c_i, M_i]$. In order to obtain the most accurate cut point, c_i, we propose to use the information of the position of the clusters.

Let us suppose that we have 3 clusters (α, β and γ) with the following z_{0l} positions after the ranking (see Figure 15):

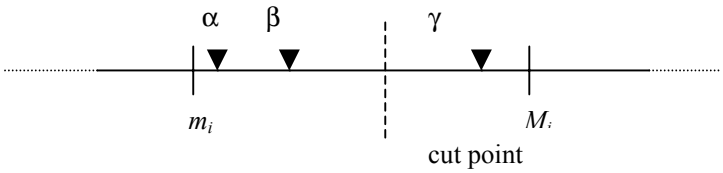

Figure 15. Selected cut point for the interval $[m_i,M_i]$

The most suitable cut point is the one between the two clusters that are more distant from each other. That is, if in Figure 15 the distance between α and β is 0.05 and between β and γ is 0.15, we decide to break up the interval just in the middle between β and γ, since the meaning of the two clusters is more different than the meaning of β with respect to α.

Each time we split a term up, two new terms are needed. The method to create new terms for the new intervals in a vocabulary is not trivial, because they should be in accordance with the rest. For this reason, we do not invent them, we introduce linguistic hedges (e.g. *very*, *not-so*, ...) in order to distinguish the different grades in the meaning of the term.

To keep the structure of the qualitative vocabularies, we have decided that the neutral label (if exists) it is never split up, since its meaning is that its negation is itself, and an split will end with this property. The rest of the vocabulary can be divided in two sets: T_{inf} and T_{sup}. T_{inf} has the labels that are smaller than the neutral value, and T_{sup} the ones that are greater than the neutral value. Then, the process is the following:

```
algorithm make_names is
   if t∈T_inf then
         if t has not been previously split then
           return {very-t, t }  being very-t < t
         else  /* this means that very-t exists */
           return  { t, not-so-t }  being t < not-so-t
         end if
   else  /* t∈T_sup */
         if t has not been previously split then
           return { t, very-t }   being t  < very-t
         else  /* this means that very-t exists */
           return  {not-so-t, t }  being not-so-t < t
         end if
   end if
end algorithm.
```

We express the grades in the meaning, introducing a new more precise term that uses the modifiers *very* or *not-so*.

This algorithm assumes that we will only cut a term once or twice. That is, we will not generate more than 3 terms from a single one. We consider that if more than 3 terms must be obtained, we should ask the decision maker (i.e. the user) in order to obtain more appropriate terms.

Regarding the global process presented in this section, it may produce bad results if there are some clusters whose positions are very close (we should consider that a difference of only the 20% of the length of the term is problematic). This situation indicates that we have two clusters that are very similar in relation to the ranking position (given by the Principal Components Analysis or by the Similarity-based Ranking) but whose elements were not considered similar enough to be assigned to the same class, during the clustering process (the aggregation stage). This is a problematic situation, since the ranking methods have not distinguished the goodness of the two different clusters in relation to the ideal alternative. However, the quality measures that we have defined (which will be detailed in section 5.2.2) will give us some idea of the trustworthiness of the ranking obtained. If the degree of quality is under some threshold, the decision maker can decide to stop the process, or to ignore the values finally given to these conflicting clusters.

5.1.3 Building the negation function of the new criterion

Once we have got a set of terms, possibly adapted to fit the consensus partition, we have to study their semantics. If the consensus partition were identical to the expert's one, the meaning of the terms would not change, but this will usually not be the case. The meaning of the terms has to be built knowing the alternatives that each term is now describing.

Following the approach based on negation functions, the meaning of each term is going to be expressed using the negation. Moreover, this is also the form in which experts have supplied their knowledge. So, they are supposed to be familiar with the negation concepts and notation. Therefore, it will be an easy and comprehensible form to express the meaning of the new terms.

To calculate the new negation function, first we have to attach a numerical interval in [0,1] to each label, $I(t_i)$. The disjoint intervals are built with the positions z_{01} of the clusters into the first principal component. Using the fuzzy approach for linguistic labels, we can say that the labels have a triangular membership function [Yuan&Shaw,1995] (except in the extremes), so the z-value is taken as the point of the label where the membership value reaches 1.

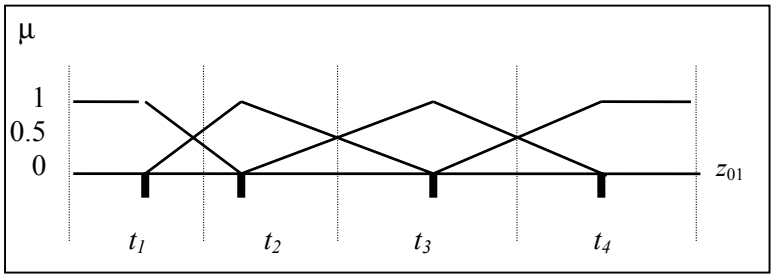

Figure 16. Fuzzy partition used to establish the semantics

If some of the terms of the vocabulary have not been used to explain the clusters obtained in the previous stages, we include a new imaginary cluster with a prototype positioned in the centre of the interval corresponding to this term. Then, the negation function is built with the real and imaginary prototypes. The additional prototypes are located in the centre in order to try to avoid the changes in the limits of the labels that are not used in the result, since we do not have any information about what should be their meaning in the new criterion.

In order to keep the neutral term centred in 0.5, we begin the process of building the fuzzy sets from the middle. If the two neighbour prototypes are not located at the same distance from 0.5, we take the nearest prototype location as the boundary of the support of the fuzzy set of the term. For example, in Figure 17 we can see 3 prototypes (marked with a bold line), the one in the left is the closest to the neutral class, so this establishes the point where the membership to the neutral cluster ends in the left. Since the similarity function of the neutral term must be symmetrical, we have that the end of the membership function in the right is established at the same distance to the centre than the prototype in the left.

Once the fuzzy set of the neutral term has been fixed, we continue with the rest of the membership functions as explained before.

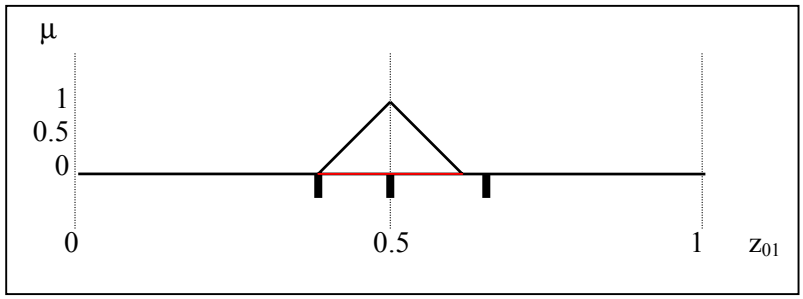

Figure 17. Negation for the neutral term

It can be observed that, in general, the middle point between two consecutive projections is the one that has membership equal to 0.5. These are the points usually corresponding to the limits of intervals, as in the example of Figure 18.

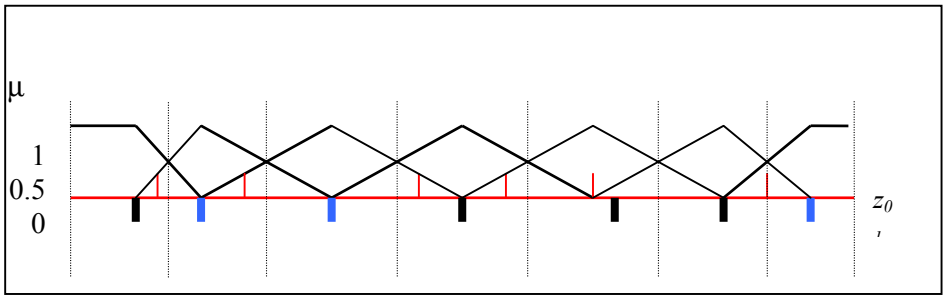

Figure 18. Fuzzy sets corresponding to an example with 4 clusters (the blue marks correspond to imaginary prototypes for unused terms, the black ones are the real positions of the clusters after the ranking, the red line corresponds to the distribution of the terms according to the original negations)

Once each term has its corresponding interval in the new criterion, $I(t_i)$, the negation of each one can be computed as:

$$Neg(t_i) = \{ t_j \mid I(t_j) \cap 1\text{-}I(t_i) \neq \varnothing \} \qquad \text{Eq. 5.2}$$

where $1\text{-}I(t_i)$ is the interval between $1\text{-}\max(I(t_i))$ and $1\text{-}\min(I(t_i))$.

Using Figure 18 we will follow an example of the negation function generation. We will see that some problems appear, and we will present some methods to sort them out.

Let us take that the original vocabulary is $\{l_1, l_2, l_3, l_4, l_5, l_6, l_7\}$, and its semantics is given by the negation function we have in the first column of Table 8. The second column shows the interval corresponding to each label according to this semantics (using Eq.3.1).

Original Negation	Original Intervals
Neg (l_1) = $\{l_7\}$	I (l_1) = [0.0, 0.11]
Neg (l_2) = $\{l_6\}$	I (l_2) = [0.11, 0.22]
Neg (l_3) = $\{l_5, l_6\}$	I (l_3) = [0.22, 0.44]
Neg (l_4) = $\{l_4\}$	I (l_4) = [0.44, 0.56]
Neg (l_5) = $\{l_3\}$	I (l_5) = [0.56, 0.67]
Neg (l_6) = $\{l_2, l_3\}$	I (l_6) = [0.67, 0.89]
Neg (l_7) = $\{l_1\}$	I (l_7) = [0.89, 1.0]

Table 8. Semantics for the example with 7 terms

Now we look at the positions of the clusters. Let us suppose that we have obtained 4 clusters. In Table 9 we have the positions of the real (black) and additional (blue) prototypes.

Class	Positions in [0,1]	Term
7th	0.09	l_1
6th	0.16	l_2
5th	0.33	l_3
4th	0.48 → 0.5	l_4
3rd	0.71	l_5
2nd	0.80	l_6
1st	0.94	l_7

Table 9. Positions of the 4 clusters in the example

After applying the methodology to build the new intervals for the terms, we obtain the result shown in Table 10. The first column is the result of the interval generation based on the fuzzy membership functions. The second column is the opposite interval corresponding to each term, which is calculated doing 1-x_i. Finally, the third column gives the negation induced by these intervals, considering that a difference of 0.02 in the value of the borders is not significant. In general, if we have 7 terms in the vocabulary each one covers a 14% of the domain, so a 0.02 is only 1/7 of the length of a term. However, this value could be changed according to the characteristics of the application domain or the decision maker opinion.

Intervals from fuzzy sets	Opposite interval	Negation induced
I (l_1) = [0.0, 0.125]	[1.0, 0.875] ≅ I(l_7)	Neg (l_1) = {l_7}
I (l_2) = [0.125, 0.245]	[0.875, 0.755] ≅ I(l_6)	Neg (l_2) = {l_6}
I (l_3) = [0.245, 0.33]	[0.755, 0.67] ≅ I(l_5)	Neg (l_3) = {l_5}
I (l_4) = [0.33, 0.67]	[0.67, 0.33] = I(l_4)	Neg (l_4) = {l_4}
I (l_5) = [0.67, 0.735]	[0.33, 0.265] ≅ I(l_3)	Neg (l_5) = {l_3}
I (l_6) = [0.735, 0.87]	[0.265, 0.13] ≅ I(l_2)	Neg (l_6) = {l_2}
I (l_7) = [0.87, 1.0]	[0.13, 1.0] ≅ I(l_1)	Neg (l_7) = {l_1}

Table 10. Result of the semantics generation

Notice that, in this example, we have obtained a new criterion with the classical negation function. In Figure 19 we can see the distribution of the intervals according to the new semantics against the original distribution. As we can see, the new intervals are more suitable to explain the clusters, because each cluster belongs to a different interval.

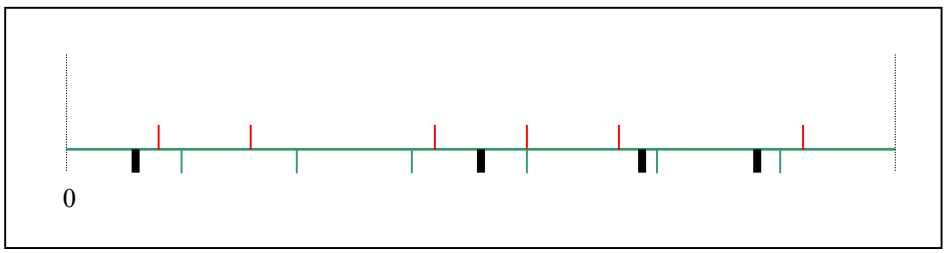

Figure 19. Comparison between the old (up) and new (down) intervals

It is worth to note that once we have established the negation function of the new criterion, the intervals induced by this negation may be slightly different to the ones we have used to build the function. In Table 11 we can see the intervals obtained (with Eq.3.1) from the classical negation function. These values can be compared to the ones calculated from the positions of the clusters according to the ranking, which are the ones in the first column of Table 10.

New Negation	New Intervals
Neg (l_1) = {l_7}	I (l_1) = [0.0, 0.143]
Neg (l_2) = {l_6}	I (l_2) = [0.143, 0.286]
Neg (l_3) = {l_5}	I (l_3) = [0.286, 0.428]
Neg (l_4) = {l_4}	I (l_4) = [0.428, 0.571]
Neg (l_5) = {l_3}	I (l_5) = [0.571, 0.714]
Neg (l_6) = {l_2}	I (l_6) = [0.714, 0.857]
Neg (l_7) = {l_1}	I (l_7) = [0.857, 1.0]

Table 11. Negation function for the new criterion

5.2 Quality Measurement

In this section we define some quality measures that can be useful for the user in order to decide the reliability of the result. In many applications where fusion techniques are required, it is interesting to know to what extent the result of the process is acceptable. In addition, if the person that is executing the process is a non-specialised end user, the ignorance about the way the result is obtained often causes a mistrust feeling, and the consequent abandon of the system to continue doing the processes by hand.

For this reason, we have studied in detail the techniques applied at each stage of this new method. In the rest of the section we will define some quality measures that use the information available at the different stages.

5.2.1 The quality of the aggregation

Remember that our aggregation method is based on a hierarchical agglomerative clustering method. At each step of the process, we find out new clusters with a lower cohesion value. This cohesion value, h_α, is an upper threshold of the similarity values between any two alternatives in the class. So that, for any cluster α,

$$h_\alpha \geq d(a_i, a_j) \qquad \text{Eq. 5.3}$$

being (a_i, a_j) any pair of alternatives that belong to this cluster α.

At the end of the clustering, we can measure the global level of cohesion in the r clusters of the selected partition with Eq. 5.4. This is the first part of the *goodness* value of the aggregation stage (i.e. G_{Agg1}). According to this definition, $0 < G_{Agg1} \leq 1$, where 1 is the best value, which is obtained when the differences between the objects in the clusters are small.

$$G_{Agg1} = 1 - \frac{\sum_{i=1}^{r} h_i}{r} \qquad \text{Eq. 5.4}$$

Another interesting value to consider is the dimension of the clusters. The alternatives that belong to the same cluster cannot be distinguished by the user, because all of them will receive the same linguistic term (i.e. category). Therefore, it is appropriate to have all clusters with similar number of objects. Entropy has been used in aggregation to evaluate dispersion of weights [Marichal,1999b]. Here, defining R_i with Eq. 5.6, we can consider the use of entropy [Shannon&Weaver,1949] to measure

how much of the information is explained by each cluster. The maximum is achieved if all the clusters explain the same amount of information, that is, we have the same number of alternatives in each one.

$$G_{Agg2} = -\frac{1}{\ln r}\sum_{i=1}^{r} R_i \ln R_i \qquad \text{Eq. 5.5}$$

where r is the number of clusters in the ranking. R_i corresponds to the proportional cardinality of the i-th cluster with respect to the total number of alternatives, m, which can be seen has the probability that a random alternative a_k belongs to the cluster C_i.

$$R_i = \frac{cardinality(C_i)}{m} \qquad \text{Eq. 5.6}$$

If R_i is 0, the measure G_{Agg2} is undefined. However, this is not possible since we do not have empty clusters. We have that this quality measure (to be maximised) is limited as follows: $0 < G_{Agg2} \leq 1$.

If we are dealing with a multi-criteria selection problem, we can also inform the decision maker about the goodness of the first cluster in the ranking. In this case, it is interesting to have got a small cluster in the best position, in order to not have many alternatives indistinguishable, which may not be very helpful for the decision maker.

Having into account this last remark, we have defined the goodness of the aggregation stage subject to the dimension of the best cluster, C_{best}. That is, if the number of alternatives in this cluster is greater than the expected number of terms, we decrease the quality of this stage as it is shown in Eq. 5.7.

$$G_{Agg} = \begin{cases} \dfrac{G_{Agg1} + G_{Agg2}}{2} & \text{if } R_{C_{best}} \leq r \\ \dfrac{G_{Agg1} + G_{Agg2} - \dfrac{R_{C_{best}}}{2}}{2} & \text{if } R_{c_{best}} > r \end{cases} \qquad \text{Eq. 5.7}$$

5.2.2 The quality of the ranking

The evaluation of this stage depends on the characteristics of the decision problem, which will determine the use of the Principal Components Analysis or the use of a Similarity Function.

Ranking based on PCA

In the application of the PCA, some of the values obtained during the process are also useful to interpret the final result. Different measures are well defined in PCA literature [Jackson,1991]. We have studied the use of these measures to qualify the ranking of alternatives in a decision-making framework. Then, we have defined a goodness measure (Eq. 5.8) that takes into account the quality of the representation of the clusters by the first principal component, as well as, the agreement of the criteria (or experts) in relation to the first component.

$$G_{PCA} = \frac{\dfrac{\sum_{j=1}^{p} s \cdot CORR_1^2(x_j)}{p} + \dfrac{\sum_{j} QLT_1(j)}{\text{number of clusters}}}{2} \qquad \text{Eq. 5.8}$$

where s depends on the direction of the first component. If the x_j is positively correlated to the first component, $s = 1$. Otherwise, $s = -1$.

The best value of G_{PCA} is 1. The worst value is 0, which would correspond to a situation where the clusters were not well represented and the criteria did not agree with the first component.

In the numerator, the first addend is measuring the correlation of the variables, using equation Eq.4.20. The second addend is related to the quality of representation of the clusters, which is measured using Eq. 4.18, which can be rewritten as Eq. 5.9 for the case of a single component. If a cluster obtains a value near to 0, it means that it is bad represented by the first component, if the value is 1, the cluster is perfectly explained by the axis.

$$QLT_1(j) = \frac{z_i^{\,2}(j)}{d^2(j,G)} \qquad \text{Eq. 5.9}$$

being d the Euclidean distance between the alternative j and the centre of gravity (0 in our case, because we work with the correlations matrix).

In addition to the goodness measure, there are other interesting information values that should be given to the decision maker. The first one regards to the agreement between the experts or criteria analysed. As it has been explained in Chapter 4, the elements of the eigenvector are giving the contribution of each variable to the formation of its corresponding axis. Therefore, we can detect when a criterion differs from the social opinion, just looking into the values of the first eigenvector. If one of them is significantly smaller than the others, we can conclude that this criterion is significantly different from the consensus.

Another indicator is based on the quality of the projection of the clusters into the principal component using Eq.4.17. This allows the user to discover objects that can not be synthesised because the experts do not agree in their descriptions. In this

situation, as the aggregation is not possible, this group of alternatives[7] is removed from the study taking an "unknown" label. This "unknown" label, in case it exists, is taken from the set of terms that experts provided; otherwise, a predefined linguistic label is used.

In Figure 20 we can see a graphical representation of the PCA result for the case of two variables. In this case, this quality value will detect those clusters that may have been positioned in a point that does not represent their real relation to the other ones (like cluster D). This will happen if the criteria give different opinions about alternatives in the class. So, with this method we can tell the user which alternatives are the conflicting ones.

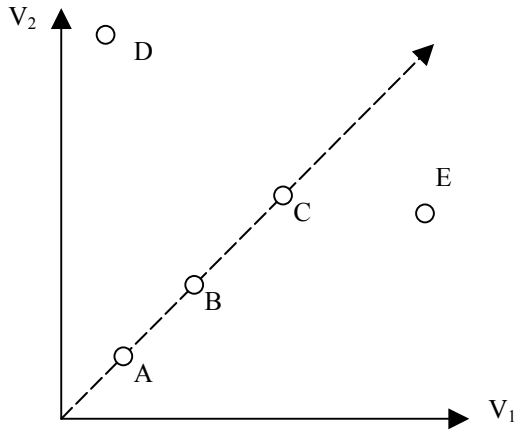

Figure 20. First principal component for a two-variable matrix

If the two variables give the same value to the alternatives, the clusters formed will be positioned in an axis that will be in the middle of the two variables, like clusters A, B and C. Alternatives that are described with different values will not be in this line. For example, alternatives in D are bad (low value) according to criterion V_1 and good (high value) according to criterion V_2. On the other hand, alternatives in E are good for V_1 but only acceptable for V_2.

Similarity-based Ranking

When the ranking is based on the similarity to an *Ideal* alternative, other quality measures have to be designed. In this case, we can have two clusters with equal similarity values but that they are quite different from one to the other. That is, the distance to the *Ideal* is the same but due to different criteria. Then, we propose to give some additional information to the decision maker about in which criteria the alternatives do not have the desired value.

In addition to this information, we have defined a goodness measure based on the agreement between the criteria for each cluster.

[7] Usually these groups are small.

$$G_{Sim} = 1 - \frac{\sum_{i=1}^{r} s_i}{r} \qquad \text{Eq. 5.10}$$

where the value that we are adding is based on the measurement of the dispersion (i.e. standard deviation, Eq. 5.11) of the values of the prototype of each cluster. The maximum value of this goodness measure is 1, which is given if all the clusters have dispersion equal to 0.

$$s_i = \sqrt{\frac{\sum_{j=1}^{r} \left(x_j - \overline{x_j}\right)^2}{r-1}} \qquad \text{Eq. 5.11}$$

5.2.3 The quality of the explanation stage

After the complete definition of the new criterion (i.e. c_{new}), we can evaluate the goodness of the new vocabulary and semantics. We should see if this new vocabulary could be misinterpreted. That is, if we are using some words that the decision maker will understand with a different meaning, we can induce him to an error. So, we propose to compare the new criterion with the ones in the initial decision matrix that have some terms in common with it, $C_{common}=\{c_i, c_j, ..., c_k\}$. Obviously, the vocabulary from which we have generated the new one will be in this set.

We propose to use the distance d_v to measure the differences in the meaning of the terms in each vocabulary. The larger the differences (remember that the distance d_v gives values in [0,0.25]), the more confusing the result may be. Therefore, when the result is 1, we have a perfect correspondence between the terms in all the experts.

$$G_{Terms} = 1 - \frac{\sum_{c_i \in C_{common}} d_v\left(c_{new}, c_i\right) \Big/ 0.25}{cardinality(C_{common})} \qquad \text{Eq. 5.12}$$

Once we have given a linguistic term to each cluster, we evaluate their appropriateness. The position of each cluster before and after the explanation stage can be compared. The ranking stage provides a numerical position in [0,1] for each set of alternatives, z_{01}, which is used to select the most appropriate label from the vocabulary. After the explanation process, the position of some clusters may have changed due to the different meaning of the terms. That is, the intervals induced by the negation function may not have the cluster at the centre of the interval.

$$G_{Neg} = 1 - \frac{\sum_{j=1}^{r} \left| z_{01}(j) - \left(m(j) + M(j)\right)/2 \right|}{r} \qquad \text{Eq. 5.13}$$

This measure compares the position of the alternatives before and after the introduction of the negation-based semantics. Being j the prototype of one cluster, $[m(j), M(j)]$ is the interval corresponding to the term assigned to this cluster using the new negation function.

Finally, we can define a global goodness measure for the whole ClusDM process.

$$G_{ClusDM} = \omega_1 G_{Agg} + \omega_2 G_{Rank} + \omega_3 G_{Terms} + \omega_4 G_{Neg} \qquad \text{Eq. 5.14}$$

where ω_i are the degrees of importance given to each step of the decision making process. For example, increasing ω_1 the user may indicate that obtaining very good and compact clusters is the best option, although it implies a change in the vocabularies and semantics. These weights must hold that $\sum \omega_i = 1$.

The ClusDM methodology pretends to be a useful recommender tool for decision makers. Our main aim has been to present the results using a linguistic vocabulary easily understandable by the user. The different goodness values can be used by the decision maker to have an idea of the quality of the different stages of the process. In addition, the overall goodness value can be also understood as the weight attached to the new preference criterion obtained.

Apart from that, our method is able to provide some additional information during the execution of the multiple criteria analysis. The importance of providing additional explanations of the results obtained with the decision model is a problem frequently considered in the Artificial Intelligence community [Papamichail,1998]. In our case, the information provided by ClusDM to the decision maker is the following:

- Which alternatives receive conflicting values from the different criteria. Those alternatives are identified during the ranking stage and do not appear in the final ranking given to the user. However, they should be presented to the decision maker in order to allow him to be aware of these special cases and perform an appropriate action if required.
- Which is the general degree of agreement (i.e. correlation) between the criteria or experts.
- Which criteria (i.e. experts) do not sufficiently agree with the result given by the system. However, this value is only available when the PCA ranking is possible.

In chapter 7, we will see some application examples where this additional information plays an important role.

Chapter 6.

ClusDM Properties

In this chapter, we will study in detail the properties of the ClusDM method. As it is a general procedure rather than a concrete algorithm, we will concentrate on the study of this method when it is applied to a set of qualitative preference criteria. In this case, this decision making technique can be seen as an aggregation operator since we are trying to capture the knowledge provided by a set of qualitative criteria and summarise it with a single overall criterion.

To study some of the properties, we will consider that the vocabulary used in the new preference criterion is specified by the user instead of being selected from the available ones. The reason for this assumption is that if the vocabulary was automatically selected, we could not study some of the properties, since the final vocabulary will not necessarily be the same after applying ClusDM to different data sets.

Formally, we will denote as Θ the ClusDM operator, which is defined over a set of qualitative preference criteria. The result, c_r, is a new qualitative preference criterion that takes into consideration the p preferences provided by the data suppliers: $c_1,...,c_p$.

$$\Theta(c_1,...,c_p) = c_r$$

It must be noted that the units that are aggregated are criteria (i.e. the columns in our decision matrix). For this reason, the property of individual independence must not be required to the rows (i.e. alternatives). In this case, it has no sense to obtain the final preference of the alternatives without considering the relationships among them, because the similarity relationships from one alternative to the others will determine its final value of preference.

For example, let us consider the two decision matrices, X1 and X2 of Figure 21. They have a unique common alternative, *a5=b5*. We can see that in the first case, X1, the alternative *a5* is placed in the second position in the ranking, with the value of

"good", whereas in the second case, X2, an identical alternative, *b5*, is placed in the first position, with a preference value of "very good". This example shows that the final preference of an alternative depends on the values of the other alternatives to which it is compared.

	c_1	c_2	c_r
a1	vgood	vgood	vgood
a2	vgood	vgood	vgood
a3	vgood	good	good
a4	vgood	good	good
a5	vgood	good	good
a6	bad	good	reg
a7	bad	good	reg
a8	bad	good	reg
a9	bad	vbad	bad
a10	bad	vbad	bad
a11	vbad	vbad	vbad

X_1

	c_1	c_2	c_r
b1	good	good	good
b2	good	good	good
b3	good	good	good
b4	good	vgood	vgood
b5	vgood	good	vgood
b6	good	bad	reg
b7	good	bad	reg
b8	good	bad	reg
b9	bad	good	reg
b10	bad	bad	bad
b11	vbad	bad	vbad

X_2

Figure 21. Qualitative matrices with a common alternative

We will now see the properties that are fulfilled by our Multiple Criteria Decision Making procedure, ClusDM, for qualitative preferences. The definition of each property is followed by an explanation to proof whether ClusDM satisfies the property or not.

6.1 Basic Aggregation Properties

We will begin with the study of three properties that are usually required to aggregation and decision making operators: symmetry, idempotency and monotonicity [Marichal,1999b].

6.1.1 Symmetry

It is also known as commutativity, neutrality or anonymity.

The symmetry property is fulfilled if the ordering of the criteria does not affect to the result. This is required when we combine criteria of equal importance or anonymous expert's opinions.

Θ is a symmetric operator if, for all criteria $c \in C$ and all $\pi \in \Pi_p$ (where Π_p corresponds to the set of all permutations of dimension p), we have:

$$\Theta(c_1,...,c_p) = \Theta(c_{\pi(1)},...,c_{\pi(p)})$$

In situations when criteria or individual opinions are not equally important, the symmetry property must be omitted.

Symmetry of ClusDM:

In the case of having equally weighted (or non-weighted) criteria, the symmetry of ClusDM depends on the clustering algorithm. If the clustering does not take into account the order of the criteria, we will obtain the same set of clusters, and the following stages will give the same results.

The clustering builds clusters according to the values in the similarity matrix. So, if the similarity function is symmetric, the grouping stage will not be affected. As we recommend the use of the Manhattan distance and this metric is symmetric, we have that ClusDM is symmetric.

However, if we would like to use another similarity function, we can easily see that the types considered in *Sedàs* [Valls et al.,1997] are also symmetric:

Similarity Calculation	***Properties***	***Sy?***
Functions based on Distances	The result is obtained from a summatory of distances. Being distances symmetric, the addition is also symmetric.	Yes
Association Coefficients	The result is based on some counters that are independent of the position of the criteria	Yes
Correlation Coefficients	Correlation is independent of the ordering of the criteria	Yes

Table 12. Symmetry of similarity functions

6.1.2 Idempotence

This property is also called unanimity. It refers to the case of having a data matrix with equal columns, that is, all the criteria give exactly the same value to the alternatives. In this case, the expected result is to have, at the end, the same value for each alternative.

$$\Theta(c,...,c) = c$$

Idempotence of ClusDM:

To check this property we will analyse the aggregation, ranking and explanation stages separately.

STAGE 1. With regard to the aggregation, the clustering will create a cluster for each term in the vocabulary of this criterion, *c*.

STAGE 2. The ranking stage can be performed using the Principal Components Analysis or the Similarity Ranking. However, if all the criteria have the same vocabulary and semantics, and they give the same value to the same alternatives, the correlation among them will be maximum. That is, we will be in CASE A and apply the PCA.

In this case, the p-dimensional space will be reduced to a 1-dimensional space, since all the dimensions are equal. Thus, the space of criteria is a line, which is the first principal component that we will obtain if we calculate the PCA. The projection of the prototypes of the clusters in this line will not modify the ranking. Moreover, the value attached to each cluster will be the central value of the interval corresponding to the term that has originated the class.
STAGE 3. In this stage we will select the vocabulary of this unique criterion, *c*, to explain the clusters (CASE D). The algorithm that selects the vocabulary will select the same terms that were originally assigned to the alternatives, because the value attached to each cluster is the central point of these intervals.

Therefore, we can say that this method is idempotent.

In addition, it is important to underline that if the vocabularies are different but they have the same granularity and the same semantics, the idempotency property will be fulfilled too.

6.1.3 Monotonicity

This property refers to the fact that increasing (respectively, decreasing) the values of one criterion in a data matrix will produce a result that is greater than the original one.

This is, when $c_k > c_k'$, where $c_k > c_k'$ means that $c_k(a_i) > c_k'(a_i), \text{for all } i = 1..m$, we will have a monotonous operator if $\Theta(c_1,...,c_k,...,c_p) \geq \Theta(c_1,...,c_k',...,c_p)$.

That is, monotonicity is satisfied if the result of applying the decision operator to a data matrix that has better preference values in one criterion, is greater or equal than the result obtained with the other matrix.

Monotonicity in ClusDM

We show with an example, that monotonicity is not satisfied. Let us consider two data matrices, X1 and X2, which only differ in the second criterion (see Figure 22). For X2, criterion c_2+ has better values for all the alternatives than the corresponding criterion c_2 in X1, so $c_2+ > c_2$.

	c_1	c_2
a_1	e	d
a_2	d	d
a_3	d	d
a_4	d	d
a_5	e	c
a_6	c	c
a_7	c	b
a_8	b	b
a_9	b	b
a_{10}	b	b
a_{11}	b	f
a_{12}	f	f
a_{13}	g	c

X1

	c_1	c_2+
a_1	e	c
a_2	d	c
a_3	d	c
a_4	d	c
a_5	e	b
a_6	c	b
a_7	c	a
a_8	b	a
a_9	b	a
a_{10}	b	a
a_{11}	b	e
a_{12}	f	e
a_{13}	g	b

X2

Figure 22. Data matrices for the monotonicity example

Let us consider the case that all criteria have the same vocabulary {g,f,e,d,c,b,a}, with g<f<d<c<b<a. The terms have the same semantics for all criteria, which is given by the following negation function, N:

N(a)={f,g}, N(b)={f}, N(c)={e}, N(d)={d}, N(e)={c}, N(f)={a,b}, N(g)={a}

Let us suppose that we want a result described with 5 terms, then, after applying the clustering method (centroid clustering with Manhattan distance), we will have 5 clusters for each data matrix. Here you have a trace of the clustering process:

STEP 1: Build a dissimilarity matrix and put together those alternatives with minimum value.

In this case, alternatives a_2, a_3 and a_4 are the components of the first class, k_1, and a_8, a_9 and a_{10} form the second class, k_2. The elements of these clusters have dissimilarity 0, which means that they are equal (indistinguishable in the space of preferences).

STEP 2: Calculate the prototype of each new class.

For matrix X1, the prototypes are:

Prototype k_1 = (d,d) = (0.5,0.5) = $a_2 = a_3 = a_4$

Prototype k_2 = (b,b) = (0.72,0.72) = $a_8 = a_9 = a_{10}$

For matrix X2, the prototypes are:

Prototype $k_1' = (d,c) = (0.5, 0.61) = a_2 = a_3 = a_4$

Prototype $k_2' = (b,a) = (0.72, 0.89) = a_8 = a_9 = a_{10}$

STEP 3: Modify the dissimilarity matrix, including the new clusters and deleting their components.

	a_1	k_1	a_5	a_6	a_7	k_2	a_{11}	a_{12}	a_{13}
a_1	0	**0.11**	**0.11**	0.33	0.44	0.55	0.61	0.45	0.44
k_1		0	0.22	0.22	0.33	0.44	0.5	0.56	0.55
a_5			0	0.22	0.33	0.44	0.72	0.56	0.33
a_6				0	**0.11**	0.22	0.5	0.78	0.55
a_7					0	**0.11**	0.61	0.89	0.66
k_2						0	0.5	1	0.77
a_{11}							0	0.5	1.05
a_{12}								0	0.55
a_{13}									0

Figure 23. Dissimilarity matrix for X1 (step3)

For matrix X1, we obtain the result in Figure 23 (the red value is the minimum). For matrix X2, we obtain the result in Figure 24.

	a_1	k_1'	a_5	a_6	a_7	k_2'	a_{11}	a_{12}	a_{13}
a_1	0	**0.11**	**0.11**	0.33	0.5	0.61	0.55	0.39	0.44
k_1'		0	0.22	0.22	0.39	0.5	0.44	0.5	0.55
a_5			0	0.22	0.39	0.5	0.66	0.5	0.33
a_6				0	0.17	0.28	0.44	0.72	0.55
a_7					0	**0.11**	0.61	0.89	0.73
k_2'						0	0.5	1	0.83
a_{11}							0	0.5	0.99
a_{12}								0	0.49
a_{13}									0

Figure 24. Dissimilarity matrix for X2 (step 3)

STEP 4: Build another level of clusters.

For matrix X1, we have that the minimum value is 0.11, which creates two new clusters: $k_3 = \{a_1, k_1, a_5\}$ and $k_4 = \{a_6, a_7, k_2\}$.

For matrix X2, we have that the minimum value is also 0.11, which creates two other clusters: $k_3' = \{a_1, k_1', a_5\}$ and $k_4' = \{a_7, k_2'\}$.

STEP 5: Calculate the prototype of each new class.

For matrix X1:

Prototype k_3 = (0.456, 0.522) = average (a_1, a_2, a_3, a_4, a_5)

Prototype k_4 = (0.676, 0.698) = average (a_6, a_7, a_8, a_9, a_{10})

For matrix X2:

Prototype k_3' = (0.456, 0.646) = average (a_1, a_2, a_3, a_4, a_5)

Prototype k_4' = (0.692, 0.89) = average (a_7, a_8, a_9, a_{10})

Notice, that clusters k_4' and k_4 are not equal. The difference is due to the fact that the interval covered by each term has not the same length, that is, we have terms whose interval of possible numerical values is smaller than others.

For matrix X1, we can stop the process because we have the alternatives in 5 groups: { a_1, a_2, a_3, a_4, a_5}, {a_6, a_7, a_8, a_9, a_{10} }, {a_{11}}, {a_{12}} and {a_{13}}.

For the case of matrix X2, we must continue a step forward, to reduce the number of clusters, now equal to 6. So, we recalculate the similarity matrix, introducing k_3' and k_4' and removing their elements (Figure 25).

	k_3'	a_6	k_4'	a_{11}	a_{12}	a_{13}
k_3'	0	**0.23**	0.48	0.52	0.49	0.47
a_6		0	0.25	0.44	0.72	0.55
k_4'			0	0.53	0.97	0.81
a_{11}				0	0.5	0.99
a_{12}					0	0.49
a_{13}						0

Figure 25. Dissimilarity matrix for X2 (step 5)

Using the dissimilarity values in that matrix, we build a new cluster with the elements of k_3' and a_6. The prototype of this new class, k_5', is (0.482, 0.647), which corresponds to the arithmetic average of a_1, a_2, a_3, a_4, a_5 and a_6.

At the end of the aggregation of X2, we have obtained the following 5 groups: {a_1, a_2, a_3, a_4, a_5, a_6}, {a_7, a_8, a_9, a_{10} }, {a_{11}}, {a_{12}}and {a_{13}}.

Notice that, the partitions generated by the same methodology with the two data matrices are different. Alternative a_6 belongs to different clusters because the increase of the values of one criterion has modified the relationships among the alternatives.

To establish a ranking of the clusters, we use the Manhattan distance with respect to the *Ideal* alternative, which in this case is I = (1.0,1.0).

In this example, we can see that if we change the values of a particular criterion (increasing them), we can modify the relations among the alternatives, which produces a new classification. In case of obtaining different clusters, some alternatives can be positioned lower in the new criterion, because now they are more similar to other alternatives with lower values. Alternative a_6, who had the best position in the ranking from X1, has the second position considering data in X2.

	Elements of the class	Prototype	Similarity to the Ideal	Rank
X1	$\{a_1, a_2, a_3, a_4, a_5\}$	(0.456, 0.522)	1.022	2
	$\{a_6, a_7, a_8, a_9, a_{10}\}$	(0.676, 0.698)	0.626	1
	$\{a_{11}\}$	(0.72, 0.22)	1.06	3
	$\{a_{12}\}$	(0.22, 0.22)	1.56	5
	$\{a_{13}\}$	(0.05, 0.61)	1.34	4
X2	$\{a_1, a_2, a_3, a_4, a_5, a_6\}$	(0.482, 0.647)	0.871	2
	$\{a_7, a_8, a_9, a_{10}\}$	(0.692, 0.89)	0.418	1
	$\{a_{11}\}$	(0.72, 0.39)	0.89	3
	$\{a_{12}\}$	(0.22, 0.39)	1.39	5
	$\{a_{13}\}$	(0.05, 0.72)	1.23	4

Table 13. Ranking of X1 and X2 based on similarities

In conclusion, ClusDM is not monotonous in general.

This is so because when the increase in the preference value of all the alternatives is not constant, the relationships among them may be modified, which produces new clusters, and some alternatives will decrease its preference in the resulting ranking.

However, if we increase the value of all the alternatives in the same degree[8], we only produce a translation in the space without affecting the relationships among the alternatives. In this case, the result will be greater or equal, depending if the increase is enough to move the clusters to receive the next term in the vocabulary. If it is the case, the property fulfilled could be stated as:

$$\Theta(v_{i1} + d, ..., v_{ip} + d) = \Theta(v_{i1}, ..., v_{ip}) + d$$

If we change each term for the one that is d positions up or down (a better one or a worse one), the result shows an increase or decrease of the same degree, d, in the value of the alternatives.

Let us now study the case that the vocabularies of our criteria have equal or different granularity but with the semantics given by the classical negation function (to assure terms equally informative).

In this case, the clustering process will produce the same clusters with the initial data and with the values increased (or decreased) in d units, because the differences between the values do not change if terms are equally informative.

The ranking stage may apply the Principal Components Analysis or the Distance Calculation. With the change of the terms, the values of the numerical prototypes suffer

[8] This case is possible only if the semantics of the criterion modified is given by the classical negation function, which gives equal informativeness to all the terms.

a transformation of Δ_i units in each criterion, where Δ_i depends on the granularity of each criterion c_i, to that:

$$\Delta_i = d \cdot length_i(t) \qquad \text{Eq. 6.1}$$

being $length_i(t)$ the length of the interval corresponding to any term t of the vocabulary of the i-th criterion.

Notice that, although we modify each criterion with a different value, Δ_i, we are only performing a translation of the points in the p-space. Let us now study in detail the ranking process for the two possible approaches:

- PCA:

The translation of the prototypes is counteracted by the use of a correlation[9] matrix to generate the principal components. Thus, the U vectors will be the same than the ones obtained with the original matrix. For this reason, although the clusters have suffered a translation in the original alternatives space, their z-scores will remain the same. However, the values of the *ideal* and *nadir* alternatives will not be modified (they do not increase or decrease d units), so, their z-scores will be different. As the values of these to extreme fictitious alternatives are used to scale the z-scores to the interval [0,1], we will obtain new positions for the clusters, although the ranking will be the same.

To measure the change in the position of the clusters, we must know the change in the position of the reference alternatives (the ideal and nadir points). Both of them will have a new z-score that differs from the one in the original matrix in λ units. Being z_1 the value in the first analysis, calculated as follows,

$$z_1 = u_{11}\left(\frac{x_1 - \overline{x_1}}{s_1}\right) + u_{12}\left(\frac{x_2 - \overline{x_2}}{s_2}\right) + \ldots + u_{1p}\left(\frac{x_p - \overline{x_p}}{s_p}\right)$$

and z_1^{Δ} the score corresponding to the second analysis (after the increment),

$$z_1^{\Delta} = u_{11}\left(\frac{x_1 - \left(\overline{x_1} + \Delta_1\right)}{s_1}\right) + u_{12}\left(\frac{x_2 - \left(\overline{x_2} + \Delta_2\right)}{s_2}\right) + \ldots + u_{1p}\left(\frac{x_p - \left(\overline{x_p} + \Delta_p\right)}{s_p}\right)$$

the difference between them, λ, is given by equation Eq. 6.2.

[9] Using the correlation matrix we work with centred and standardised values.

$$\lambda = -\sum_{i=1}^{p} \frac{\Delta_i}{s_i} u_{1i} \qquad \text{Eq. 6.2}$$

Consequently, if the z-scores of the clusters have not been modified, and the ones of the ideal and nadir points are increased in λ units (see that λ is a negative value), the clusters will be nearer the ideal than in the first analysis.

However, this difference, λ, is given in units without having into account the scaling of the z-scores to the interval [0,1], if we perform this scaling, we have equation Eq. 6.3.

$$\lambda_{01} = \frac{|\lambda|}{z_1(a_{ideal}) - z_1(a_{nadir})} \qquad \text{Eq. 6.3}$$

Following with the ClusDM procedure, the explanation stage will use the vocabulary given by the user to explain the result based on the position of each cluster in the interval [0,1].

Being t any term of this vocabulary, and $length(t)$ the length of the interval corresponding to each term, if $(d+1) \cdot length(t) > \lambda_{01} \geq d \cdot length(t)$, the terms selected will be exactly the ones that are d positions up or down with respect to the ones selected with the original data.

To check this condition, we will study each inequality separately. So, we must check if the following conditions are true:

Condition 1: $\lambda_{01} \geq d \cdot length(t)$

Condition 2: $(d+1) \cdot length(t) > \lambda_{01}$

Beginning with condition 1, we substitute the value of λ_{01} in equation Eq. 6.3, obtaining:

$$\frac{\sum_{i=1}^{p} \frac{\Delta_i}{s_i} u_{1i}}{z_1(a_{ideal}) - z_1(a_{nadir})} \geq d \cdot length(t)$$

which can be rewritten as:

$$\sum_{i=1}^{p} \frac{\Delta_i}{s_i} u_{1i} \geq d \cdot length\ (t) \cdot \left(z_1(a_{ideal}) - z_1(a_{nadir})\right)$$

if we expand the expression corresponding to Δ_i, we have:

$$\sum_{i=1}^{p} \frac{d \cdot length_i(t)}{s_i} u_{1i} \geq d \cdot length(t) \cdot \left(z_1(a_{ideal}) - z_1(a_{nadir})\right)$$

which can be simplified as:

$$\sum_{i=1}^{p} \frac{length_i(t)}{s_i} u_{1i} \geq length(t) \cdot \left(z_1(a_{ideal}) - z_1(a_{nadir})\right)$$

Substituting $z_1(a_{nadir})$ and $z_2(a_{nadir})$ by their corresponding expression in terms of U (Eq. 4.7), we can easily see that if the ideal is an alternative with the highest values (1,1,....,1) and the nadir is an alternative with the lowest values (0,0,....,0), we have that $z_1(a_{ideal}) - z_1(a_{nadir})$ is equal to:

$$u_{11}\left(\frac{1-\overline{x_1}-(0-\overline{x_1})}{s_1}\right)+u_{12}\left(\frac{1-\overline{x_2}-(0-\overline{x_2})}{s_2}\right)+\ldots+u_{1p}\left(\frac{1-\overline{x_p}-(0-\overline{x_p})}{s_p}\right)=\sum_{i=1}^{p}\frac{1}{s_i}u_{1i}$$

Substituting this result in the previous inequality, we obtain that:

$$\sum_{i=1}^{p} \frac{length_i(t)}{s_i} u_{1i} \geq \sum_{i=1}^{p} \frac{length(t)}{s_i} u_{1i}$$

$$\sum_{i=1}^{p} \frac{length_i(t)}{s_i} u_{1i} - \sum_{i=1}^{p} \frac{length(t)}{s_i} u_{1i} \geq 0$$

If we separate *length*(*t*), which is the value we are analysing, we have:

$$\sum_{i=1}^{p} \frac{u_{1i}}{s_i} length_i(t) \geq length(t) \cdot \sum_{i=1}^{p} \frac{u_{1i}}{s_i}$$

So, the length of the terms is constrained according to equation Eq. 6.4.

$$\frac{\sum_{i=1}^{p} \frac{u_{1i}}{s_i} length_i(t)}{\sum_{i=1}^{p} \frac{u_{1i}}{s_i}} \geq length(t) \qquad \text{Eq. 6.4}$$

Now, we proceed to study the condition number 2, to select exactly the terms *d* position up (or down) and not another greater (or smaller).

Substituting λ_{01} we have:

$$\frac{\sum_{i=1}^{p} \frac{\Delta_i}{s_i} u_{1i}}{z_1(a_{ideal}) - z_1(a_{nadir})} < (d+1) \cdot length(t)$$

Then, we can perform an analysis equal to the one done with the other inequality, to find out the following expression:

$$d \cdot \sum_{i=1}^{p} \frac{length_i(t)}{s_i} u_{1i} < (d+1) \cdot length(t) \cdot \sum_{i=1}^{p} \frac{1}{s_i} u_{1i}$$

We can rewrite this expression and obtain:

$$d \cdot \sum_{i=1}^{p} \frac{length_i(t)}{s_i} u_{1i} - d \cdot length(t) \cdot \sum_{i=1}^{p} \frac{1}{s_i} u_{1i} < length(t) \cdot \sum_{i=1}^{p} \frac{1}{s_i} u_{1i}$$

which is the same that:

$$d \cdot \left(\sum_{i=1}^{p} \frac{length_i(t) - length(t)}{s_i} u_{1i} \right) < \sum_{i=1}^{p} \frac{length(t)}{s_i} u_{1i}$$

$$\sum_{i=1}^{p} \frac{length(t)}{s_i} u_{1i} - \sum_{i=1}^{p} \frac{d \cdot length_i(t) - d \cdot length(t)}{s_i} u_{1i} > 0$$

Now, we will separate the *length*(*t*) variable to know what is it constrained for. So, we separate the constant elements from the ones depending on the variable *i*.

Putting all together, we have:

$$length(t)\cdot\sum_{i=1}^{p}\frac{u_{1i}}{s_i}+d\cdot length(t)\cdot\sum_{i=1}^{p}\frac{u_{1i}}{s_i}-d\cdot\sum_{i=1}^{p}\frac{u_{1i}}{s_i}length_i(t)>0$$

$$(d+1)\cdot length(t)\cdot\sum_{i=1}^{p}\frac{u_{1i}}{s_i}>d\cdot\sum_{i=1}^{p}\frac{u_{1i}}{s_i}length_i(t)$$

We obtain that the length of the terms in the vocabulary must be:

$$length(t)>\frac{d}{(d+1)}\cdot\frac{\sum_{i=1}^{p}\frac{u_{1i}}{s_i}length_i(t)}{\sum_{i=1}^{p}\frac{u_{1i}}{s_i}} \qquad \text{Eq. 6.5}$$

Consequently, we have that the property will be true iff the equations Eq. 6.4 and Eq. 6.5 hold.

$$\frac{\sum_{i=1}^{p}\frac{u_{1i}}{s_i}length_i(t)}{\sum_{i=1}^{p}\frac{u_{1i}}{s_i}}\geq length(t)>\frac{d}{(d+1)}\cdot\frac{\sum_{i=1}^{p}\frac{u_{1i}}{s_i}length_i(t)}{\sum_{i=1}^{p}\frac{u_{1i}}{s_i}} \qquad \text{Eq. 6.6}$$

The expressions that are restricting the length of the terms in the final vocabulary are weighted averages. It is interesting to note that the weights are proportional to the contribution of the criteria (i.e. experts) to the formation of the first principal component (which is the meaning of the U-vectors, see section 4.1.4). Moreover, we can see that these weights are inversely proportional to the standard deviation of the criterion, which is a measure of the data dispersion.

- Similarity:

The translation of the prototypes makes them to be nearer (or farther) from the ideal alternative in λ units. Thus, if $(d+1)\cdot length(t)>\lambda_{01}\geq d\cdot length(t)$, the terms selected in the final vocabulary will be exactly the ones *d* positions up (or down).

In this case λ depends on the similarity function. Let us study the case of the Manhattan distance, which is the one we recommend. We will see that the interpretation of the result in this case is straightforward.

Taking the Manhattan distance (Eq.3.3), we can see that the distance to the ideal point will be modified with a value equal to:

$$\lambda = \sum_{i=1}^{p} \Delta_i$$

However, to work with values in the interval [0,1], we scale the distance obtained with the Manhattan calculation by dividing it by the distance between the nadir and the ideal points, which is:

$$\lambda_{01} = \frac{\sum_{i=1}^{p} \Delta_i}{\sum_{i=1}^{p} |1-0|} = \frac{\sum_{i=1}^{p} \Delta_i}{p} \qquad \text{Eq. 6.7}$$

Now, we can check the two conditions required:

Condition 1: $\lambda_{01} \geq d \cdot length(t)$

Condition 2: $(d+1) \cdot length(t) > \lambda_{01}$

Beginning with condition 1, we substitute the value of λ_{01} in Eq. 6.7, obtaining:

$$\frac{\sum_{i=1}^{p} \Delta_i}{p} \geq d \cdot length(t)$$

Substituting the value of Δ_i, we have:

$$\frac{\sum_{i=1}^{p} d \cdot length_i(t)}{p} \geq d \cdot length(t)$$

Which can be simplified as:

$$\frac{\sum_{i=1}^{p} length_i(t)}{p} \geq length(t) \qquad \text{Eq. 6.8}$$

Taking, now, condition number 2, we repeat the same analysis:

$$\frac{\sum_{i=1}^{p} \Delta_i}{p} < (d+1) \cdot length(t)$$

$$\frac{\sum_{i=1}^{p} d \cdot length_i(t)}{p} < (d+1) \cdot length(t)$$

Having,

$$\frac{\sum_{i=1}^{p} length_i(t)}{p} < \frac{(d+1)}{d} \cdot length(t) \qquad \text{Eq. 6.9}$$

Finally, we obtain that the two conditions (Eq. 6.8 and Eq. 6.9) can be written as:

$$\frac{\sum_{i=1}^{p} length_i(t)}{p} \geq length(t) > \frac{d}{(d+1)} \cdot \frac{\sum_{i=1}^{p} length_i(t)}{p} \qquad \text{Eq. 6.10}$$

This equation, Eq. 6.10, means that if we want to have a monotonous operation, the length of the terms in the final vocabulary must be less or equal to the arithmetic average of the lengths of the terms used by the different experts (i.e. criteria), and greater than this average length multiplied by a factor related to the increment or decrement applied. Notice, that again we are constrained by an average of the lengths of the terms in the vocabularies of the criteria, however, now we make an arithmetic average while in the Principal Components based ranking we have to perform a weighted average.

After the analysis of the two ranking approaches, we can say that under some conditions, ClusDM is monotonous. In particular, if the length of the terms in the final vocabulary is constrained according to Eq. 6.6 or Eq. 6.10, the property is fulfilled.

6.2 Other Properties

In this section we review other properties studied for aggregation operators: the stability of ClusDM to some modifications in the data values. In particular, we study the behaviour of the method with respect to the negation of all the values in the data matrix, and with respect to the inversion of the preference values of one criterion.

6.2.1 Stability for the negation

An aggregation operator is stable for the negation if the reversal of the scale has no effect on the evaluation.

$$\Theta\left(N(c_1),...,N(c_p)\right) = N\left(\Theta\left(c_1,...,c_p\right)\right)$$

being N the application of a negation operator to all the values in one column of the decision matrix (i.e. to one criterion).

In the numerical case, the negation operator can be the classical one, $N(x_{ij}) = 1 - x_{ij}$, or a strong negation operator of the form $N(x_{ij}) = \varphi^{-1}\left(1 - \varphi(x_{ij})\right)$.

The rationale of this property is that if we assume that the experts give us values of their non-preference (or distaste), the result should be the opposite to the one obtained in terms of preference.

This property expresses self-duality of Θ, equivalently to the De Morgan laws in fuzzy sets theory [Klir&Yuan,1995].

Stability for the negation of ClusDM

This property can only be applied if the negation of each term is a single term, that is, if it is the classical negation. In this case, the property holds because the relations among the terms will be the same, so the clusters will be the same, and the terms selected will be the ones corresponding to the negation of the ones obtained with the original data. This is proven below.

Instead, if more than one term belong to the negation of another, the substitution of this term by its negation is not possible, since we do not allow to have more than one value in each cell of the decision matrix.

To prove that the terms selected will be the negation of the terms obtained, in case of using the data matrix with the real values, we will study the ranking results.

- PCA: Since all the terms in the vocabulary are equal informative (i.e. have equal interval lengths), we have that the values used to compute the principal

components are, 1-x_{ij}, being x_{ij} the numerical value corresponding to the term attached to the i-th alternative for the j-th criterion. As the relationships among the data do not change (they are only inverted), the use of the correlation matrix assures that the eigenvalues and eigenvectors will be the same, since the standardisation of the data will remain the same. Consequently, the z-scores of the cluster prototypes will only change their sign. However, the z-scores of the ideal and nadir alternatives, will not suffer this modification since the negation is not applied to their values.

Therefore, the distance of a z-score to the ideal will be now the distance to this score to the nadir. So, it is as we considered as the reference point 1 the position of the nadir, and as reference point 0 the position of the ideal. With this interchange we will have that all the positions of the clusters will suffer the following modification when they are scaled to the unit interval:

$$z_{01}^{N}(x_i) = 1 - z_{01}(x_i)$$

In the following step, if all the terms are equally informative, the assignment algorithm will select the opposite terms.

– Similarity-Based Ranking: When all the values of the matrix are negated, the objects suffer a translation in the variables space. However, the ideal alternative remains at the same point. So, the clusters obtained with the aggregation method will be the same, but their prototypes will have the negations of the values of the original ones. Now, what we want to prove is that the distance of a cluster prototype in the negated matrix, x_i^N, is equal to 1-z_{01}, where z_{01} is the distance of the same cluster with the original values, x_i.

$$z_{01}\left(x_{i}^{N}\right) = 1 - z_{01}(x_i)$$

Considering the MCD distance, we can write this equation as:

$$\sum_{i=1}^{p} \frac{\left|1 - x_i^N\right|}{p} = 1 - \sum_{i=1}^{p} \frac{\left|1 - x_i\right|}{p}$$

According to the fact that x_i^N corresponds to the negation of x_i, this expression corresponds to:

$$\sum_{i=1}^{p} \frac{\left|1 - (1 - x_i)\right|}{p} = 1 - \sum_{i=1}^{p} \frac{\left|1 - x_i\right|}{p}$$

Thus, we have:

$$\sum_{i=1}^{p} \frac{|x_i|}{p} = 1 - \sum_{i=1}^{p} \frac{|1 - x_i|}{p}$$

which can be rewritten as:

$$\sum_{i=1}^{p} \frac{|x_i|}{p} + \sum_{i=1}^{p} \frac{|1 - x_i|}{p} = 1$$

As $0 \leq x_i \leq 1$, we can ignore the absolute value calculation:

$$\sum_{i=1}^{p} \frac{x_i + 1 - x_i}{p} = 1$$

This equation is always true since $\sum_{i=1}^{p} \frac{1}{p}$ is equal to 1. Consequently, we have that the position of the prototype with negated values will be exactly the negation of the position of the prototype with the original values. Therefore, the algorithm for selecting the terms, will select the terms corresponding to the negation of the ones selected in the study with non-negated values.

6.2.2 Stability for the Opposition

In a data matrix with two criteria, if the two criteria have opposite preferences, then the result of the aggregation should be a null degree of preference over the alternatives.

Opposition in ClusDM

This property holds when the semantics of the terms is based on the classical negation function. In this case, if two criteria give completely opposite preferences to all the alternatives in the decision matrix, the result of the aggregation will be a single cluster with the neutral label (the one corresponding to position 0.5 in the interval [0,1]).

What is happening is that the opposite terms compensate each other. So if all the terms in the vocabulary are used, the distances between the objects will be constant and they will be put together in a single cluster with a neutral prototype.

In Figure 26, we can see an example with two criteria with a vocabulary of 5 terms (from best to worst): {a, b, c, d, e}.

	c_1	c_2
a_1	a	e
a_2	b	d
a_3	c	c
a_4	d	b
a_5	e	a

Figure 26. Decision matrix with opposite preferences

The first step in the clustering procedure will calculate the Manhattan distance considering the classical negation function, so we will obtain:

	a_1	a_2	a_3	a_4	a_5
a_1		0.4	0.8	1.2	1.6
a_2			0.4	0.8	1.2
a_3				0.4	0.8
a_4					0.4
a_5					

Figure 27. Dissimilarity matrix for opposite preferences

With this values, we have to build a cluster with objects: a_1, a_2, a_3, a_4 and a_5. They all have a dissimilarity value of 0.4. The prototype of this cluster will have the following values: {0.5,0.5} = {c,c} whose aggregation label will be the neutral one, c.

6.3 Properties with respect to the alternatives

Although our decision making operator is applied to the criteria, it is also interesting to study the results from the point of view of the alternatives, since what we are trying to do is an analysis of the alternatives in order to know which is their individual preference in an overall criterion. For this reason, we will now study the behaviour of the ClusDM methodology according to the values of the alternatives.

We will denote as Φ the application of the ClusDM operator to an alternative. Although we do not mention that the aggregation result of an alternative depends on the value of the other alternatives, it is implicitly taken into account. In fact, to study these properties we assume that the rest of the data matrix does not change. Formally we have,

$$\Phi(a_i) = \Phi(v_{i1}, v_{i2}, ..., v_{ip}) = v_{ir}$$

Considering this new view of the ClusDM operator, in this section we study three properties that are usually fulfilled by aggregation operators: increasingness or monotonicity, positive association and neutrality.

6.3.1 Increasingness

This property is fulfilled when an alternative with a better rating for each criterion evaluates better in the final rating. This is, if there is an alternative a_k such that for all the other alternatives, a_l, satisfying $c_i(a_k) > c_i(a_l)$ for all criteria i=1..p, then we have that $\Phi(a_k) > \Phi(a_l)$.

Increasingness of ClusDM for alternatives:

Let us consider a data matrix where all the alternatives have the same value for all criteria.

When an alternative a_k is better than all the others, it will be also the best one in the result, c_r, because for the idempotency property of ClusDM, the preference value of each alternative in the result, will be the same one that they have in all the criteria.

However, if the values of the alternatives are different according to the criteria, the alternative with best value cannot receive the best term in the result. It will depend on the relationships among the alternatives and the groups produced by the clustering.

Here we have an example of the non-monotonicity for the case of two different columns. Let us consider the case that all criteria have the same vocabulary (from worse to best): {g,f,e,d,c,b,a}. The terms have the same semantics for all criteria, which is given by the following negation function, N:

N(a)={f,g}, N(b)={f}, N(c)={e}, N(d)={d}, N(e)={c}, N(f)={a,b}, N(g)={a}

	c_1	c_2
a_1	a	a
a_2	b	a
a_3	c	a
a_4	d	a
a_5	d	b
a_6	a	d
a_7	f	c
a_8	e	e
a_9	f	f
a_{10}	g	g
a_{11}	d	b
a_{12}	d	b
a_{13}	d	b
a_{14}	d	b

Figure 28. Decision matrix for the example of increasingness of alternatives

The trace of applying the clustering method (centroid clustering with Manhattan distance) is given here:

STEP 1: Build a dissimilarity matrix and put together those alternatives with minimum value. In this case, alternatives a_5, a_{11}, a_{12}, a_{13} and a_{14} are the components of the first cluster because these alternatives are identical. We will denote this cluster k_1.

STEP 2: Calculate the prototype of the new cluster.

Prototype k_1 = (d,b) = (0.5, 0.72) = $a_5 = a_{11} = a_{12} = a_{13} = a_{14}$

STEP 3: Build a new dissimilarity matrix and group the alternatives with minimum value.

	a_1	a_2	a_3	a_4	k_1	a_6	a_7	a_8	a_9	a_{10}
a_1	0	0.17	0.28	0.39	0.55	0.39	0.94	1.0	1.33	1.67
a_2		0	**0.11**	0.22	0.39	0.55	0.78	0.83	1.17	1.5
a_3			0	**0.11**	0.28	0.67	0.67	0.72	1.05	1.39
a_4				0	0.17	0.78	0.55	0.61	0.94	1.28
k_1					0	0.61	0.39	0.44	0.78	1.11
a_6						0	0.78	0.61	0.94	1.27
a_7							0	0.39	0.39	0.72
a_8								0	0.33	0.67
a_9									0	0.33
a_{10}										0

Figure 29. Dissimilarity matrix in the 3rd. step

STEP 4: Calculate the prototype of the new cluster.

Prototype k_2 = (0.61, 0.89) = average (a_2, a_3, a_4)

STEP 5: Modify the dissimilarity matrix, including the new cluster and deleting its components.

	a_1	k_2	k_1	a_6	a_7	a_8	a_9	a_{10}
a_1	0	**0.28**	0.55	0.39	0.94	1.0	1.33	1.67
k_2		0	**0.28**	0.67	0.67	0.72	1.06	1.39
k_1			0	0.61	0.39	0.44	0.78	1.11
a_6				0	0.78	0.61	0.94	1.27
a_7					0	0.39	0.39	0.72
a_8						0	0.33	0.67
a_9							0	0.33
a_{10}								0

Figure 30. Dissimilarity matrix in the 5th. step

STEP 6: Build another level of clusters.
We have that the minimum value is also 0.28, which creates another class: $k_3 = \{a_1, k_2, k_1\}$.

STEP 7: Calculate the prototype of this class.
Prototype $k_3 = (0.58, 0.795)$ = average $(a_1, a_2, a_3, a_4, a_5, a_{11}, a_{12}, a_{13}, a_{14})$

We stop the process because we have 6 clusters. Then, to establish a ranking of the clusters, we can use the similarity with respect to the *Ideal* alternative, which in this case is I = (a,a) or the Principal Components Analysis (see Table 14).

If we consider that the two criteria are correlated, we can apply the ranking based on the Principal Components Analysis. In this case, the position of the clusters in the ranking is the same that the one obtained with the similarity function (Table 14). We have that the correlation degree between the two criteria is 83%. Moreover, we can see that cluster $\{a_6\}$ and cluster $\{a_7\}$ are not properly represented by this ranking, since they have a quality value of 0.207 and 0.593, respectively. Therefore, although a_6 is given as the best alternative, we should not rely on this result, because this alternative has different preference values for the two criteria: criterion 1 says that it is the best (value a) while criterion 2 says that it is normal (value d).

Elements of the class	**Prototype**	**Similarity to the Ideal**	**PCA projection**	**Rank**
$\{a_1, a_2, a_3, a_4, a_5, a_{11}, a_{12}, a_{13}, a_{14}\}$	(0.58, 0.795)	0.405	0.66	2
$\{a_6\}$	(0.89, 0.5)	0.39	0.79	1
$\{a_7\}$	(0.22, 0.61)	0.95	-0.91	3
$\{a_8\}$	(0.39, 0.39)	1.0	-1.0	4
$\{a_9\}$	(0.22, 0.22)	1.34	-1.97	5
$\{a_{10}\}$	(0.05, 0.05)	1.68	-2.94	6

Table 14. Ranking of the alternatives in the increasingness proof

With this example, we can see that ClusDM is not monotonous with respect to the alternative's preference. In the initial data, a_1 was the most preferred (in fact, it is the ideal) and in the result a_6 is considered better than a_1.

However, this situation occurs when, for two alternatives a_k and a_l such that, $a_k > a_l$, we have that the aggregation makes that a_k become part of a cluster whose centroid is worse than the cluster that a_l belongs to (see Figure 31).

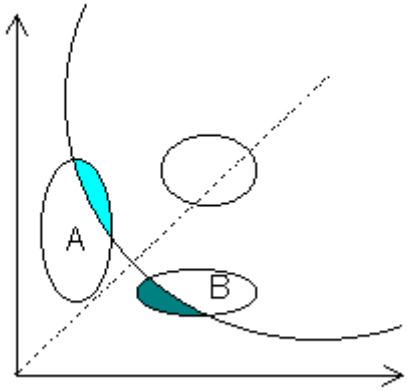

Figure 31. Two clusters with objects that do not fulfil the increasingness property

In this figure, we can see that some elements of cluster A are better that others of cluster B. However, the prototype of A is worse than the one of cluster B.

This situation is more difficult to arise in the case of having qualitative criteria. This is due to the fact that the alternatives can only take a linguistic value that is covering an interval in the numerical domain. In this case, the possibility of generating clusters that produce this effect decreases, since we have to build elliptical clusters, rather than spheripherical ones, which means that we need alternatives with conflicting evaluations (i.e. one criteria says that all the alternatives have the same preference, while the other distinguish quite different preference degrees). It can be seen in the example used to demonstrate that ClusDM in not monotonous.

6.3.2 Positive Association

This property holds when having a set of preference criteria (i.e. experts) whose resulting decision criterion establishes that a_i is preferred to a_j, then making a_i better or a_j worse than before, implies that, in the overall decision criterion, a_i remains preferred to a_j. This can be formalised as follows.

Let $a_i = (v_{i1}, v_{i2}, ..., v_{ip})$ and $a_j = (v_{j1}, v_{j2}, ..., v_{jp})$ such that $\Phi(a_i) > \Phi(a_j)$. Then for all $a'_i > a_i$ (i.e. $v'_{ij} > v_{ij}$ for some j) it holds $\Phi(a'_i) > \Phi(a_j)$.

Positive Association in ClusDM:

In Figure 32 we can see a two-variable example of the non fulfilment of this property for the case of ranking according to the PCA results. Note that each dimension on this

figure corresponds to a variable and each painted cell represents one or more alternatives (the darker the grey is, the more alternatives with the same value are). In particular, we want to study the behaviour of the x and y alternatives. The alternative y has the values (b,g), which is marked in green colour in the figure. Otherwise, the orange cell corresponds to alternative x, with (c,e) values. According to the first picture, x is preferred to y. Then, if we increase the value of the second criterion of x, obtaining (c,d), this alternative is now closer to another class, whose projection is worse than the one of y. So, in the second picture, y is preferred to x.

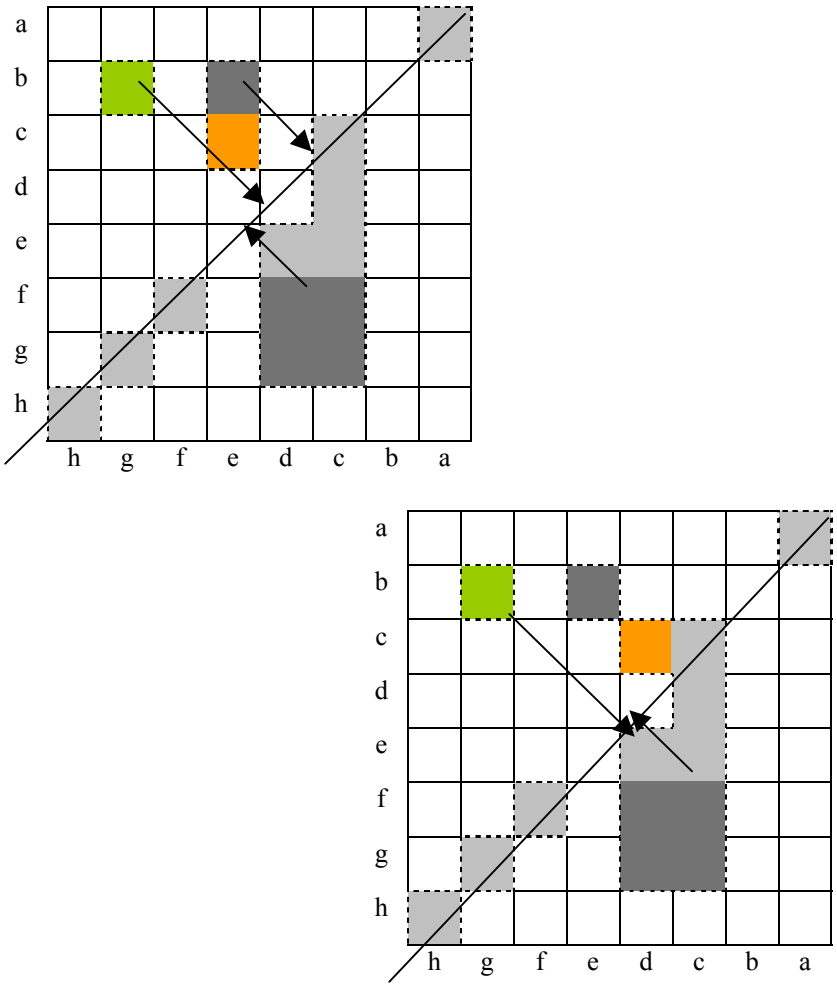

Figure 32. Representation of alternatives that do not fulfil the positive association

As it can be seen in the pictures above, in order to make that an alternative, x, which belongs to a cluster better projected than the one of y, to be moved to another cluster whose projection is worse than the one of y, we need that the cluster that receives the alternative x has an elliptical form, in order to be near the alternative x after the increase of its values but having the gravity centre lower than the one of the cluster of y. Therefore, we believe that it is a non-common case for decision making problems, since it means that alternative with very different values are put together in an elliptical class. With the quality measures we would detect such a cluster with a very low intra-cluster cohesion, which will decrease the confidence on the value attached to it.

In case of using the similarity-based ranking, we need an elliptical cluster whose gravity centre is farther than the one of cluster y (green). So, if an alternative x of the orange cluster receives a better value in the second criterion, it will can move to the big cluster (grey) and then, it will become worse than y. See Figure 33.

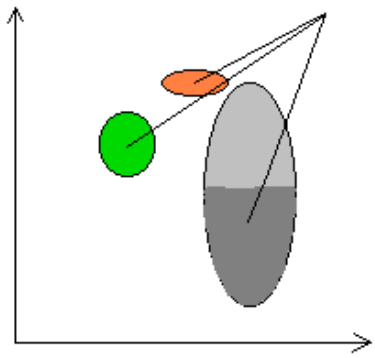

Figure 33. Clusters that do not fulfil the positive association property

This case is similar to the one explained before, in which we have some strange clusters, since one of them is covering a widespread set of alternatives.

In general, it seems reasonable that if an alternative moves to a situation in which there is a large cluster with low intra-cluster cohesion, the new label attached to this alternative is not appropriate, since it is not clear that this cluster can have a unique prototype that determines its value. Therefore, although positive association is not fulfilled, the quality measures that ClusDM uses, probably will show us that the result obtained is not reliable. Thus, although it cannot be proved, in the general case, and specially if criteria are correlated, we will have that the positive association of the alternatives holds.

6.3.3 Neutrality with respect to alternatives

In decision making procedures it is required that any two identical alternatives (such that they have the same value for all the criteria) receive the same preference value in the final ranking.

Having two alternatives $a_i = (v_{i1}, v_{i2}, ..., v_{ip})$ and $a_j = (v_{j1}, v_{j2}, ..., v_{jp})$ so that $v_{ik} = v_{jk}$ for all *k* in 1..*p*, this property can be stated as:

$$\Phi(a_i) = \Phi(a_j)$$

Neutrality in ClusDM

Our method guarantees this property, since the first clusters that are generated in the aggregation process are the ones that put together those indistinguishable objects, because they have a dissimilarity value equal to 0. Therefore, even in the case of cutting the tree at the lowest level, we will have those alternatives in the same class, which means that they will be attached to the same term.

Chapter 7.

Applications

The methodology developed in this thesis is general enough to be used in different kinds of problems. As we have already said, ClusDM can be used as a decision making tool or as an aggregation operator. In this chapter, we will show how to use the methodology in different frameworks. In particular we will explain the following applications:

- *Journal Review*. This is a selection problem in which the decision maker has to distinguish the best papers to be published. We have used real data provided by the editor of a special issue of a scientific journal. Each of the 22 papers submitted was reviewed by 3 experts according to 22 preference criteria. The experts evaluated the papers using a form that included quantitative and qualitative preferences as well as non-ordered categorical criteria.
- *Organ Transplant Receiver*. This decision-making problem consists in assisting the coordinator of organ transplants of a hospital in the determination of the most suitable receivers for a given organ. The goal is to obtain a ranking of the list of waiting patients according to their matching with the characteristics of the organ that is available at a particular time. Although not all the criteria can be analysed by ClusDM, the preference list obtained can be used by the medical specialists in order to make a better selection and increase the transplant success. In this case, at the moment, we have only been able to make a simplified test of ClusDM with artificial data.
- *Statistical Disclosure Control*. This is an application of our method as an aggregation operator for heterogeneous criteria. The goal is to re-identify the real values of a set of records that have been masked using different techniques. Statistical Offices must protect the data published (using masking methods) in order to preserve personal confidentiality. The degree of re-identification achieved by ClusDM gives an idea of the risk of publishing those data sets. In this application we have used the public data of the *American Housing Survey 1993* provided by the *U.S. Census Bureau*.

The following sections are devoted to these three application examples. The results obtained show that our methodology is able to give good results in many different frameworks. The first problem will be used to explain in detail the use of our methodology. For the rest of application examples, we will devote more time to comment the results than to the process itself.

7.1 Journal Review

Research publications are usually reviewed by a group of experts who give their opinion about the quality of different aspects of a set of papers. The evaluations of the experts are collected by a committee who is in charge of the selection of the best papers to be published. This problem is known as Multiple Expert - Multiple Criteria Decision Making (ME-MCDM) [Yager, 1993] because we have multiple experts that provide multiple criteria for evaluating the papers.

In the following section we will explain how we can sort out these ME-MCDM problems. The solution consists of aggregating the information of the experts and criteria at two different stages. Then, in the next section we will see with great detail the use of ClusDM in the selection of the best papers of a real journal.

7.1.1 Making Multiple Experts - Multiple Criteria Decisions

To sort out a ME-MCDM problem we have to deal with the information provided by each expert about a set of criteria. Thus, we have a data matrix for each expert, as it is shown in Figure 34. The ranking or selection of the best alternatives must be done using all this information. A two-stage process can be designed in order to aggregate the data at two different levels. In [Yager,1993] the author proposes to find an overall evaluation function for each individual expert and, in a second stage, a MCDM method is applied to aggregate these evaluations to obtain an overall value for each alternative.

We propose to interchange these two processes. In Yager's proposal, the aggregation of the data matrix provided by an expert gives us the global opinion of the expert. However, the criteria that are aggregated can refer to very different aspects of the problem (i.e. different properties, qualities, preference evaluations, etc.), so the result is putting together a huge variety of questions. Moreover, depending on the aggregation operator and the number of criteria in the matrix, the result may not reflect some important evaluations given by the expert. Our proposal consists of starting by making an aggregation of the information about each criterion given by the different experts. The result will be the consensus of the experts' opinions about a specific aspect of the problem. Then, the second stage consists of applying a MCDM method to the consensued criteria in order to find the overall evaluation for the alternatives. With this approach we pretend to reduce the loss of valuable information during the process. The consensus of the opinions about a single criterion is also interesting to detect the aspects of the problem in which the experts do not agree, or to study the ranking of the alternatives considering only a single criterion. Therefore, with our approach we are

able to offer more information about the data to the decision maker. This will be illustrated with an example in the next section.

e_1	...	c_k
a_1		v_{11}
a_2		v_{11}
a_3		v_{11}
a_4		v_{11}
a_5		v_{12}
a_6		v_{12}

Expert e_1's data matrix

e_2	...	c_k
a_1		v_{21}
a_2		v_{21}
a_3		v_{22}
a_4		v_{22}
a_5		v_{21}
a_6		v_{22}

Expert e_2's data matrix

Figure 34. Data matrices about the same domain provided by e_1 and e_2

c_i	e_e	e_2
a_1	v_{11}	v_{21}
a_2	v_{11}	v_{21}
a_3	v_{11}	v_{22}
a_4	v_{11}	v_{22}
a_5	v_{12}	v_{21}
a_6	v_{12}	v_{22}

Figure 35. Data matrix to build the consensus of the i-th criterion

With our approach, in the first stage we use all the information provided by the experts to aggregate each criterion separately (see Figure 35). The preferentially independence of the 0criteria is assumed.

Notice that now the problem of synthesising this data matrix corresponds to the same problem we solve in MCDM. Thus, the same methods can be used. Nevertheless, some difficulties may arise: (i) not all the criteria are used by all the experts, and (ii) the alternatives analysed by the group of experts are not the same for all of them.

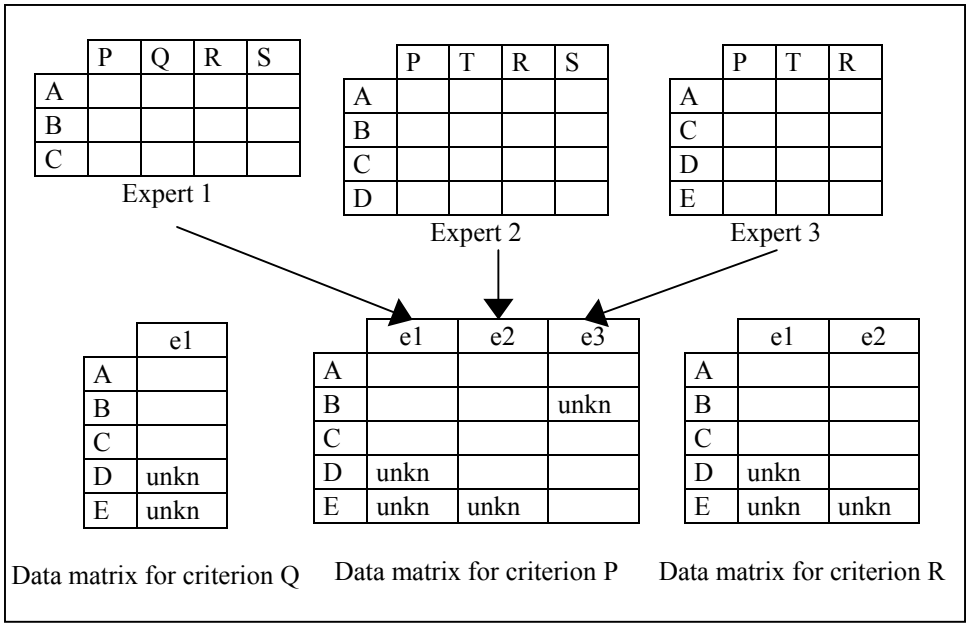

Figure 36. The decision matrix of each criterion (with missing values)

The first case is easily solved because we only put a column in the data matrix of criteria c_i if there is an expert that can fill it (see data matrix of the P and R criteria, in Figure 36). If a criterion is only provided by a single expert, there is no consensus process to be done (step 1 is not applicable, as for the Q criterion in Figure 36).

The second problem is solved using missing values, denoted as "unknown". Therefore, the process of building the matrices is as follows. First, we put in the data matrix of c_i all the alternatives considered by the experts that use c_i. When an expert does not have a value for an alternative (because he does not know it or is not able to give his opinion about it, etc.) we introduce a special value that indicates that it is not known. Figure 36 also illustrates this procedure ("unkn" denotes a missing or unknown value). Note that this construction requires the aggregation method to be able to deal with this kind of values (as our method based on the classifier *Sedàs* does [Valls et al.,1997]).

The aggregation method to be used depends on the type of criterion (i.e. numerical, qualitative, Boolean, ...). In case of having heterogeneous criteria we can use the ClusDM methodology to find a new qualitative preference criterion. In addition, we will obtain a goodness value that can be used to weight this criterion in the next step of the process.

Once we have obtained the synthesis of each criterion, we proceed to build a data matrix with these new social criteria (Figure 37). Then, an appropriate MCDM method is used again to aggregate and rank the alternatives, and solve the decision problem.

	"P"	"Q"	"R"	"S"	"T"
A					
B					
C					
D					
E					

Figure 37. Data matrix with the consensued criteria (indicated with "")

7.1.2 Selecting the best papers for the journal with ClusDM

The call for papers for this special issue of the journal had two steps. Firstly, the authors sent an extended abstract to the editors. These submissions were numbered from 1 to 33. After a period of time, the authors were required to send the complete paper. Some of the authors did not send their papers, so finally only 22 papers were received. The papers submitted were the ones with the following identifiers: 1, 3, 4, 5, 6, 8, 9, 10, 13, 14, 16, 17, 18, 20, 21, 22, 24, 26, 29, 31, 32 and 33.

A group of 26 experts on the subject evaluated a subset of papers according to a predefined form with 22 questions. In Appendix A we can see the questions of the form. In Table 15 we have a brief description of the criteria. Ten of the questions receive a numerical mark, two are non-ordered qualitative properties (i.e. categories) and the rest, 10, are qualitative preferences over many different aspects of the paper. For some of the qualitative preferences we assumed a non-classical negation semantics. In Table 16 we show those criteria with the corresponding negation function.

Name	subject	research	relevant	are agents?	MAS descript.	original	sound.	technical limits
Type	QN	QN	QO	QO	QO	QO	QO	QO
Domain	2 terms	3 terms	4 terms	4 terms	3 terms	5 terms	3 terms	4 terms

Name	approach	applicat. descript.	applicat method	method. descript.	method. applicab	abstract	introduct.	conclu.
Type	QO	N	N	N	N	N	N	N
Domain	4 terms	[1,5]	[1,5]	[1,5]	[1,5]	[1,7]	[1,7]	[1,7]

Name	organis.	readable	figures	English	referen.	overall
Type	N	N	N	QO	QO	QO
Domain	[1,7]	[1,7]	[1,7]	5 terms	4 terms	5 terms

Table 15. Summary of the criteria for evaluating the papers of the journal

Criterion	Vocabulary	Negation function for the semantics[10]
relevance	no somewhat quite very	(lambda (et) (case et (no '(quite very)) (somewhat '(somewhat)) (quite '(no)) (very '(no))))
agents?	no doubts arguable yes	(lambda (et) (case et (no '(yes)) (doubts '(yes)) (arguable '(arguable)) (yes '(no doubts))))
tech-limits	not-discussed poorly briefly adequately	(lambda (et) (case et (not-discussed '(adequately)) (poorly '(adequately)) (briefly '(briefly)) (adequately '(not-discussed poorly))))
approach	not-discussed poorly briefly adequately	(lambda (et) (case et (not-discussed '(adequately)) (poorly '(adequately)) (briefly '(briefly)) (adequately '(not-discussed poorly))))
references	poor basic old complete	(lambda (et) (case et (poor '(complete)) (basic '(complete)) (old '(old)) (complete '(poor basic))))

Table 16. Non-classical negation based criteria for evaluating journal papers

Consensus of the opinions of the different judges for each criterion

Using the preferences given by the 3 experts who had evaluated each of those papers, we built a matrix for each criterion. Some papers only received two evaluations, thus they had an unknown value in the third column (the one corresponding to the 3rd evaluation). It is important to note that we put together the first evaluation of each paper in the first column of the matrix although it was not provided by the same person. That is, we were assuming that all the experts had the same interpretation of the

[10] The functions are written in Lisp. This is the language used to implement *Radames*, which is a system that follows the ClusDM methodology in qualitative and heterogeneous data sets.

vocabularies and semantics of the criteria. This seems a hard assumption but, in fact, when the editors analyse the evaluations given by the experts, they are using their own interpretation of the values, which is the same for all the expert's questionnaires.

The first step was the execution of a decision-making operation for each of the 22 data matrices according to the nature of the values. Using *Radames* we consensued the values using the following operators:

- The Arithmetic Average operator for numerical values
- ClusDM for each of the qualitative criteria (ordered and non-ordered ones)

Let us now follow the ClusDM execution and analyse the results obtained. For the non-ordered criteria, the process consists only of performing the aggregation stage and produces a partition. In relation to the number of clusters obtained for the qualitative criteria, the *subject* criterion generated two clusters, which received two artificial identifiers to distinguish them. The number of clusters obtained from the matrix corresponding to the *research* criterion was so big (about 10). This is an indicator of the disagreement among the experts about the status of the research (preliminary, mature or completed). So, we decided to remove this criterion from the analysis. For the rest of the criteria, the number of clusters was approximately the same than the number of values in the initial domain.

These partitions were obtained using the clustering tool called *Sedàs* (included in *Radames*). The clusters were generated using the Manhattan similarity function to compare the values of the different alternatives, and the Centroid method to build the hierarchical classification.

Criterion	Num. of clusters	Degree of agreement	G_{PCA}	Comments
Relevance	4	62 %	0.627	
Agents?	4	70 %	0.678	
MAS-desc	3	55 %	0.532	High Disagreement We need more than 1 component
Originality	5	58 %	0.660	Expert 3 disagrees with the result
Soundness	4	47 %	0.568	
Tech-limits	4	48 %	0.538	High Disagreement We need more than 1 component
Approach	4	41 %	0.468	High Disagreement We need more than 1 component
English	5	59 %	0.504	High Disagreement We need more than 1 component
References	4	63%	0.556	
Overall	5	62%	0.526	Expert 3 disagrees

Table 17. PCA results for each criterion of the paper's evaluation

The next step was to apply the Principal Components Analysis to each criterion. The prototypes of the partitions were ranked using the first component of each data set. Table 17 shows the number of clusters obtained for each criterion and the quality of the PCA ranking.

We can observe that the quality is low for the majority of the criteria. Moreover, in 4 of them, we cannot use the result obtained because the ranking using only the first principal component may be wrong. Using the quality of the representation of each of the clusters in the first component (Eq.4.18) we could notice that these clusters were really small (see Table 18). Analysing their elements, we can distinguish the conflicting alternatives: 10, 13, 16, 21, 24 and 33.

Relevan	Agent	Original	Sound	Tec.Lim	Appro	English	References	Overall
6,9,16	1,10	10,24	13	10	4,24	21,29,33	13,16,18,21,33	24

Table 18. Conflicting papers of each criterion

According to the editors, the papers with numbers 10, 16, 24, 29, 31 and 33 had received very different marks. After a more exhaustive review, they considered them of poor quality. We want to stress that, in this test, the analysis and selection of papers made for the editors was not influenced by our results because our study was posterior.

We can see that the majority of alternatives that ClusDM discards are the ones that needed a deepest reviewing process by the editors. This shows that this methodology can also help decision makers to identify the problematic alternatives.

At the light of the low quality of the results at this stage, we decided to repeat the process removing the conflicting alternatives from the decision matrices of the criteria. Table 19 shows the new results with only 16 papers.

Criterion	**Num. of clusters**	**Degree of agreement**	G_{PCA}	**Comments**
Relevance	4	78 %	0.67	
Agents?	4	90 %	0.89	Highest Agreement and Quality
MAS-desc	3	76 %	0.69	
Originality	5	85 %	0.84	Good Agreement and Quality
Soundness	4	68 %	0.70	
Tech-limits	4	49 %	0.40	High Disagreement
Approach	4	64 %	0.58	
English	5	62 %	0.59	
References	5	79 %	0.65	
Overall	4	81 %	0.76	Good Agreement and Quality

Table 19. Aggregation and ranking with PCA for each journal preference criterion

Notice that the degree of agreement and the overall quality of the ranking has significantly increased when the conflicting alternatives where not disturbing the clustering and ranking processes. The single criterion whose result is not acceptable enough is the one referring to whether the *Technical Limits* of the work explained in the paper are well established or not. In this case, following what we have proposed in Chapter 4, we use the Similarity-based Ranking to compare the prototypes with the ideal alternative. The quality of the ranking obtained with this method is 0.82.

Criterion	Vocabulary		z_1^{01}	Terms selected
Relevance	no	[0.0,0.4]	C1 = 0.76	C1 - very
	somewhat	[0.4,0,6]	C2 = 0.69	C2 - quite
	quite	[0.6,0.8]	C3 = 0.56	C3 - *unknown*
	very	[0.8,1.0]	C4 = 0.29	C4 - no
Agents?	no	[0.0,0.2]	C1 = 0.78	C1 - yes
	doubts	[0.2,0.4]	C2 = 0.52	C2 - arguable
	arguable	[0.4,0.6]	C3 = 0.34	C3 - doubts
	yes	[0.6,1.0]	C4 = 0.25	C4 - no
MAS-desc	bad	[0.0,0.33]	C1 = 0.56	C1 - normal
	normal	[0.33,0.67]	C2 = 0.40	C2 - *unknown*
	well	[0.67,1.0]	C3 = 0.26	C3 - bad
Originality	not	[0.0,0.2]	C1 = 0.80	C1 - very
	mostly-not	[0.2,0.4]	C2 = 0.61	C2 - mostly
	somewhat	[0.4,0.6]	C3 = 0.54	C3 - *unknown*
	mostly	[0.6,0.8]	C4 = 0.27	C4 - mostly-not
	very	[0.8,1.0]	C5 = 0.10	C5 - not
Soundness	no	[0.0,0.33]	C1 = 0.70	C1 - very-yes
	somewhat	[0.33,0.67]	C2 = 0.60	C2 - yes
	yes	[0.67,1.0]	C3 = 0.41	C3 - no
			C4 = 0.17	C4 - very-no
Tech-limits	not-discussed	[0.0,0.2]	C1 = 0.60	C1 - very-adequately
	poorly	[0.2,0.4]	C2 = 0.57	C2 - adequately
	briefly	[0.4,0.6]	C3 = 0.44	C3 - briefly
	adequately	[0.6,1.0]	C4 = 0.37	C4 - *unknown*
Approach	not-discussed	[0.0,0.2]	C1 = 0.62	C1 - *unknown*
	poorly	[0.2,0.4]	C2 = 0.57	C2 - briefly
	briefly	[0.4,0.6]	C3 = 0.38	C3 - poorly
	adequately	[0.6,1.0]	C4 = 0.30	C4 - not-discussed
English	deficient	[0.0,0.2]	C1 = 0.86	C1 - correct
	typo&gramm	[0.2,0.4]	C2 = 0.70	C2 - typo
	gramm	[0.4,0.6]	C3 = 0.69	C3 - very-typo
	typo	[0.6,0.8]	C4 = 0.49	C4 - *unknown*
	correct	[0.8,1.0]	C5 = 0.42	C5 - gramm
References	poor	[0.0,0.2]	C1 = 0.76	C1 - very-complete
	basic	[0.2,0.4]	C2 = 0.64	C2 - complete
	old	[0.4,0.6]	C3 = 0.55	C3 - *unknown*
	complete	[0.6,1.0]	C4 = 0.43	C4 - old
			C5 = 0.29	C5 - basic
Overall	not-accepted	[0.0,0.2]	C1 = 0.73	C1 - accept-few-modif
	doubts	[0.2,0.4]	C2 = 0.53	C2 - accept-with-modif
	accept-with-modif	[0.4,0.6]	C3 = 0.40	C3 - doubts
	accept-few-modif	[0.6,0.8]	C4 = 0.16	C4 - not-accepted
	def-accepted	[0.8,1.0]		

Table 20. Explanation of the clusters using the terms in the vocabulary. Green: neutral term; Blue: term generated using the negation function; Red: term generated by splitting the term that should be used by more than one class.

The first results of the explanation stage of the ClusDM process are shown in Table 20. The values of the third column are the positions in the unit interval of the ordered clusters of papers. The second column shows the vocabulary and intervals of the terms

that where used by the experts to judge the papers. With these intervals the `Explain_Result` algorithm selects the most appropriate term for each cluster or generates new ones. The term attached to each cluster can be seen in the last column. The clusters that receive the *unknown* value are the ones that have a representation quality lower than 0.4 (Eq.4.18) or the ones with a dispersion higher than 0.2 (Eq.5.11).

Criteria	Initial Vocab.		New Vocab.		Negation
Relevant	no	[0.0,0.4]	no	[0.0,0.4]	quite,very
	somewhat	[0.4,0,6]	somewhat	[0.4,0.6]	somewhat
	quite	[0.6,0.8]	quite	[0.6,0.73]	no
	very	[0.8,1.0]	very	[0.73,1.0]	no
Agents?	no	[0.0,0.2]	no	[0.0,0.3]	yes
	doubts	[0.2,0.4]	doubts	[0.3,0.42]	yes
	arguable	[0.4,0.6]	arguable	[0.42,0.58]	arguable
	yes	[0.6,1.0]	yes	[0.58,1.0]	no,doubts
MAS descript.	bad	[0.0,0.33]	bad	[0.0,0.38]	well
	normal	[0.33,0.67]	normal	[0.38,0.62]	normal
	well	[0.67,1.0]	well	[0.62,1.0]	bad
Original	not	[0.0,0.2]	not	[0.0,0.25]	very
	mostly-not	[0.2,0.4]	mostly-not	[0.25,0.45]	mostly
	somewhat	[0.4,0.6]	somewhat	[0.45,0.55]	somewhat
	mostly	[0.6,0.8]	mostly	[0.55,0.7]	mostly-not
	very	[0.8,1.0]	very	[0.7,1.0]	not
Sound.	no	[0.0,0.33]	very-no	[0.0,0.29]	very-yes
	somewhat	[0.33,0.67]	no	[0.29,0.45]	yes
	yes	[0.67,1.0]	somewhat	[0.45,0.55]	somewhat
			yes	[0.55,0.64]	no
			very-yes	[0.64,1.0]	very-no
Technic. limits	not-discussed	[0.0,0.2]	not-discussed	[0.0,0.27]	very-adequately
	poorly	[0.2,0.4]	poorly	[0.27,0.47]	adequately
	briefly	[0.4,0.6]	briefly	[0.47,0.53]	briefly
	adequately	[0.6,1.0]	adequately	[0.53,0.58]	poorly
			very-adequately	[0.58,1.0]	not-discussed
Appro-ach	not-discussed	[0.0,0.2]	not-discussed	[0.0,0.34]	adequately
	poorly	[0.2,0.4]	poorly	[0.34,0.44]	adequately
	briefly	[0.4,0.6]	briefly	[0.44.0.56]	briefly
	adequately	[0.6,1.0]	adequately	[0.46,1.0]	not-discus.,poorly
English	deficient	[0.0,0.2]	deficient	[0.0,0.2]	correct
	typo&gramm	[0.2,0.4]	typo&gramm	[0.2,0.4]	typo,very-typo
	gramm	[0.4,0.6]	gramm	[0.4,0.6]	gramm
	typo	[0.6,0.8]	very-typo	[0.6,0.7]	typo&gramm
	correct	[0.8,1.0]	typo	[0.7,0.78]	typo&gramm
			correct	[0.78,1.0]	deficient
Referen.	poor	[0.0,0.2]	poor	[0.0,0.23]	very-complete
	basic	[0.2,0.4]	basic	[0.23,0.43]	complete
	old	[0.4,0.6]	old	[0.43,0.57]	old
	complete	[0.6,1.0]	complete	[0.57,0.70]	basic
			very-complete	[0.70,1.0]	poor
Overall	not-accepted	[0.0,0.2]	not-accepted	[0.0,0.28]	def-accepted
	doubts	[0.2,0.4]	doubts	[0.28,0.45]	accept-few-mod
	accept-with-mod	[0.4,0.6]	accept-with-mod	[0.45,0.55]	accept-with-mod
	accept-few-mod	[0.6,0.8]	accept-few-mod	[0.55,0.75]	doubts
	def-accepted	[0.8,1.0]	def-accepted	[0.75,1.0]	not-accepted

Table 21. Old and new vocabulary and semantics of the qualitative criteria

After the ranking and selection of the terms that describe each of the clusters, we build the new vocabularies and their semantics. As it was explained in sections 5.1.2, the new vocabulary has all the terms of the vocabulary selected as more appropriate (which in our case is the same for all the experts) and also the new terms generated during the explanation process. Comparing the first and second columns of Table 21, we can see the changes in the vocabulary and the semantics of the terms, which is expressed with their corresponding numerical intervals. Using the intervals of the new vocabulary (given by the fuzzy sets built with the positions of the prototypes of the clusters), we defined the negation function of each term, which are shown in the last column of Table 21.

Using the new semantics we know the numerical value that would correspond to each paper according to the interval assigned to each cluster (which can be calculated using the negation function). In Table 22 we can see the numerical value for each cluster before and after defining the negation function.

The results show that the order is kept in the new semantics although there are some small differences in the position of the cluster in the unit interval. These variations are due to the adaptations of the intervals when the new negation function is defined, in order to fulfill the conditions of a negation-based semantics (definition in section 3.1).

	Relevance		Agent?		MAS desc.		Originality		Soundness	
	bef	*aft*	*bef*	*aft*	*bef*	*aft*	*bef*	*aft*	*bef*	*aft*
C1	0.76	0.9	0.78	0.8	0.56	0.5	0.80	0.9	0.70	0.9
C2	0.69	0.7	0.52	0.5	0.40	unkn.	0.61	0.7	0.60	0.7
C3	0.56	unkn.	0.34	0.3	0.26	0.17	0.54	unkn.	0.41	0.3
C4	0.29	0.2	0.25	0.1			0.27	0.3	0.17	0.1
C5							0.10	0.1		

	Techn. Limits		Approach		English		References		Overall	
	bef	*aft*	*bef*	*aft*	*bef*	*aft*	*bef*	*aft*	*bef*	*aft*
C1	0.60	0.9	0.62	unkn.	0.86	0.93	0.76	0.9	0.73	0.7
C2	0.57	0.7	0.57	0.5	0.70	0.78	0.64	0.7	0.53	0.5
C3	0.44	0.5	0.38	0.3	0.69	0.64	0.55	unkn.	0.40	0.3
C4	0.37	unkn.	0.30	0.1	0.49	unkn.	0.43	0.5	0.16	0.1
C5					0.42	0.5	0.29	0.3		

Table 22. Numerical values that represent each cluster

At this moment, we have a new consensus criterion for each of the aspects evaluated by the different experts. Thus, the papers can be studied comparing all these criteria to find out which are the ones that should be selected to be published in the journal.

Before starting the second stage of this ME-MCDM process, we must have a look at the goodness of the new qualitative criteria. To calculate the global goodness we have given the same weight to each step of the ClusDM process, since we had no extra information from the user.

	G_{ClusDM}	G_{Agg1}	G_{Agg2}	G_{Agg}	G_{PCA}	G_{Sim}	G_{Terms}	G_{Neg}
Relevance	0.83	0.97	0.77	0.71	0.67		1.0	0.92
Agents?	0.93	0.96	0.96	0.88	0.89		1.0	0.94
MAS-desc	0.85	0.96	0.89	0.80	0.69		1.0	0.92
Originality	0.92	0.95	0.88	0.91	0.84		1.0	0.94
Soundness	0.73	0.96	0.75	0.78	0.70		0.56	0.88
TechLimits	0.80	0.91	0.75	0.83		0.82	0.71	0.84
Approach	0.84	0.96	0.82	0.89	0.58		1.0	0.88
English	0.79	0.97	0.97	0.97	0.59		0.69	0.93
References	0.77	0.95	0.86	0.81	0.65		0.71	0.93
Overall	0.90	0.95	0.85	0.90	0.76		1.0	0.94

Table 23. Goodness of ClusDM in the consensus of the criteria

The first column of Table 23 shows that we have achieved very encouraging quality values for all the new social criteria (the smallest is 0.73 and most of them are over 0.8). Although the data were provided by 26 different experts, it seems that we have been able to summarise their opinions for each criterion separately.

Joint analysis of the social criteria

The second stage of the ME-MCDM process consists of aggregating and ranking the consensued data of the new decision matrix. This matrix is built with the new social criteria obtained in the previous stage. In our case, the new matrix has 21 columns, since one of the criteria (the *research* status) have been removed because the system was not able to find a coherent result. Moreover, the number of alternatives has been reduced to 16 after dropping out those that had conflicting evaluations. To aggregate this data we will use again the ClusDM methodology because we must deal with a wide range of data types with different domains.

Before starting the ClusDM decision-making process, we established a predefined vocabulary to explain the result. As it has been explained in section 5.1.1, when the vocabularies of the criteria are not appropriate to describe the overall preference of the alternatives, we must define a suitable vocabulary. In this case the set of terms chosen are: *terrible, bad, poor, borderline, acceptable, good* and *excellent*. The semantics of them is the classical negation, that is, *borderline* is the neutral term, and we have 3 labels for giving negative values and 3 labels for positive qualifications.

The aggregation of the decision matrix using clustering produces a partition of the papers in 6 groups. At the next step, the Principal Components Analysis builds an axis that is able to explain the 68.5 % of the information of the matrix. The global goodness of the ranking is only 0.54 over 1.0. Moreover, the stopping criterion is saying that we

need 4 axes to have a good view of the data, although the first one is pretty better than the others. For this reason, we perform another ranking using the similarity to an ideal paper. With this method, we achieve a quality value of 0.82, which is acceptable enough to consider this ranking as good.

In the explanation step, we select the terms of the vocabulary to describe each cluster (Table 24). The clusters with a variance greater than 0.2 are considered as conflicting ones, because they have significantly different preference values for the criteria. In this case, the cluster with conflicting value has only a paper, number 31, which is one of the papers that were deeply reviewed for the editors, as it has been previously said.

Initial Vocabulary		z_1^{01}	**Terms selected**	**Paper's id.**
terrible	[0.0, 0.14]	C1 = 0.78	C1 - excellent	4 - 14 - 26
bad	[0.14, 0.29]	C2 = 0.67	C2 - good	18
poor	[0.29, 0.43]	C3 = 0.63	C3 - acceptable	3 - 29 - 32
borderline	[0.43, 0.57]	C4 = 0.58	C4 - borderline	6 - 8 - 9 - 17 - 20 - 22
acceptable	[0.57, 0.71]	C5 = 0.38	C5 - poor	1 - 5
good	[0.71, 0.86]	C6 = 0.36	C6 - *unknown*	31
excellent	[0.86, 1.0]			

Table 24. Qualitative description of the papers at the end of the process

To finish the ClusDM process we must obtain the new semantics of the terms. Using the similarities of the clusters to the ideal alternative in [0,1], we build the new negation function that will give meaning to the terms. In Table 25 we can see the intervals corresponding to the classical negation function, which are the original ones of the vocabulary given by the decision maker. The following columns show the intervals generated by the fuzzy sets attached to the terms, which are the ones used to determine the negations given in the last column.

Initial Vocab.		**New Vocab.**		**Negation**
terrible	[0.0, 0.14]	terrible	[0.0,0.14]	excellent
bad	[0.14, 0.29]	bad	[0.14,0.29]	excellent
poor	[0.29, 0.43]	poor	[0.29,0.44]	acceptable, good
borderline	[0.43, 0.57]	borderline	[0.44,0.56]	borderline
acceptable	[0.57, 0.71]	acceptable	[0.56, 0.65]	poor
good	[0.71, 0.86]	good	[0.65,0.72]	poor
excellent	[0.86, 1.0]	excellent	[0.72, 1.0]	terrible, bad

Table 25. Vocabulary and negation values of the papers selection criterion

Comparing the intervals corresponding to the terms before and after the process, we can see that the meaning of the positive terms of the vocabulary has changed. The coverage of term new "acceptable" is smaller than the initial one, while the term excellent has now a broader meaning.

Evaluation of the Results

The papers selected by the editors of this special issue of the journal were: 4, 14, 22 and 26. In addition, two other papers, 18 and 21, were recommended to be included in other numbers of the same journal due to the lack of space in this issue. Regarding to the last positions of the expert's preference, 1 and 5 were the worst papers.

If we analyse the results obtained with ClusDM, we can see that the papers greatly recommended for inclusion in the journal were indeed selected by the editors. The paper number 22, which was also included in the journal, was selected after another careful review of the paper by the editors, who considered that the marks given by one the referees were too low. Moreover, this work was about a subject of great interest for the research community. Those additional factors determined the final inclusion of this paper.

Concerning the low positions of the ranking, the worst papers according to ClusDM are the same than the ones indicated by the editor, number 1 and 5. Our method gives them a value of "poor" while the experts qualify them as "bad" and "terrible". This is due to the bad impression of the marks of these papers in comparison to the other ones. However, these marks are not too close to 0 as the editors thought. In spite of not obtaining such a bad qualification, we can see that the method is able to separate them and give them a low quality value.

After this rough analysis, let us pay our attention to the quality of this result. Remember that the confidence on the result is subject to the goodness values obtained in the different stages of the process. For this reason, we have detailed the calculation of these quality values in Table 26.

Measure	Value	Partial values for each element
G_{Agg}	0.90	$G_{Agg1} = 1 - \frac{0.128 + 0.139 + 0.124 + 0.137 + 0.0 + 0.0}{6} = 0.91$ $G_{Agg2} = -\frac{-0.314 * 2 - 0.173 * 2 - 0.26 - 0.368}{\ln 6} = 0.89$
G_{Rank}	0.82	$G_{Sim} = 1 - \frac{0.166 + 0.222 + 0.128 + 0.128 + 0.177 + 0.230}{6}$
G_{Terms}	0.81	$G_{Terms} = 1 - (0.0469/0.25)$
G_{Neg}	0.96	$G_{Neg} = 1 - \frac{0.08 + 0.015 + 0.025 + 0.08 + 0.015}{5}$
G_{ClusDM}	**0.87**	$G_{Neg} = 0.25 * 0.90 + 0.25 * 0.82 + 0.25 * 0.81 + 0.25 * 0.96$

Table 26. Goodness study of the second stage of the ME-MCDM

At the end of the ME-MCDM process, we have been able to rank the papers according to their global preference for being included in the journal. Moreover, the measure of confidence on the result is 0.87 (an 87%).

Nevertheless, as this is a selection problem we are really interested in knowing if the first class is good enough. For this reason we have a look at its size and we can see

that, considering the rough parameter of our method, it is the ideal one (i.e. 16 papers in 6 clusters: 16/6, that makes a rounded value of 3). In this particular application, the number of papers that should be selected was 4. With this extra information we can see that we have no problem in presenting to the user the 4 best alternatives: 4, 14, 26 and 18.

With this application, we have seen that the selection made by ClusDM is very similar to the one done by the editors. The only exception is number 22, which is a special case, as mentioned above. Other interesting results have been obtained during the process. For example, the detection of the papers that receive very different evaluations for the different experts.

Having into account that this was a complex problem because we were dealing with very different types of criteria as well as multiple experts and multiple criteria at the same time, the results are very encouraging for the use of this methodology in decision making.

7.2 Organ Transplant Receiver

The Research Group on Artificial Intelligence of *Universitat Rovira i Virgili* (to which I belong) is working on a prototype to support the communication and negotiation layers of the organ transplant co-ordination process [Aldea et al., 2001]. Organ transplants have an increasing importance in Medicine. Nowadays, surgery techniques and medical treatments allow to make transplant operations to many people. However, before an operation is performed a very complex co-ordination process takes place, which could be improved with the help of an intelligent computer system.

The process starts when an organ is available to be transplanted. Then, the most appropriate receiver for this organ must be found and this search must be done in a very brief period of time (in hours). Different organisations co-ordinate all the stages of the donation and transplant process according to the local, regional, national, and international norms and laws. There is a complex co-ordination model that must be followed. For the moment, all the tasks are made by people from different hospitals, who get in touch using the telephone and fax machines. However, many hospitals are interested in automating this process using Internet. This automation could reduce the time needed to find a receiver for an organ, which is important because the organs degrade through time.

The evaluation of the list of possible receivers and their ranking according to their compatibility with the donated organ is a very important task. The transplant co-ordinator must take into account many different criteria: time in the waiting list, physical characteristics, emotional state, etc. Moreover, the decision must be made under critical time constraints. For these reasons we have proposed the use of MCDA methods to help the co-ordinator [Valls et al., 2001]. In particular, the ClusDM methodology is interesting for two reasons: (1) the information is of heterogeneous nature and (2) we can give quality measures about the degree of trust on the preference ranking obtained.

In the initial prototype that we are developing, we have considered 6 criteria: the time the patient has been in the waiting list (the patients that have waited longer, have a high priority), the distance between the hospital of the donor and the hospital of the receiver (because the farther, the more difficult will be the carriage and transplant before the life deadline of the organ), the rest of the criteria are related to some physical characteristics of the person: weight, size of the organ needed/donated, antigens typology and age of the person. With all this information, the donor is compared to the possible receivers[11] and we obtain a decision matrix with the following preference criteria:

- difference between the weights of the donor and the receiver
- difference between the size of the donor's organ and the size of the organ needed by the receiver
- number of different antigens
- difference of age between the donor and the receiver
- distance to cover to bring the organ to the receiver
- amount of time that the receiver has been waiting for this organ

The preferences are expressed with linguistic values in the vocabulary chosen by the hospital transplant co-ordinator (see Table 27).

	worst value . . . *best value*
Weight	inadequate, feasible, good, optimum
Size	inadequate, feasible, good, optimum
Antigens	different, similar, identical
Age	more_thn_20,more_thn_17,more_thn_14, more_thn_11,more_thn_8,more_thn_5,the_same
Distance	country, zone, region, city, hospital
Waiting time	very_short, short, acceptable, long, very_long

Table 27. Vocabularies of the criteria for comparing the receivers of an organ

Although all the criteria are qualitative, they refer to very different aspects that are important in a transplant. Therefore, their vocabularies are not appropriate to describe the global suitability of the receivers. Instead of using the vocabularies of those criteria, we have build a new preference vocabulary. To avoid having to generate new terms, the vocabulary has 15 terms (with the classical negation semantics), indicating different degrees of compatibility with the donated organ, which are: *the_worst, terrible, very_bad, bad, not_recommendable, inappropriate, borderline, acceptable, adequate, recommendable, fairly_good, good, very_good, excellent, optimum*.

[11] The list of possible receivers is obtained from the hospitals of the rest of the country following a complex hierarchical procedure that is detailed in [Aldea et al.,2001].

As the process of searching the best receiver is done hierarchically, we have fixed the desired number of groups of patients to 7. After ranking these groups using the similarity method, each group receives one of the 15 terms of the vocabulary. The Principal Components Analysis is not appropriate in this case because the criteria talk about concepts that are not correlated.

The transplants co-ordination prototype. Agentifying ClusDM.

A prototype of the system that could be used for the transplant co-ordinator is being developed. The system must deal with different distributed data sources, different knowledge levels and a complex set of rules and norms. For this reason, the multi-agent systems technology is particularly appropriate to solve this problem. Before giving more details about the prototype, let me introduce the concept of agent and multi-agent systems:

An *agent* [Wooldridge,2002] is a computer system capable of flexible autonomous action in some environment. An agent has its own goals and the tools to be able to achieve them. The main properties of agents are:

- *Social ability*: an agent must be able to communicate with other agents, and co-operate with them to solve complex tasks.
- *Reactivity*: an agent is aware of the changes in the environment and responds to them in a timely fashion.
- *Autonomy*: the agent may decide whether to fulfil a given request or not, and may decide which is the best way to achieve his goals.

There are particular problems that cannot be solved by a single agent because different resources, knowledge or tools are needed. In this case, agents must cooperate, co-ordinate or negotiate with other agents to achieve their goals. This is a Multi-Agent System (MAS) [Weiss,1999]. MAS are interesting for large and complex systems in several senses: (i) with geographically distributed data, (ii) with many components or entities, possibly with particular interests, (iii) with a broad scope and huge amounts of information to consider. The use of intelligent, distributed agents is a suitable approach for this type of problems.

In the case of organ transplants, the selection of the best receiver must be done in a very short period of time (only some hours) because they cannot be frozen as we do with tissues and bones. For this reason, there is a great interest in having a tool to help in this process. In collaboration with some hospitals we are designing a Multi-Agent System that will follow the national and international rules established for transplants and will try to model the process that is done at the moment, known as the Spanish co-ordination model [Matesanz&Miranda, 1995]. This model is centred in the figure of the Hospital Transplant Co-ordinator (HTC) of each hospital. In each hospital a list of people waiting for an organ is maintained. When a donor is recognised, the searching of the most appropriate receiver starts. This process involves different organisations

that collaborate at different levels (see Figure 38). In our MAS we will respect this hierarchical organisation in order to find a receiver.

Concerning the use of MCDM techniques, we must concentrate on the agents working inside a particular hospital. In Figure 39 we can see the architecture of the internal MAS of a hospital. The doted square contains the agents that belong to the same hospital.

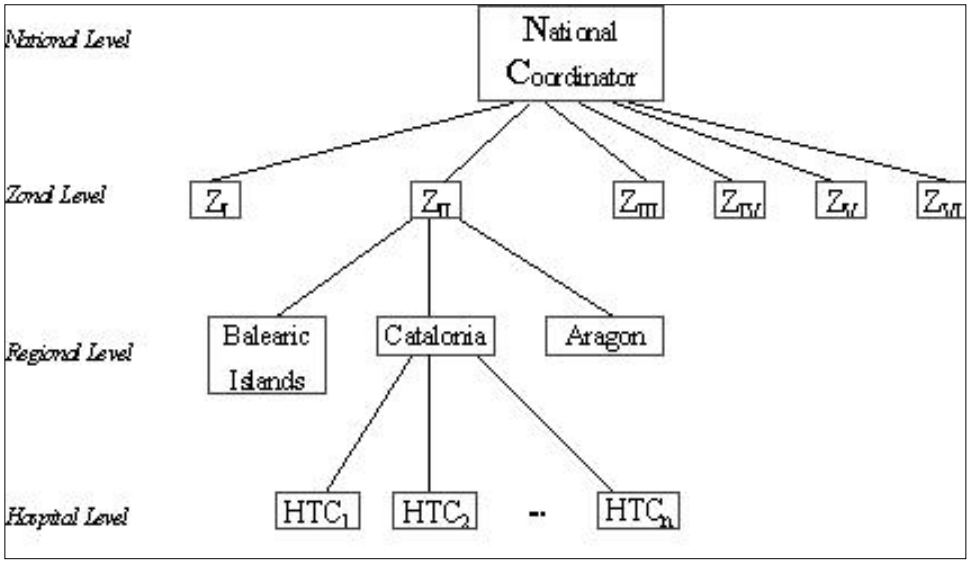

Figure 38. Hierarchical dependencies among transplant co-ordinators in Spain

The agent that is in contact with the medical personnel (and, in particular, with the hospital transplant co-ordinator) is called TCA (Transplant Co-ordinator Agent). This agent receives the characteristics of the donor and the organ that can be transplanted and starts the process to recommend the best possible receivers. First of all, he searches in the local database for potential receivers at the same hospital. This search is made with the help of the Medical Database Wrapper, which is the agent that is in charge of the access to the hospital database. Then, TCA sends a request for other candidates to the other TCAs in the same region (via the regional co-ordinator) or to the same zone (via the zonal co-ordinator). If no adequate receiver is found, he continues the search to other regions or zones, following some fixed rules.

When TCA has obtained a list of candidates, he sends them to the Transplant Specialist (TS) agent, which has knowledge about the field of organ transplants. This agent discards the patients that do not fulfil some basic compatibility conditions (e.g. the blood types of the donor and the potential receiver are not compatible). After this initial filtering, TS compares the attributes of the donor with the ones of the candidates

and builds a preference matrix using qualitative criteria, which is sent to an agent that is able to execute a MCDA method.

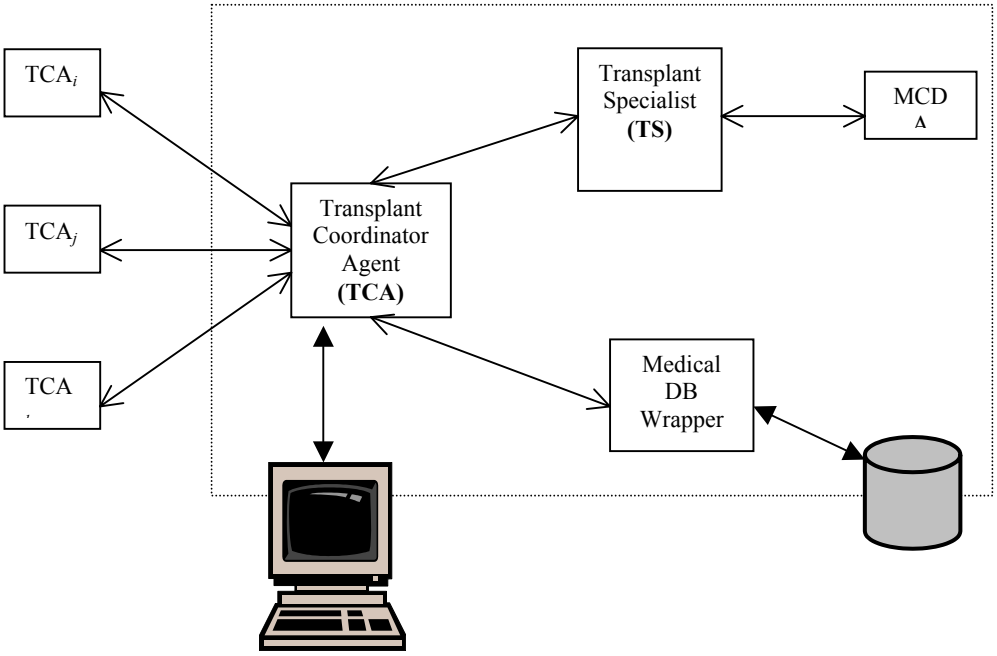

Figure 39. Intra-hospital multi-agent system architecture

We have already implemented an agent called *ClusDMA (ClusDM Agent)*. It is an agent that offers a very specific service: ranking a set of alternatives using multiple criteria. This service may be requested by agents that have to solve a decision problem or agent that have to aggregate heterogeneous data.

The *agentification* of a method consists of building an agent that is basically specialised in using this method properly. The agentification of MCDA methods is useful because multi-criteria decision aid is not a simple task. In the real world, it is usually done by experienced analysts who know how to apply the methodology and how to interpret the obtained results. Moreover, not all the MCDA methods can be successfully applied to the same kind of problems, it depends on the properties of the method and the characteristics of the problem. Thus, the first question that the analyst has to solve is the selection of which MCDA technique to apply. In a multi-agent system we could have different MCDA agents, where each of them was an expert in a particular technique. In [Valls&Torra, 2002a] we argue that it may be useful to

generate agents that are experts in solving MCDA problems. These agents would receive requests of any other agent that has to face a decision problem, regardless of the particular application or multi-agent system to which it belongs.

In this case, ClusDMA, has the three properties that define an agent:

- ClusDMA is able to communicate with the other agents engaged in the solution of a more complex process.
- ClusDMA aborts the clustering process if the time at his disposal is near to expire, in which case the result is the ordering of the classes built up to that moment. On the other hand, if it detects that the result will not be good (quality measurement stage), *ClusDMA* aborts the process and communicates it to the requester.
- At the reception of a request, ClusDMA can decide if he will make the ranking or not, depending on the characteristics of the message received (the data matrix is correct, the information about the semantics of the criteria is correct and the amount of data is tractable); thus, it also shows a certain degree of autonomy.

ClusDMA has been implemented using Jade, which is a collection of Java libraries that ease the implementation of FIPA-compliant[12] multi-agent systems. We are running this prototype on Windows in standard PCs, although it could be used in any other platform that supports Java. In Figure 40 we can see the interface that the Transplant Co-ordinator Agent shows to the hospital transplant expert. Before requesting the ranking of the set of patients, the user must assign a weight to each criterion. After processing the patients' data (which is stored in the file indicated at the top of the window) the system will display the result at the bottom. Then, the user can express his agreement with this result using the vocabulary listed at the bottom-right side. This information will be interesting to evaluate the ClusDM methodology when the system works with real data. Moreover, we have prepared the system to easily include other agentifications of MCDA methods in order to compare their performances.

This MCDA agent will be working with the multi-agent transplants co-ordination prototype (shown in Figure 38). At the moment, we have done some local tests with artificial data.

[12] FIPA (*Foundation for Intelligent Physical Agents*) is a non-profit association that provides internationally agreed specifications for developing agent-based applications.

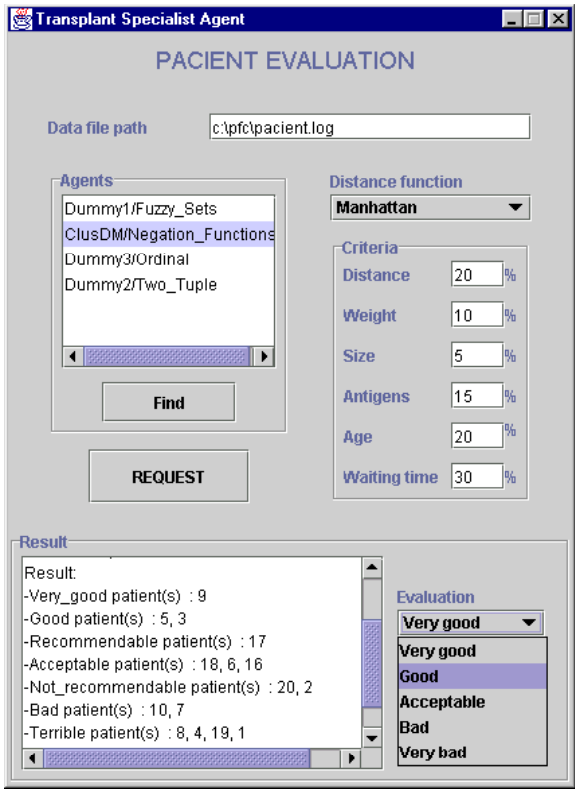

Figure 40. Transplant Co-ordinator Agent Interface

ClusDMA has different parameters (user's preference vocabulary for the description of the result, number of desired groups), which can be fixed in advance, in order to facilitate the use of the system by the transplant co-ordinator. Moreover, we do not show to the user the degree of quality of the result, because we do not want to influence his evaluation of the list of patients. Remember that this is only a prototype to test the possibility of obtaining automatic recommendations in such a critical medical problem. Therefore, we want to be careful in evaluating the appropriateness of the use of MCDA methods.

7.3 Statistical Disclosure Control

In recent years, the so-called information explosion has caused the development of new techniques for data analysis and information management. One class of techniques where this improvement can be found is the one related with information fusion and knowledge integration. As the number of available information sources and the amounts of information increase, the need of these techniques also increases. Applications of these techniques are now as diverse as scientific fields. One of the particular applications of information fusion techniques is Statistical Disclosure Control [Doyle et al., 2001].

National Statistical Offices (NSOs) are devoted to collect information from respondents and to their posterior publication. In fact, data dissemination is a requirement for National Statistical Offices as is the main justification for the resources spent and of their existence. However, data dissemination is usually a sensitive task because of re-identification risk. National Statistical Offices should process data prior to publication so that published data ensures that particular individuals or organizations cannot be re-identified. This is, no sensible data is published in a way that can be re-identified with a particular respondent (see [Torra, 2000] for a review of re-identification methods). Thus, data has to be protected (this is the so-called disclosure control problem) to avoid possible re-identification. Failure of protection can cause major problems due to legal norms and because respondents would refuse to new collaborations with the NSOs.

To avoid disclosure, masking methods are applied (see [Domingo&Torra,2002b] for a comparative study on masking methods performance). Masking methods introduce distortion to the data prior to its publication so that the information is not disclosured. Distortion should be kept small so that published data is valid for researchers and users (they can infer the same conclusions that would be inferred from the original data) but on the other hand should be protected enough so that disclosure is not possible. Statistical Disclosure Control (SDC) studies methods that attempt to perform such a nontrivial distortion.

When different microdata methods are applied to the same original file, different masked files are generated. In some cases, multiple protected versions of the same confidential data set are released, each one protected to minimize information loss for a particular use. In this case, an additional thread for re-identification risk appears due to the formation of coalitions of users. This is so because data fusion techniques can integrate the information contained in n different distorted versions of the data set. Thus, compromising statistical confidentiality. Note that, the better the reconstruction, the larger the disclosure risk. This suggests that data fusion tools can be applied to multiple masked data files to evaluate to what extent the original data file can be reconstructed.

We have studied the problem of fusing categorical data and evaluating the reidentification achieved [Valls&Torra, 2002b], [Domingo et al., 2002b]. We propose the use of the ClusDM as fusion (i.e. aggregation) method for categorical values in order to evaluate the degree of reconstruction achievable in this kind of data.

ClusDM as a fusion operator in Statistical Disclosure Control

We have considered a situation in which several masked versions of a single variable have been published. Our goal is to know if we can re-construct the original values of the alternatives from the fusion of the different releases.

For the application of the system to the SDC problem, we assume that each masking method corresponds to one criteria and that the aggregated criterion obtained by our system corresponds to an approximation of the original values.

To test the behaviour of ClusDM with heterogeneous qualitative variables, we have used 20 records extracted from the *American Housing Survey* of 1993 [Census Bureau, 1993]. Seven releases of the *Degree* variable have been generated using the most common masking methods for categorical data: Top coding (*Tp*), Bottom coding (*Bp*), Global recoding (*Gp*), Rank swapping (*Rp*) and Post-Randomization method (*Pp*). Different parameterisations have been considered, whose value is indicated by *p*. Parameterisations are based on the study of performance comparison for different masking methods with respect to information loss and re-identification risk [Domingo&Torra, 2002a].

Let us now briefly explain the masking methods we have included in this study:

- **Top Coding**. This method, applicable only to variables in ordinal scales, consists of the recoding of the highest *p* values of the variable into a new category. We have used the symbol '&' to denote the new term of the vocabulary, which substitutes the *p* values fusionated. A recoding of 4 categories has been considered (T4 in Table 28). Top coding is applied to avoid the re-identification of largest values as they are frequently easy to re-identify.

- **Bottom Coding**. This masking method is similar to the previous case but now the lowest *p* categories are recoded into a new one. As before, we have selected p=4 and the new category is codified by '&'. As in the case of Top coding, this masking method is applied to avoid the re-identification of the smallest values when the availability of this information allows the re-identification of the individuals.

- **Global Recoding**. Global recoding consists of the recodification of some categories by some other ones. Selection of categories is done on the basis of increasing the number of individuals that match a particular category. For example, if there is a record with "Marital status = Widow/er" and "Age = 17", global recoding could be applied to "Marital status"' to create a broader category "Widow/er or divorced", so that the probability of the above record being unique would diminish. In our experimentation, the following parameterisation has been considered: recode the *p* lowest frequency categories into a single one. We have used p=4.

- **Rank Swapping**. This method is better explained from their operational point of view. First, values of variable c_i are ranked in ascending order; then each ranked value of c_i is swapped with another ranked value randomly chosen within a restricted range (e.g. the rank of two swapped values cannot differ by more than p% of the total number of records). We have used p=10%.

- **Post-RAndomization Method or PRAM**. This is a perturbative probabilistic method in which the value of a given individual is changed according to a prescribed probability mechanism (a Markov Matrix). This method reduces the number of matching for all categories (reduction depends on the Markov matrix). The selected Markov matrix is based on the approach described in [Kooiman et. al., 1998]. This approach is as follows: Let $T_V=(T_V(1), ..., T_V(K))^t$ be the vector of frequencies of the K categories of variable V in the original file (assume without loss of generality that $T_V(k) \geq T_V(K) > 0$ for $k < K$) and let θ be such that $0 < \theta < 1$. Then the PRAM matrix for variable V is defined as:

$$p_{kl} = \begin{cases} 1-\theta T_V(K)/T_V(k) & \text{if } l = k \\ \theta T_V(K)/((K-1)T_V(k)) & \text{if } l \neq k \end{cases}$$

  In our example we have considered different parameterisations p = 4, 8, 9 and p=10θ.

The original variable *Degree* have a negation-based linguistic vocabulary with 7 terms: L={*coldest, cold, cool, mixed, mild, hot*}. In Table 28, the original values are replaced by the position of the category in the set L, for the sake of clarity. Thus, value 1 stands for *coldest*, 2 for *cold*, 3 for *cool* and so on. The first column corresponds to the identifier of the record (i.e. alternative), the second column is the original value of the variable (o.v), columns 3-9 are masked variables, column 10 is aggregated value (a.v.).

	o.v.	B4	T4	G4	R10	P8	P9	P4	a.v.			o.v.	B4	T4	G4	R10	P8	P9	P4	a.v.
a	3	&	&	3	3	3	3	3	3		k	3	&	&	3	4	3	3	3	3
b	3	&	&	3	2	3	3	3	3		l	3	&	&	3	2	3	3	3	3
c	3	&	&	3	3	3	3	3	3		m	3	&	&	3	3	3	3	3	3
d	3	&	&	3	3	3	3	3	3		n	2	&	2	2	2	2	2	2	2
e	4	4	&	4	4	4	4	4	4		o	3	&	&	3	3	3	3	3	3
f	4	4	&	4	4	4	4	3	4		p	2	&	2	2	2	2	2	2	2
g	4	4	&	4	3	4	4	4	4		q	3	&	&	3	3	3	3	3	3
h	4	4	&	4	4	4	4	4	4		r	5	&	&	n	5	3	3	4	3
i	4	4	&	4	4	4	4	4	4		s	2	&	2	2	2	2	2	2	2
j	1	&	1	n	1	1	1	1	1		t	2	&	2	2	2	3	4	2	2

Table 28. Records used in the re-identification test

Using the classifier *Sedàs* we obtain the dendrogram in Figure 41. Then, a cut level has to be selected in the tree to obtain a partition of the elements. The cut is done so that the number of clusters is equal to 4 because this is the average number of linguistic labels used in columns B4-P4. This cut is also displayed in Figure 41. The obtained partition is defined by 4 sets (named α, β, δ and γ) as follows: α={n,t}, β={a,b,k,r}, δ={j}, γ={e,f,g}. This partition satisfies the conditions required in [Domingo et al., 2002] for a correct partition selection in this context:

- records with all the variables with the same value should correspond to different clusters (e.g. record a and e),
- clusters should be defined according to the dendrogram.

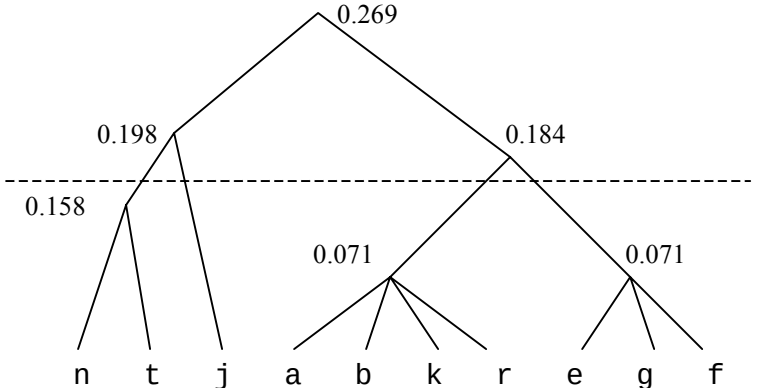

Figure 41. Dendrogram for the clustering of the statistical data

Note that for the sake of simplicity, we only include in the dendrogram and in the partition one of those elements that are indistinguishable (i.e., it appears the element a but does not appear c because it has the same values for all columns).

The 4 clusters obtained have been ranked using the similarity to the best possible alternative, the one that has the largest value for all categories. The ranking with PCA was not possible because the stopping criterion selects two axes instead of a single one.

Following the explanation stage process, each class is given a category from the original vocabulary, L: δ is *coldest* (1), α is *cold* (2), β is *cool* (3) and γ is *mixed* (4). This result can be seen in Table 28. The goodness values for each step are the following: 0.8 for the aggregation, 0.93 for the ranking, 0.63 for the vocabulary and 0.95 for the representation of the clusters by the new semantics. The overall confidence on the result is 0.83. The lowest quality is for the stage of building the new vocabulary and semantics, this indicates that the meaning of the terms has been changed with respect to the original one.

In this example, we can see that we have re-identified the original value of each record, except one, record r. Therefore, the publication of these 7 versions of the same data is very dangerous due to the proved ability of discovering the original value of the variable. However, although we re-construct the "labels" attached to the records, we have seen that the semantics of the terms is not completely re-identified.

We can see that the ClusDM methodology can be successfully used as an aggregation or fusion operator for the re-identification of statistical data. However, we must mention that other tests have not been so good. If the masking methods produce some outliers, the clustering method would create a class for those "different" records, which will make that the rest of records ought to be put in a smaller number of clusters than it should. In this case, the reconstruction is more difficult. Nevertheless, we can detect this situation using the information provided by ClusDM about the conflicting records (the outliers), and we can repeat the process increasing the number of clusters in the partition or removing the outliers from the analysis (as we have shown in section 7.1).

Chapter 8.

Summary and Future Work

After having explained in detail the difficulties of multiple criteria decisions, the different approaches to facilitate the work of the decision maker and having presented a new methodology called ClusDM, this chapter is devoted to review the main characteristics of our approach and to give some future research lines to improve it.

8.1 Summary and conclusions

This thesis proposes a new methodology for Multiple Criteria Decision Aid. This work is the result of some years of research in order to develop a method to deal with complex multicriteria decisions. The difficulties that we have faced up are the management of: criteria of different nature (numerical, qualitative and Boolean), different scales (different vocabularies or categories) and missing values. We considered a new approach to this heterogeneous data that does not require the transformation of all the values into a unified scale. The use of clustering to perform the aggregation of the values has been proved to be a good solution for the integration of heterogeneous data. The approach based on similarities (inherent in clustering techniques) allows us to compare the alternatives and understand the relationships among them. These global preference relationships cannot be found if we assume the independence of the alternatives, as classical MCDA methods do.

The second aim of our research is the development of tools that the user can understand and apply easily. The negation-based semantics seems quite appropriate for this purpose because it is based in the *antonym* concept, which is nothing new for people. During chapter 5, we have seen how to use this semantics representation to attach a suitable linguistic label to each alternative. This is a crucial point, because the decision maker will base their final decision on these values, having into account that

the relative preferences over these values are expressed by the negation function attached to its vocabulary.

Nevertheless, we have gone a step further. We have analysed in detail the process in order to extract useful knowledge about the elements of the decision framework. Conflicting alternatives and criteria are detected and presented to the decision maker. This additional information together with a quality evaluation of the process is of great value for understanding and successfully applying the solution obtained.

To end this overview, we would like to mention some drawbacks of ClusDM. The first one is that a minimum number of alternatives are required to obtain sufficiently good results. As alternatives are compared with each other during the first stage of the process, if the number of alternatives is small (e.g. less than 7) the clusters will not be very significant because their number of elements will be low. Therefore, ClusDM is a good method to be used in decision problems that involve a large set of alternatives. However, a second drawback is related to the first: if we study the temporal complexity of the ClusDM process, we can see that it is $O(m,p) = m^2 p$, being m the number of alternatives and p the number of criteria. That is, the number of alternatives in the set has great influence in the time consumed by the process.

8.2 Future directions

In chapter 2, some methods that work with uncertain information in MCDA have been presented. Some of them are able to deal with heterogeneous data sets. However, as it has been pointed out before, they perform a transformation of the original values of the criteria into a unified framework, where the decision analysis is done. Some of them define processes to put different linguistic vocabularies into a common one but do not consider the case of including numerical data, others handle the possibility of having qualitative and numerical criteria in the same decision matrix. The two cases are interesting in order to study the behaviour of ClusDM in a qualitative framework with different vocabularies or in the case of mixing numerical and qualitative criteria.

In Table 29 we have made a classification of the methods presented in section 2.5 in terms of the possibility of managing different qualitative vocabularies or mixed types of criteria versus the type of semantics given to the linguistic terms. An exhaustive comparison of these methods would be interesting.

	Many Vocabularies	**numerical + Qualitative**
Fuzzy Sets	LOWA (after translations explained in [Herrera et al.,2000b])	LOWA (after translations explained in [Delgado et al.,1998])
Negation functions	ClusDM Antonyms-based aggregation	ClusDM
2-tuple	2-tuple Weighted Mean 2-tuple OWA (after translations explained in [Martínez,1999])	2-tuple Weighted Mean 2-tuple OWA (after translations explained in [Herrera&Martínez,2000a])
Ordinal scale	QWM (after translations explained in [Torra&Godo,1999])	

Table 29. Aggregation operators for heterogeneous criteria

Methods for ordinal linguistic values given in section 2.5 can be classified according to the kind of semantics they deal with: explicit semantics (like the use of fuzzy sets or negation functions), implicit semantics (like the 2-tuple linguistic values) or direct computation on the ordinal scale. For applying the methods based on fuzzy sets, we must know the fuzzy membership function for each term. Considering that qualitative criteria have a negation-based semantics, we can use the intervals induced by the negations to build the fuzzy set corresponding to each linguistic value. Following [Yuan&Shaw,1995] (as it has been done in the explanation stage, section 5.1.3), the centre of each interval may be the point of maximum membership to the corresponding term. The rest of the triangular membership function is defined by these points. Obviously, with this approximation, there is a modification of the information given by the terms, which will influence the results when the two semantics are compared.

Once we have a set of criteria described using fuzzy values, negation functions and 2-tuples, we could use the methods in Table 29. In [Zimmermann,1990] some guidelines to compare and classify MCDA methods are given. Zimmermann mentions 5 different dimensions: generality (i.e. the degree of general applicability of the method), discrimination (i.e. the capability of differentiating alternatives with slightly different values), fuzzification (i.e. treatment of uncertainty), information requirements (i.e. if the method needs a standard representation of the inputs) and sophistication (i.e. mathematical complexity). In a recent book ([Triantaphyllou, 2000]), the author compares some classical methods of the utility-based and the outranking approaches. For example, he makes a comparison of the methods in terms of two evaluative criteria: (i) an MCDA method that is accurate in multi-dimensional problems should also be accurate in single-dimensional problems and (ii) an effective MCDM method should not change the indication of the best alternative when an alternative (not the best) is replaced by another worse alternative. Moreover, we should also make comparisons of the goodness of the result obtained for problems with a known solution.

After this comparative analysis, we would like to perform more accurate tests in the application domains presented in chapter 7. For example, we can obtain more

statistical public data or we can use ClusDM in other journal or conference reviewing process. In addition, we are considering other application domains. In fact, we are developing a multiagent system to help companies to make personnel selection (an initial prototype is explained in [Batet,2002]).

With the use of our methodology in various domains, we could improve the explanation stage. We pretend to present to the decision maker a more user-friendly view of the quality measures and knowledge extracted during the process. The use of natural language will be of great interest as is argued by people ([Greco et al., 2001], [Bana e Costa, 1990]).

Another future research line is the adaptation of the ClusDM methodology to deal with dynamic environments. As [Olson et al., 2001] pointed out, decision making problems usually deal with changing elements. It would be interesting that ClusDM could include or drop alternatives during the process. In fact, the modification of the alternatives would affect the clusters obtained in the aggregation stage. If the modification of the alternatives set is done after the aggregation, the prototypes should be recalculated before the ranking stage, because both the Principal Components Analysis and the Similarity-based ranking are based on the prototypes values. Once the ranking stage has finished, the inclusion and deletion of alternatives should be carefully studied, for its implication not only in the vocabulary and semantics but also in the ranking itself.

In the same line, it is possible to have to evaluate new alternatives after the analysis of the initial data set. In this case, we would like to study the work on automatic rules generation [Riaño,1998]. Then, the partition induced by the new preference-ordered qualitative criterion given by ClusDM could be explained using if-then rules. These rules will be used to classify (to know the linguistic preference value) corresponding to a new alternative.

The results obtained until now encourage us to continue our work in MCDA. We hope to be able to develop interesting solutions for the open-problems outlined in this section.

APPENDIX A. Review Form of the Journal

The following review form was designed to test the ClusDM methodology. We adapted the model provided by the editors of the journal. We included different types of criteria: numerical, ordinal qualitative and categorical.

The form is divided up in 6 parts; the first one identifies the paper (this information is not used in the reviewing process) and provides some information about the main characteristics of the work, which are two of the criteria included in the analysis. The following two sections (A and B) are devoted to evaluate the content and presentation of the paper. All the questions were considered as qualitative preference criteria in the test, except for question A.4 that is answered in natural language. For the same reason, section C could not be included to the ClusDM analysis. Section D shows the overall evaluation of the decision makers, which was included as another ordinal qualitative criterion. Finally, the last section identifies the reviewer and their confidence on the subject. This additional information was not considered in the test.

AUTHOR(S):

TITLE:

The paper reports on: [ ] A methodology [ ] Applications

The emphasis of the paper is on:
[ ] Preliminary research
[ ] Mature research, but work still in progress
[ ] Completed research

A. CONTENT

1. How relevant is the content of the paper to the theme of the special issue? (is the described system really composed of "AGENTS" that are applied in an interesting way to any aspect of "HEALTHCARE"?)

[ ] Very Relevant [ ] Quite Relevant
[ ] Somewhat Relevant [ ] Not Relevant

2. Is the paper really concerned with "agents" (i.e. autonomous, intelligent, communicative, cooperative, proactive entities)?

[ ] I definitely think so.
[ ] It might be arguable whether "agent" is the best expression for the elements of the described system.
[ ] I have strong doubts regarding the usage of the word "agent" in this work.
[ ] The paper abuses the use of the word "agent", i.e. it tries to "agentify" an otherwise standard AI application.
3. If the paper describes a multi-agent system, are communication, co-ordination and/or negotiation techniques described?

[ ] They are well described.
[ ] They should be explained in more depth.
[ ] They are not clearly explained.

4. What are the main contributions of the paper?

5. How original is the research reported?

[ ] Very Original [ ] Mostly Original
[ ] Somewhat Original [ ] Mostly Unoriginal [ ] Not Original

6. Quality of the Research:

Is the research technically sound? [ ] Yes [ ] Somewhat [ ] No

About the technical limitations/difficulties ...
[ ] They are adequately discussed
[ ] They are briefly discussed
[ ] They are poorly discussed
[ ] They are not discussed

About the approach ...
[ ] It is adequately evaluated
[ ] It is briefly evaluated
[ ] It is poorly evaluated
[ ] It is not evaluated
7. For papers focusing on applications:
(Give a numerical evaluation from 1 to 5, 1 is the worst value)

Is the application domain adequately described? (1..5)
Is the choice of a particular methodology discussed? (1..5)

8. For papers describing a methodology:
(Give a numerical evaluation from 1 to 5, 1 is the worst value)

Is the methodology adequately described? (1..5)
Is the application range of the methodology adequately described, e.g. through clear examples of its usage? (1..5)

B. PRESENTATION

(Give a numerical evaluation from 1 to 7, 1 is the worst value)

1. Are the title and abstract appropriate? (1..7)
2. Does the introduction show the intentions of the paper and presents the rest of the article? (1..7)
3. Does the last section give the conclusions or the most relevant results of the work? (1..7)
4. Is the paper well organized? (1..7)
5. Is the paper easy to read and understand? (1..7)
6. Are figures/tables/illustrations sufficient? (1..7)

7. Is the paper free of typographical/grammatical errors?
[ ] The English is correct
[ ] There are some typographical errors
[ ] There are some grammatical errors
[ ] There are both typographical and grammatical errors
[ ] The English is deficient

8. Is the references section complete?
[ ] Yes
[ ] The basic work is referenced but recent work is not.
[ ] There are missing relevant basic references
[ ] It is very poor

C. SUGGESTED/REQUIRED MODIFICATIONS & ANY OTHER COMMENTS

D. OVERALL

The paper ...
[ ] is definitely recommended for inclusion in the special issue
[ ] is recommended for inclusion in the special issue after a few modifications
[ ] could be recommended for inclusion only after important modifications
[ ] is interesting, but not mature enough to be included in this issue
[ ] is definitely not recommended for inclusion in the special issue

REVIEWER'S NAME:
Reviewer's confidence in the subject area of the paper: [ ] High [ ] Medium [ ] Low

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