Benefits of Combinatorial Auctions with Transformability Relationships

Andrea Giovannucci[1] and **Jesús Cerquides**[2] and **Juan Antonio Rodríguez-Aguilar**[3]

Abstract. In this paper we explore whether an auctioneer/buyer may benefit from introducing his transformability relationships (some goods can be transformed into others at a transformation cost) into multi-unit combinatorial reverse auctions. Thus, we quantitatively assess the potential savings the auctioneer/buyer may obtain with respect to combinatorial reverse auctions that do not consider tranformability relationships. Furthermore, we empirically identify the market conditions under which it is worth for the auctioneer/buyer to exploit transformability relationships.

1 Introduction

Consider a company devoted to sell manufactured goods. It can either buy raw goods from providers, transform them into some other goods via some manufacturing process, and sell them to customers; or it can buy already-transformed products and resell them to customers. Thus, either the company buys raw goods to transform via an in-house process at a certain cost, or it buys already-transformed goods. Figure 1 graphically represents an example of a company's inner manufacturing process, more formally *Transformability Network Structure* (TNS), fully described in [1]. This graphical description largely borrows from the representation of Place/Transition Nets (PTN), a particular type of Petri Net [2]. Each circle (corresponding to a PTN *place*) represents a good. Horizontal bars connecting goods represent manufacturing operations, likewise *transitions* in a PTN. Manufacturing operations are labeled with a numbered *t*, and shall be referred to as *transformation relationships* (*t-relationships* henceforth). An arc connecting a good to a transformation indicates that the good is an *input* to the transformation, whereas an arc connecting a transformation to a good indicates that the good is an *output* from the transformation. The labels on the arcs connecting *input goods* to transitions, and the labels on the arcs connecting *output goods* to transitions indicate the units required of each *input good* to perform a transformation and the units generated per *output good* respectively. Each transformation has an associated cost every time it is carried out.

Say that a buying agent requires to purchase a certain amount of goods g_3, g_5, g_6, g_7, g_8, g_9, and g_{10}. For this purpose, it may opt for running a combinatorial reverse auction with qualified providers. But before that, a buying agent may realise that he faces a decision problem: shall he buy g_1 and transform it via an in-house process, or buy already-transformed goods, or opt for a *mixed-purchase* solution and buy some already-transformed goods and some to transform

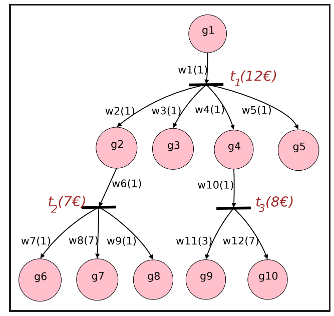


Figure 1. Example of a Transformability Network Structure.

in-house? This concern is reasonable since the cost of g_1 plus transformation costs may eventually be higher than the cost of already-transformed goods.

The work in [1] addresses the possibility of expressing transformability relationships among the different assets to sell/buy on the bid-taker side in a multi-unit combinatorial reverse auction. The new type of combinatorial reverse auction (the Multi-Unit Combinatorial Reverse Auction with Transformability Relationships among Goods (MUCRAtR)) provides to buying agents: (a) a language to express required goods along with the relationships that hold among them; and (b) a winner determination problem (WDP) solver that not only assesses what goods to buy and to whom, but also the transformations to apply to such goods in order to obtain the initially required ones. It is shown that, if the TNS representing the relationships among goods is acyclic, the associated WDP can be modeled by an integer linear program.

The purpose of this paper is twofold. On the one hand, we quantitatively assess the potential savings the auctioneer/buyer may obtain with respect to combinatorial reverse auctions that do not consider tranformability relationships. On the other hand, we empirically identify the market conditions under which it is worth for the auctioneer/buyer to exploit transformability relationships. Thus, we provide rules of thumb for an auctioneer/buyer to help him decide when to run a MUCRAtR instead of an MUCRA.

2 Empirical Evaluation

Our experiments artificially generate different data sets. Each data set shall be composed of: (1) a TNS; (2) a Request for Quotations (RFQ) detailing the number of required units per good; and (3) a set of combinatorial bids. Then, we solve the WDP for each auction problem regarding and disregarding *t-relationships*. This is done to quantitatively assess the potential savings that a buyer/auctioneer may obtain thanks to *t-relationships*, as well as the market conditions where such savings occur. Thus, the WDP for an MUCRA will only con-

[1] IIIA - CSIC, Campus UAB, 08193, Bellaterra, Spain, andrea@iiia.csic.es
[2] WAI, Dept. de Matemàtica Aplicada i Anàlsi, Universitat de Barcelona, cerquide@maia.ub.es
[3] IIIA - CSIC, Campus UAB, 08193, Bellaterra, Spain, jar@iiia.csic.es

sider the last two components of the data set, whereas the WDP for a MUCRAtR will consider them all. In order to solve the WDP for an MUCRA we exploit its equivalence with the multi-dimensional knapsack problem [3].

2.1 Experimental Settings and Results

Our goal is to determine under which market conditions MUCRAtR leads to savings when compared to MUCRA. At this aim, we empirically measure the differences in outcome cost between MUCRA and MUCRAtR. Thus, we define the *Savings Index*(SI) as:$SI = 100 \cdot (1 - \frac{C^{MUCRAtR}}{C^{MUCRA}})$; where C^{MUCRA} and $C^{MUCARtR}$ are the costs associated to the optimal solutions found respectively by MUCRA and MUCRAtR WDAs.

With the aim of assessing the most sensitive parameters with respect to SI, we employ a fractional factorial experiment design [4], assigning to each parameter different values.

We run 5756 instances of the experiments and for each run we sampled SI. In 150 cases (2.606%) the optimizer could not find an optimal solution within the time limit for MUCRA. In 289 cases (5.02%) the solver could not find an optimal solution within the time limit for MUCRAtR. As explained above, the total number of samples that have been considered are $5756 - 150 = 5606$. Among these new samples, the optimizer could not find an optimal solution for MUCRAtR for 191 (3.407%) tests.

We empirically observe that the savings of MUCRAtR with respect to MUCRA go: (1) up to 44%; (2) beyond 3.29% in 50% of the cases; (3) beyond 8.59% in 30% of the cases. Next, we perform a sensitivity analysis in order to determine which parameters most affect an auction's outcome.

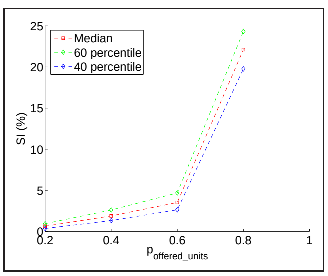


Figure 2. Variation of SI for different providers' capacities.

Sensitivity Analysis.

Figure 2 shows how the SI vary as the providers capacities increase. From it we can conclude that

C1: *The higher the providers' capacities, the higher the expected savings when introducing t-relationships.*

For this reason, when analysing the behavior of SI with respect to the remaining parameters, we will differentiate two cases: (1) $p_{offered\_units} < 0.8$; and (2) $p_{offered\_units} = 0.8$.

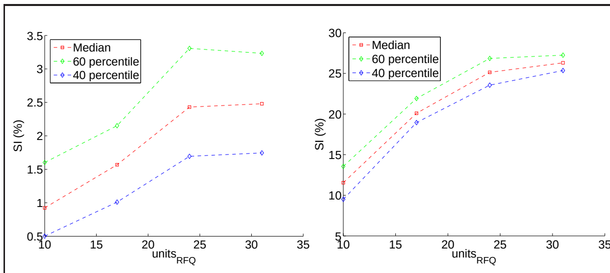
Figure 3 shows the variation of SI with respect to the number of required units. Our second conclusion is thus,

C2: *The finer the granularity of the transformations, the higher the expected savings when introducing t-relationships.*

The third factor significantly affecting SI is the relationship between the transformation costs of a buying agent with the providers' ones. Experimental results confirm that, as expected,

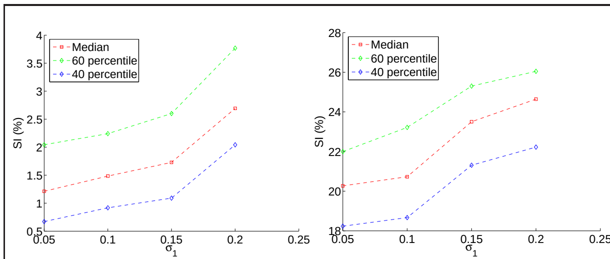
The behaviour of SI when changing the number of transformations within the TNS can be summarized by,

C4: *The more the number of transformations, the more the expected savings with respect to a MUCRA.*



(a) $p_{offered\_units} < 0.8$ (b) $p_{offered\_units} = 0.8$

Figure 3. SI with respect to the number of required units.



(a) $p_{offered\_units} < 0.8$ (b) $p_{offered\_units} = 0.8$

Figure 4. SI with respect to the variance of providers' prices.

Figure 4 shows the effect of varying the prices of goods among providers. The fifth conclusion follows:

C5: *The larger the market's prices spread, the higher the expected savings.*

To summarise, we can indeed confirm, based on the observation above, that there are market conditions (identified by **C1**, **C2**, **C3**, **C4**, and **C5**) wherein it is worth using MUCRAtR instead of MUCRA.

3 Conclusions and Future Work

In this paper we have performed a set of experiments to quantitatively assess the potential savings in employing a MUCRAtR instead of a MUCRA. Furthermore, we have also identified the market conditions for which MUCRAtR is expected to lead to better auction outcomes to the auctioneer/buyer, namely: (1) markets with high-capacity providers; (2) auctions whose number of required units per good is large with respect to the units required by transformations (i.e. the likelihood of exploiting transformations is high); (3) auctions run by a buyer whose transformation (production) costs are cheaper than the providers' ones; and (4) markets where providers' competitiveness is not high (the more scattered the providers' competitiveness, the larger the expected savings).

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