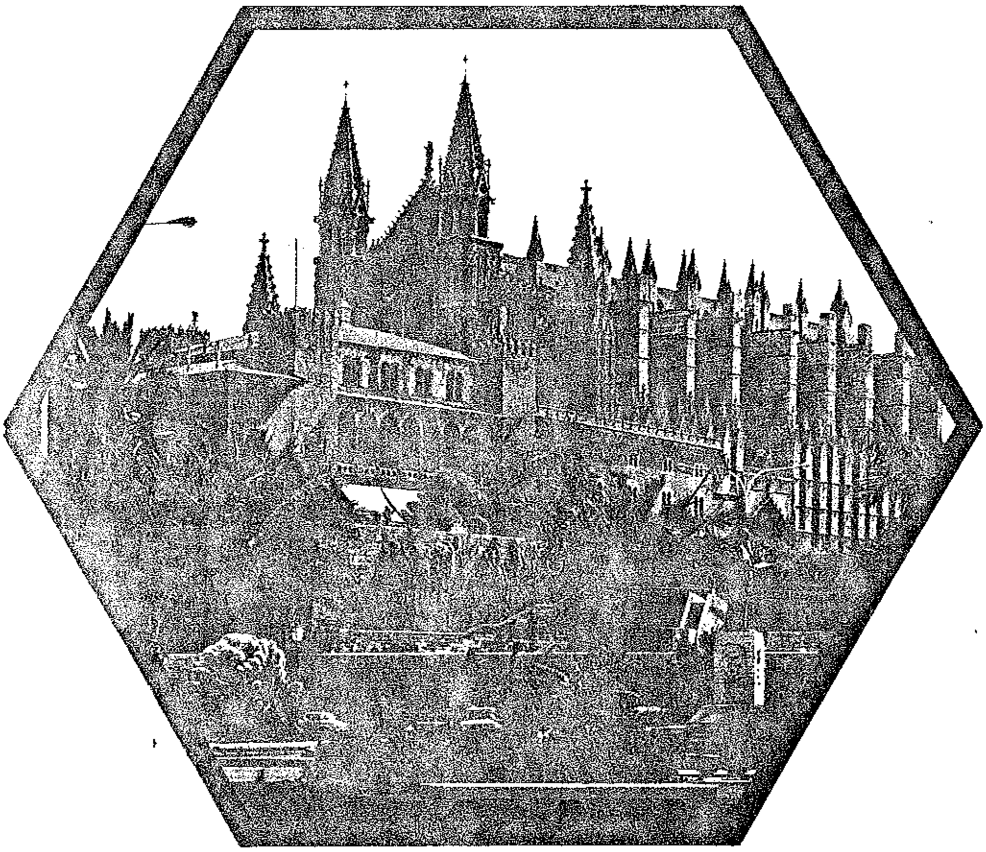
PROCEEDINGS

THE EIGHTEENTH INTERNATIONAL SYMPOSIUM ON MULTIPLE-VALUED LOGIC



MAY 24-26, 1988
PALMA DE MALLORCA, SPAIN

COMPUTER SOCIETY

UNIVERSIDAD DE LAS ISLAS BALEARES

CONSEJO SUPERIOR DE INVESTIGACIONES CIENTIFICAS

MVL TECHNICAL COMMITTEE

ISSN 0195-623X
Computer Society Order Number 859
Library of Congress Number 79-64110
IEEE Catalog Number 88CH2546-0
ISBN 0-8186-0859-5

THE COMPUTER SOCIETY

IEEE THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, INC.

COMPUTER SOCIETY PRESS

Session 7: Invited Address

Session 8a: Logic Design

Session 8b: Artificial Intelligence

Session 9a: Logic Design

Session 9b: Fuzzy Sets

*Abstract only

A New Approach to Connective Generation in the Framework of Expert Systems using Fuzzy Logic.

Lluís Godo, Carlos Sierra

C.S.I.C.-Centre d'Estudis Avançats de Blanes
Camí de Santa Bàrbara 17300 Blanes Girona Spain

Abstract.

This paper presents a tecnique to model uncertainty in Expert Systems. Operators are defined using linguistic terms to avoid any numerical representation. These operators consider linguistic term set ordering, constraints that are the counterpart of properties fullfilled by triangular norms in fuzzy logic, and additional restrictions created by the expert's procedure to combining certainty.
This method avoids the usual problems that arise in other treatements of certainty linguistic terms, where they are represented as fuzzy numbers or fuzzy truth labels. One of the most significative problems is the lack of consensus in the representation of each term by a group of experts, due to the necessity of representing the terms in a pseudo-numerical scale.

1 Introduction.

The management of uncertainty in Expert Systems (Representation and Propagation) is an area of increasing interest. Systems using certainty linguistic terms are very useful in capturing human reasonning. (Bonissone, 1979; Bonissone and Becker 1985; Godo et al., 1987). Fuzzy Sets Theory provides tools that represent linguistic terms as fuzzy sets, or as fuzzy labels, and to use triangular functions to model the logic connectives that perform the propagation of certainty.

1.1Representation with fuzzy sets.

Each term T_i is represented by a fuzzy interval in the truth space represented by [0,1]. The membership function of each interval is trapezoidal and is characterized by four parameters $T_i=(a_i,b_i,c_i,d_i)$, corresponding to figure 1.

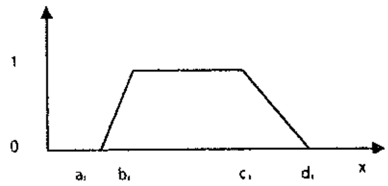


Fig 1 Trapezoidal function

In the application PNEUMON-IA (López de Mántaras et al., 1987) the linguistic term set and its definition in this representation schema is shown in Figure 2 and as:

IMPOSSIBLE	=(0, 0, 0, 0)
ALMOST IMPOSSIBLE	=(0, 0,0.05,0.08)
SLIGHTLY POSSIBLE	=(0.05,0.07,0.14,0.17)
MODERATELY POSSIBLE	=(0.10,0.15,0.35,0.45)
POSSIBLE	=(0.25,0.35,0.55,0.65)
QUITE POSSIBLE	=(0.45,0.55,0.75,0.85)
VERY POSSIBLE	=(0.65,0.75, 1, 1)
ALMOST SURE	=(0.95,0.98, 1, 1)
SURE	=(1, 1, 1, 1)

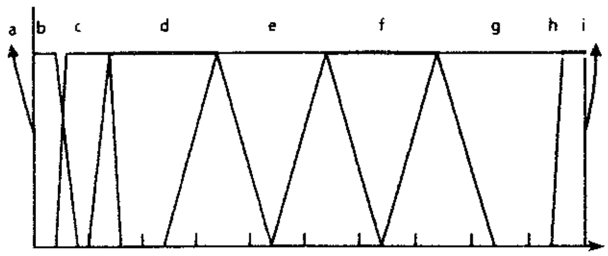


Fig. 2 Term set of PNEUMON-IA. a Impossible, b Almost Impossible, c Slightly Possible, d Moderately Possible, e Possible, f Quite Possible, g Very Possible, h Almost Sure, i Sure.

1.2 Representation with fuzzy labels

A fuzzy label is defined as the function $\tau:[0,1]\rightarrow[0,1]$ such that the proposition "(X is **A**) is τ" is equivalent to "X is **A'**", where **A** and **A'** are fuzzy subsets of an universe U, with $\mathbf{A'}=\tau o \mathbf{A}$, in the sense of the usual functions composition (Figure 3). Here **A** and **A'** are identified with their associated possibility distributions (Zadeh,1983; Baldwin,1979).
From this definition, fuzzy labels are understood as linguistic modifiers of the meaning of the term represented by a fuzzy subset **A**. The modification makes the fuzzy subset more or less restrictive over the set to which it is applied. They can therefore be used as certainty labels propositions "X is **A**"

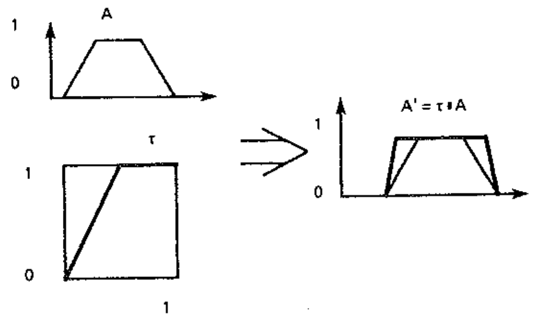


Fig. 3 Modification of a fuzzy set by a fuzzy label

In MILORD (Godo et al, 1987) two families of linguistic fuzzy labels are considered. They have a two-parameter representation :

$$\tau_{a,b}(x)=\begin{cases}0, & 0\leq x\leq a\\ \dfrac{x-a}{b-a}, & a<x<b \\ 1, & b\leq x\leq 1\end{cases} \qquad \text{si } a\neq b$$

$$\tau_{0,0}(x)=\begin{cases}0, & x=0\\ 1, & 0<x\leq 1\end{cases}$$

$$\tau_{1,1}(x)=\begin{cases}0, & 0\leq x<1\\ 1, & x=1\end{cases}$$

where a=0 in the first family and b=1 in the second. The first family transforms a possibility distribution into a less restrictive one (Figure 3) and the second into a more restrictive one. Depending on the degree of modification it is possible to define an ordering of the linguistic labels as performed using the trapezoidal representation.
Using this representation the certainty linguistic terms in PNEUMON-IA are defined as:

IMPOSSIBLE	=	(0,0)
ALMOST IMPOSSIBLE	=	(0,0.05)
SLIGHTLY POSSIBLE	=	(0,0.10)
MODERATELY POSSIBLE	=	(0,0.65)
POSSIBLE	=	(0,1)
QUITE POSSIBLE	=	(0.35,1)
VERY POSSIBLE	=	(0.9,1)
ALMOST SURE	=	(0.95,1)
SURE	=	(1,1)

1.3 Linguistic aproximation and logic operators construction

To combine and propagate uncertainty, it is necessary to define logic operators on the set of linguistic terms. This is performed in two steps:
The first step is to obtain operators over the representations above mentioned, adapting the logic connectives of the systems of Multivalued Logic related to Fuzzy Set Theory (t-norms, t-conorms, implication functions ...).
The second step is a linguistic aproximation process. The operators obtained in the first step produce another trapezoid or fuzzy label when applied to elements of the term set. This depends on the representation schema used. To mantain set closure, the result is matched to a term in the term set with the closest meaning (representation). Having the term set closed allows storage the logic operators as matrices that can be computed off-line.

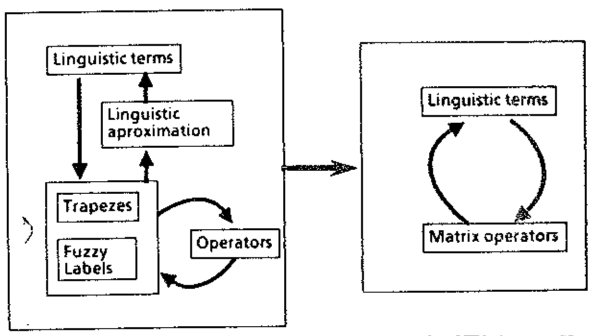


Fig 4 Logical operators construction.

2. Alternative aproach to connectives computation.

2.1 Motivations

Despite the theoretical model being flexible and expressive, sometimes it is difficult for the knowledge engineer to translate the abstract meaning of a linguistic term to one of the possible representations presented above (i.e. very possible). These difficulties include:

* Often the expert is not able to define the exact meaning of his linguistic terms using a numerical scale.

* Experts do not agree with the representation of a concrete linguistic term. Studies show a great dispersion in the definition given by experts (Kong et al, 1986). It means that the same certainty linguistic term set will produce different operators for different experts. This problems arises despite the expert having no problem in determining how the linguistic terms are combined.

The objective of the proposed approach is to obtain logical operators defined over a set of certainty linguistic terms, as before, but without any explicit representation of the meaning of each term. A supposed certainty ordering in the term set is used to define the logical operators, joint with some natural constraints depending on the kind of logical connective being modeled.

In what follows, let **E** be an ordered set of linguistic terms $\mathbf{E}=\{E_1, ..., E_n\}$. This means that if $i \leq j$ the certainty level represented by E_i is lower than the certainty level of E_j.

2.2 Unary operator: the negation

The problem of finding an unary operator,(i.e. a function N:E->E), that models a negation connective is very simple, because there is only one possibility that defines such a function. In fact, if the corresponding properties of strong negation functions (Esteva,1981) have to be fulfiled, that is:

N1. Monotonically decreasing
$N(E_i) \leq N(E_j)$, if $i \geq j$ $i,j = 1...n$
N2. Involution
$N(N(E_i)) = E_i$ $i = 1...$
(in particular, $N(E_1) = E_n$ and $N(E_n) = E_1$)

then, the function N can only be defined in the following way:

$$N(E_i) = E_k \text{ with } k = n + 1 - i$$

It should be noted that the negation conective is understood in the following way: if E_i is the certainty linguistic term associated with the proposition "X is A", then $N(E_i)$ is the certainty linguistic term associated with "X is NOT A".

The negation operator associated to a set **E** of linguistic terms will be called $N_{\mathbf{E}}$.

2.3 Binary operators.

The following describes the construction of the AND operator. The construction of the OR operators is its dual. Operators are desired, defined as functions $t{:}E \times E \rightarrow E$ that model the connective AND, satisfying properties of triangular norms. These are the most general family of two-place functions in [0,1]x[0,1] onto [0,1] that satisfy the requirements of conjunction operators. The translated properties are:

T1 *Associativity.*
$t(t(E_i,E_j),E_k) = t(E_i,t(E_j,E_k))$ $i,j,k = 1..n$
T2 *Conmutativity.*
$t(E_i,E_j) = t(E_j,E_i)$ $i,j = 1..n$
T3 *Absortion*
$t(E_1,E_i) = E_1$ $i = 1..n$
T4 *Neutral*
$t(E_n,E_i) = E_i$ $i = 1..n$
T5 *Monotonically increasing*
$t(E_i,E_j) \geq t(E_{i-1},E_j)$ $i = 2..n, j = 1..n$

Other desirable properties are:

T6 *Strictness*
$t(E_i,E_j) \neq E_1$ If $i,j \neq 1$
T7 *Smoothness*
if $t(E_i,E_j) = E_k$, $t(E_{i-1},E_j) = E_p$, and $t(E_i,E_{j-1}) = E_q$,
then the next restriction must be satisfied for a given $\alpha \in N$ (common values are 1,2):
$$k\text{-}\alpha \leq p, q \leq k$$

These properties act as constraints over the set of functions f:E x E -> E to reduce the number of possible solutions (n^{n^2}, initially). However, the number of matrices that satisfy these constraints is still large when the number of linguistic terms is more than five (Figure 5).

Conversely, these are the most general constraints that provide a standard behaviour of an AND connective. Depending upon the context in which the logical operator is determined, additional constraints are considered to decrease the number of possible solutions.

Two types of additional constraints are discussed. One describes the generation of matrices or functions, noting how incremental generation can reduce the number of solutions (section 3). The second describes the use of an expert as a guide in the generation of logical operators to better "fit" the model (section 4). Note that these two kinds of constraints are not exclusive.

n	A	B	C	D
3	2	1		
4	7	2	6/7	2/2
5	42	7	24/41	6/7
6	429	42	120/376	24/41
7		429	720/5033	120/376
8			5040	720/5033
9			40320	5040

Fig. 5 Number of operators depending on the properties satisfied
Besides T1,T2,T3,T4,T5:
A: -
B: T6
C: T7
D: T6,T7
Note: first numbers in columns C and D are for $\alpha = 1$ in property T7, and the seconds for $\alpha = 2$.

3.Incremental generation.

Incremental generation consists in defining a logical operator t_E on a set of linguistic terms **E**, by extension of another operator t_F, previously defined on a subset **F** of **E**. This extension is defined as:

1) $t_E(A,B) = t_F(A,B)$, if $A,B \in F$
2) $N_E(A) = N_F(A)$, if $A \in F$

where N_E and N_F are the associated negation operators of **E** and **F**, respectively. The first condition defines the operator on **F** to act as a pattern that ensures the consistency of the new operator with respect to the old one. This is useful in an expert system development phase

if, at a given moment, the expert needs to refine his certainty evaluations, extending his term set but without rejecting those previously made.This condition also affects the smoothness of the operator as it bounds the range of possible values in a particular place in the t_E operator matrix. The second condition extends the term set in a symetrical way with respect to the central element(s) of the set. However,this condition does not occur if the negation operator is not defined.

The above conditions lead to a tree of possibilities describing the extension of this term set, depending on the number of new elements and where they are placed. Figure 6 shows how a branch of such tree is used to generate an AND operator on a set of nine certainty linguistic terms starting from the classical connective AND in a boolean logic. The total number of possible solutions, following the specified branch, are also presented (Figure 7).

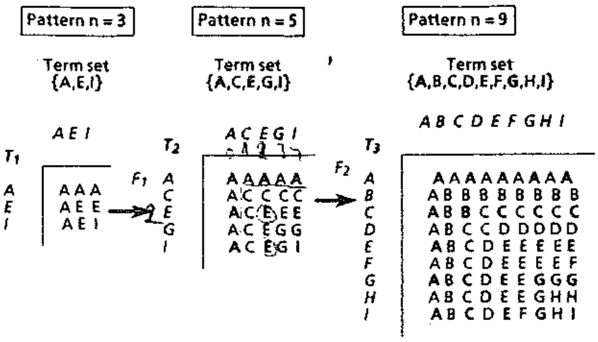


Fig. 6 Matrix generation using patterns.
F1: Add two elements simetrically to the central one.
F2: Add four elements intercalated between the others.
A possible certainty linguistic term set may be:
A:Impossible; B: Almost Impossible; C: Slightly Possible; D: Moderately Possible; E: Possible; F: Quite Possible; G: Very Possible; H: Almost Sure; I: Sure.

n \ α	1	2	3
3	1	1	1
5	2	2	2
9	48	126	132

Fig.7 Number of patterns generated following the mechanism expressed in figure 6. α stands for α-value of property T7 and n for the dimension of the matrix.

4 Application of the new approach in the field of Expert Systems.

An interesting consideration in the design of Expert Systems using aproximate reasoning is the selection of functions used to model the combination of evidence, for example, for example, the functions of "and" and "or" connectives, that best fit the expert's reasonning.
With this new approach the selection consists of the election of one matrix, from amongst the set of matrices that satisfy the more general properties mentioned in sec. 2 and the more concrete properties mentioned in section 3.As we can see in Figures 5 and 7, the final number of matrices can be very important. It is then necessary to establish a help mechanism to select the most suitable matrix to the expert's way of reasoning. These helps can be of two types:

1) Classification of the set of matrices into significatively different classes.

This approach detects differentiated prototypes and presents them to the user so the expert chooses one prototype of this set . The classification algorithm that has been used is described in (Aguilar and Piera, 1987; Desroches, 1987). Observing the result of the classifications it is found that there are some coincidences between the number of detected different classes and the number of matrix generated using trapezoidal representation with a linguistic aproximation process (Bonissone,.1979, 1986).

2) Goal matrix point establishing.

In this approach the expert fixes points of the matrix, (i.e. "quite possible" or "quite possible" will have to be "very possible"). Fixing a set of matrices values, the number of elements is reduced until the best matrix is found.
This second method gives raise to a series of questions: Which is the minimum number of questions to put the user in order to obtain the goal matrix ?; Which questions are the most discriminating ?; Which strategy of interaction with the expert will be used?. The answer to the

α	nº Elem.	nº Class.
1	48	3
2	126	7
3	132	6

$$\begin{matrix} E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 \\ E_1 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 \\ E_1 & E_2 & E_3 & E_3 & E_3 & E_3 & E_3 & E_3 & E_3 \\ E_1 & E_2 & E_3 & E_3 & E_4 & E_4 & E_4 & E_4 & E_4 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_5 & E_5 & E_5 & E_5 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_5 & E_5 & E_6 & E_6 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_5 & E_7 & E_7 & E_7 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_7 & E_8 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_8 & E_9 \end{matrix}$$

Prototype 1

$$\begin{matrix} E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 \\ E_1 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 \\ E_1 & E_2 & E_3 & E_3 & E_3 & E_3 & E_3 & E_3 & E_3 \\ E_1 & E_2 & E_3 & E_3 & E_3 & E_3 & E_4 & E_4 & E_4 \\ E_1 & E_2 & E_3 & E_3 & E_5 & E_5 & E_5 & E_5 & E_5 \\ E_1 & E_2 & E_3 & E_3 & E_5 & E_5 & E_6 & E_6 & E_6 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_7 & E_7 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_7 & E_8 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_8 & E_9 \end{matrix}$$

Prototype 2

$$\begin{matrix} E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 & E_1 \\ E_1 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 & E_2 \\ E_1 & E_2 & E_3 & E_3 & E_3 & E_3 & E_3 & E_3 & E_3 \\ E_1 & E_2 & E_3 & E_3 & E_3 & E_3 & E_3 & E_3 & E_4 \\ E_1 & E_2 & E_3 & E_3 & E_5 & E_5 & E_5 & E_5 & E_5 \\ E_1 & E_2 & E_3 & E_3 & E_5 & E_5 & E_5 & E_6 & E_6 \\ E_1 & E_2 & E_3 & E_3 & E_5 & E_5 & E_7 & E_7 & E_7 \\ E_1 & E_2 & E_3 & E_3 & E_5 & E_6 & E_7 & E_8 & E_8 \\ E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_8 & E_9 \end{matrix}$$

Prototype 3

Fig 8. Table representing the number of classes obtained by classification of the set of matrices considered in figure 7 with n = 9. Prototypes from the classes obtained with α-value of prop. T7 = 1.

first questions requires the construction of discriminant trees (Quinlan,.86) from the results of the classification process explained in point number 1. The last question can be solved using knowledge adquisition tecniques, i.e. using examples from the expert's domain to help him stablish matrix points in a more coherent way.

As a result of the interaction with the expert of PNEUMON-IA the next list of constraints that must satisfy an AND operator, has been obtained:

Moderately Possible AND *Moderately Possible* = *Slightly Possible*

Quite Possible AND *Quite Possible* = *Possible*

Moderately Possible AND *Slightly Possible* = *Almost Impossible*

Slightly Possible AND *Slightly Possible* = *Almost Impossible*

Possible AND *Moderately Possible* = *Slightly Possible*

Very Possible AND *Very Possible* = *Quite Possible*

Very Possible AND *Possible* = *Possible*

Quite Possible AND *Almost Sure* = *Quite Possible*

Very Possible AND *Almost Sure* = *Very Possible*

Possible AND *Almost Sure* = *Possible*

Very Possible AND *Quite Possible* = *Quite Possible*

Even with this large set of fixed points the final number of possible operators satisfying properties from T1 to T7 with an α-value =1 for T6 is quite important.

5 Conclusions and future work.

A new approach to construct operators with linguistic term sets has been developed. This approach does not use any numerical representation describing terms as has been suggested previously. By requiring specified properties of the operators, a set of possible operators of manageable size is obtained.
Different methodologies select the optimal operator from amongst those defined that best expresses the expert's reasoning. The optimal operator is obtained by either classification of the set or by assignment of points.
This approach is superior to the previous methods in that it frees the experts of defining a representation with which he is not familiar. It is also unnecesary to decide which analytic function will model his behaviour (Lukaciewicz, product, etc.).
The set of options from which the expert chooses is broad. The expert can communicate his perception more clearly to the knowledge engineer. with the introduction of examples, the expert is able to progressively define the operator that interest him. It is precisely in this interaction where much work remains to be done to provide answers to different open questions: Which methodology is preferable: classification and introduction of the prototypes to the expert, points fixing, or a combination of both?; Which operator is easier for the expert to determine, the "and" or the "or"?; What should be done if the expert violates properties imposed over the operators and no operator is found that satisfies his needs?...
With this approach it is possible to define new operators such as "modus ponens", "compatibility",and "composition". Each of these new operators has properties that must be fulfilled. The selection of the best operator must use the information provided by the expert for the determination of the operators that model the connectives "and" and "or".

Bibliography.

Aguilar, J. Piera, N.(1987). Connectives of Fuzzy Logic in Multivariable Data Classification. Proc. Int. Workshop on Aprox. Reasoning and Art. Int. Granada

Alsina, C. & Trillas, E. & Valverde, L. (1983). On some logical Connectives for Fuzzy Set Theory, Journal of Mathematical Analysis and Applications, Vol. 93, nº 1, pp. 15-26.

Baldwin, J.F. (1979). A new approach to approximate reasoning using fuzzy logic. Int. J. Fuzzy Sets and Systems, 2, pp: 309-325.

Bonissone, P.P. (1979). The problem of Linguistic Aproximation in System Analysis, Ph.D. dissertatin, EECS Dept., U.C.Berkeley. In University Microfilms International Publications #80-14, 618, Ann Arbor, Michigan.

Bonissone, P.P. & Decker, K.S. (1985). Selecting Uncertainty and Granularity: An Experiment in Trading-off Precision and Complexity, KBS Working Paper, General Electric Corporate Research and Delopment Center, Schenectady, New York.

Desroches, P. (1987). SYCLARE: Système de classification par apprentissage et reconaissance des formes. Manuel d'utilization. Internal Report CEAB/20 Blanes.

Esteva, F. (1981). On the form of negations in posets. Proc. of XI th I.S.M.L., pp:228-231 Oklahoma.

Godo, Ll. Lopez de Mantaras, R. Sierra, C. Verdaguer, A. (1987). Managing Linguistically expressed Uncertainty in MILORD-application to Medical Diagnosis. Proc.Seventh Int. Workshop on Expert Systems and their applications. Avignon.

Kong, A., Barnett, O., Mosteller, F., Youtz, C. (1986) How medical Professionals evaluate Expressions of Probability. The New England Journal of Medicine. Vol 315, No 12 pp: 740-744.

Quinlan, J.R. (1986). Induction of Decision Trees. Machine Learning nº 1. pp. 81-**106**

Zadeh, L. (1983). The role of Fuzzy Logic in the management of Uncertainty in Expert Systems, Int. J. Fuzzy Sets and Systems, **11**, pp: 199-227.