Journal of Global Optimization manuscript No.
(will be inserted by the editor)

A stochastic goal programming model to derive stable cash management policies

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Received: date / Accepted: date

Abstract In this paper, we consider cash management systems with multiple bank accounts described by a given particular relationship between accounts and by a linear state transition law. Since cash managers may simultaneously consider a number of possibly conflicting goals, we provide a general stochastic goal programming model that is able to handle multiple goals and also the inherent uncertainty introduced by expected cash flows. We describe in detail an instance of our general model that considers the optimization of three different criteria such as cost, risk and cash balance stability. We claim that cash balance stability is an interesting goal to deal with the inherent uncertainty of expected cash flows. We also provide useful instructions for cash managers to set the main parameters of our model in practice. Our model provides a systematic approach to multiobjective cash management that is ready to be implemented in decision support systems for cash management.

Keywords Multiple criteria · multiple accounts · stochastic goal programming · uncertainty

1 Introduction

Many papers have proposed analytical models to deal with cash management operations assuming that cash holdings are summarized in a single cash ac-

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count and an alternative investment account [1, 2]. However, cash managers usually deal with multiple banks to receive payments from customers and to send payments to suppliers, employees and other creditors. Managing multiple bank accounts implies a number of transactions between accounts to maintain the system in a state of equilibrium, meaning that there exists enough cash balance to face payments and avoid an overdraft. As a result, the challenge that face cash managers on a daily basis is finding the policy (a set of control actions) that optimizes cost and possibly additional goals.

Mathematically speaking, the cash management problem is a particular case of dynamic programming [3, 4], an optimization technique that aims to break down a multistage problem into simpler subproblems. This optimization method has found applications in almost every scientific field. An example of a similar problem in the field of water resource management is the multireservoir problem [5, 6, 7]. A reservoir is a natural or artificial place where water is collected for use, supplying a community. In this case, a system of connected reservoirs is to be optimized to maximize energy generation and satisfy irrigation demands. An initial storage, inflows and outflows for each reservoir determine the law of motion of water storage over different time periods in a similar way to a system of cash accounts. To solve the multireservoir problem different techniques such as constrained differential dynamic programming [5], control theory [8], or evolutionary algorithms [9] have been proposed. Since we here consider multiple criteria in the selection of cash management policies for a finite planning horizon, we follow a multiobjective optimization approach.

Cash management schemes with multiple accounts have received little attention of the research community with the exception of [10] and [11]. A first extension of the typical two-assets setting in the literature was proposed by [12], who considered a single bank account and two different sources of short-term funds. Later on, [10] proposed a multidimensional impulse control approach to provide optimal control restricted to continuous fluctuations of cash balances given by homogeneous diffusion processes. However, the continuous time framework and the assumption on the cash flow process seem to be far from matching the needs of real-world cash management. First, real-world cash flows are neither completely certain nor completely unpredictable [13, 14]. Second, a discrete time framework is more appropriate due to the fact that common planning and control practices in most organizations are typically performed in discrete intervals [15]. Finally, most optimization models proposed in the literature have focused only on a single objective, namely, on minimizing costs. However, cash managers may also be interested in the risk associated to cash policies due to the uncertainty introduced by cash flow forecasts as proposed by [16] and [11].

In this paper, we extend this body of work by generalizing cash management systems affected by stochastic cash flows. More precisely, we follow the recommendations in [11] to represent cash management systems as: (i) a set of bank accounts; (ii) a set of allowed transactions between accounts; and (iii) a multidimensional cash flow process for each account as a source of uncertainty about the near future. Since we are dealing with the uncertainty about cash flows in the in the near future, we rely on stochastic goal programming (SGP). Indeed, this paper differs from previous approaches in that we provide a generalized SGP model as a way to account for multiple criteria for optimization purposes. A further advantage of the use of SGP is that practitioners are allowed to set soft constraints which can be violated without generating unfeasible solutions [17]. Goal programming (GP) [18, 19, 20, 21, 22] aggregates goals to obtain a solution that minimizes the sum of deviations between achievement and the aspiration levels (or targets) of the goals. The underlying idea behind goal programming is that the decision-maker follows a satisfying logic expressed by means of targets. By establishing an achievement objective function, goal programming aims to conciliate the achievement of a set of goals instead of optimizing every goal.

In order to better deal with the uncertainty introduced by expected cash flows, we propose to consider the combination of three goals: cost, risk and cash balance stability. On the one hand, we measure cost by averaging daily cost as it is customary in the literature. On the other hand, we measure risk using the conditional-cost-at-risk (CCaR) recently proposed in [23] since cash managers are usually more interested in deviations above mean values. Finally, we introduce the concept of cash balance stability as a third desirable goal for short-term financial planning. Indeed, cash balance deviations from a given reference have been recently used in [24] and [11]. By minimizing this deviation, cash managers ensure that policies outputs stable balances. As a result, we here consider cash balance stability as an additional goal for optimization purposes within a generalized SGP approach to cash management.

Assuming that cash flow is a random variable, we need to take into account its probability distribution to make decisions under some degree of randomness. A suitable way to manage cash flow distributions for optimization optimization purposes is chance constrained programming [18, 21, 25]. Thus, we formulate the cash management problem as chance constrained program. More precisely, we follow a data-driven approach in which some constraints of the problem are based on past observations. In addition, we assume that cash managers are able to produce predictions with known accuracy or lying in a given prediction interval. Finally, we reduce the chance constrained program to an equivalent linear deterministic problem, ready to be solved by state-of-the-art solvers such as CPLEX or Gurobi. Besides its computational tractability, this approach allows us to encode the initial stochastic problem as a linear program with soft constraints.

Summarizing, the main purpose of this paper is to provide cash managers with a tool to deal with the inherent uncertainty of future cash flows by means of stable policies. To this end, we rely on SGP as a suitable technique to consider not only cost but also risk and stability measures. As a result, we highlight the following contributions:

1. We provide a generalized stochastic goal programming model as a systematic approach to multiobjective decision-making within the context of cash management systems with multiple bank accounts.

2. We show that stochastic goal programming can deal with the inherent uncertainty of expected cash flows by considering cash balance stability as an interesting third goal.
3. We provide useful instructions for cash managers to set the main parameters of our model.

In addition to extend the existing cash management literature, the previous contributions build primarily on the practical side of decision support systems for cash managers without disregarding the mathematical rigor.

In what follows, we formulate the cash management problem with multiple bank accounts in Section 2. Next, we introduce our generalized stochastic goal programming model for cash management in Section 3. We present an instance of the general model considering both cost and risk in Section 4, which we illustrate by means of a numerical example. In Section 5, we provide recommendations for cash managers to set the main parameters of our model in practice. Finally, we conclude in Section 6, suggesting natural extensions of this work.

2 Formulation of the problem

The purpose of this section is to provide a mathematical formulation of the cash management problem with multiple bank accounts derived from the single bank account formulation described in [11]. As an introductory example, consider a cash management system with two bank accounts and an investment account as the one shown in Figure 1. A hypothetical cash manager receives payments from customers and manages payments to suppliers through bank accounts 1 and 2. Daily net cash flows are summarized in variables $f_{1,t}$ and $f_{2,t}$. Temporary idle cash balances can be invested in short-term marketable securities and bonds through an investment account 3.

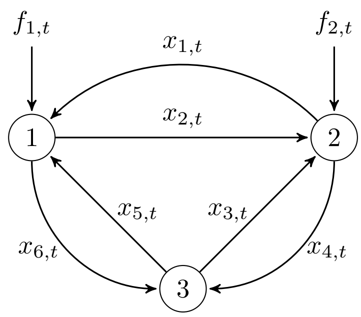

Fig. 1 A cash management system with two bank accounts and an investment account.

Let us consider a cash management system defined as set $\mathcal{M}$ of m accounts and a set $\mathcal{N}$ of n possible transactions such as the one depicted in Figure 1.

The state of the system is represented by an $m \times 1$ vector $\boldsymbol{b}_t$ at time t with cash balance holdings. Let $\{\boldsymbol{f}_t : t = 1, 2, \ldots, \tau\}$ a sequence of m-dimensional cash flow process. At each time step, cash managers aim to obtain policy $\boldsymbol{x}_t$ with n possible transactions between accounts to maintain the system in a desired state. The state transition of a cash management system with n transactions between m different bank accounts taken at time t, is determined by the following system of linear equations:

$$\boldsymbol{b}_t = A \cdot \boldsymbol{b}_{t-1} + B \cdot \boldsymbol{x}_t + C \cdot \boldsymbol{f}_t \tag{1}$$

where A is an $m \times m$ matrix; B is an $m \times n$ incidence matrix with element B_{ij} set to: 1 if transaction j adds cash to account i, -1 if transaction j removes cash from account i, and zero otherwise; and C is another $m \times m$ matrix. In what follows, we set matrix A to the identity matrix by assuming that balances do not change from one period to the following. Matrix C specify which accounts are affected by external cash flow processes. For instance, only accounts 1 and 2 in Figure 1 receive external flows. Then, matrix C is set to:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{2}$$

This setting accepts any form of the process $\{\boldsymbol{f}_t\}$ that outputs real cash flows $\boldsymbol{f}_t$ from a probability density function, an empirical data set, or any other cash flow process. In practice, since decisions are made in advance to real cash flows, both predicted cash flows $\hat{\boldsymbol{f}}_t$ and balances $\hat{\boldsymbol{b}}_t$ are used instead.

In the usual case of linear transaction costs between accounts with a fixed part γ_0, and a variable part γ_1, the transaction cost function $\Gamma(\boldsymbol{x}_t)$ at time t is defined as:

$$\Gamma(\boldsymbol{x}_t) = \boldsymbol{\gamma}_0^T \cdot \boldsymbol{z}_t + \boldsymbol{\gamma}_1^T \cdot \boldsymbol{x}_t \tag{3}$$

where $\boldsymbol{z}_t$ is an $n \times 1$ binary vector with element z_i set to one if the i-th element of $\boldsymbol{x}_t$ is not null, and zero otherwise; $\boldsymbol{\gamma}_0$ is a $n \times 1$ vector of fixed transaction costs for each transaction; and $\boldsymbol{\gamma}_1$ is a $n \times 1$ vector of variable transaction costs. On the other hand, the expected holding cost function at time t is usually expressed as:

$$H(\hat{\boldsymbol{b}}_t) = \boldsymbol{v}^T \cdot \hat{\boldsymbol{b}}_t \tag{4}$$

where $\boldsymbol{v}$ is an $m \times 1$ column vector with the j-th element set to the holding cost per money unit for account j. In this paper, we assume the common situation in practice for many companies in which penalty costs for negative cash balances are much higher than holding costs in vector $\boldsymbol{v}$. This fact is equivalent to restrict feasibility of balances to positive values.

As a result, given a cash planning horizon of τ time steps and an initial cash balance state, the solution to the problem is the $\tau \cdot n \times 1$ policy vector $\boldsymbol{x} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_\tau]^T$, obtained through vertical concatenation, that optimizes some objective function. Within the field of multiple criteria decision-making, objective functions are usually expressed through a set of q goals (see e.g. [17]), according to the particular criteria defined by decision-makers:

$$\text{opt}\, g(\boldsymbol{x}) = [g_1(\boldsymbol{x}), \ldots, g_i(\boldsymbol{x}), \ldots, g_q(\boldsymbol{x})] \tag{5}$$

subject to:

$$\boldsymbol{x} \in S \tag{6}$$

where $g_i(\boldsymbol{x})$ is the mathematical expression of the i-th criterion, and S is the set of all feasible solutions given by equation (1) and non-negativity constraints for balances $\boldsymbol{b}_t$ and controls $\boldsymbol{x}_t$. This formulation is a mathematical representation of the multiobjective cash management problem with multiple bank accounts. Although computationally tractable, this model presents practical limitations since it is unable to deal with uncertainty and feasibility is based on hard constraints. To solve these limitations, we propose a new stochastic goal programming model.

3 A generalized stochastic goal programming model for cash management

In this section, we approach the cash management problem from a multiobjective perspective in which not only cost but also additional goals are important for cash managers. A large number of engineering and economic problems require that decisions are made in the presence of uncertainty. The main approaches to optimization under uncertainty are: stochastic programming, fuzzy programming and stochastic dynamic programming [26]. In stochastic programming [25, 27], uncertainty is modeled through a probability distribution. Fuzzy programming considers random parameters as fuzzy numbers and constraints as fuzzy sets [28]. Finally, stochastic dynamic programming dates back to the seminal work by [3], where decisions are made within a dynamic environment and uncertainty is modeled as a shock or disturbance process.

Since cash management is a multistage decision process, we follow a stochastic dynamic approach. Unlike fuzzy programming, we aim to ensure linearity of objective functions and constraints for computational tractability. By introducing GP as a variant of stochastic dynamic programming, we further provide cash managers with the following advantages: 1) the possibility to consider multiple criteria to account not only for cost but also for risk and other important attributes; 2) the use of soft constraints that can be violated without producing unfeasible but yet realistic solutions; and 3) the inherent satisficing logic of GP in the Simonian sense [29] established by a set of targets defined by cash managers.

In the next subsections, we first outline a general weighted GP model, we next describe a GP model to deal with uncertainty, and we finally introduce a generalized stochastic goal programming model for cash management (SGP-CM) as a natural extension of GP in our context. This model generalizes previous cash management works approaches such as [10, 16, 30] and [11] to the cash management problem to multiple bank accounts, multiple objectives and random cash flow processes through SGP.

3.1 A weighted goal programming model

GP was introduced by [18, 19] and its popularity has been extended up to recent dates [20, 21, 22, 31, 32, 33]. GP aggregates multiple objectives to obtain the solution that minimizes the sum of deviations between achievement and the aspiration levels of the goals. Then, for each goal g_i, it is necessary to specify aspiration level or target $G_i \in \mathbb{R}$, with $i = 1, 2, \ldots, q$. Next, positive and negative deviation auxiliary variables are introduced to connect goal achievement and targets. Then, we express a general weighted goal programming model as follows:

$$\min \sum_{i=1}^{q} (w_i^+ \delta_i^+ + w_i^- \delta_i^-) \tag{7}$$

subject to:

$$g_i(\boldsymbol{x}) + \delta_i^- - \delta_i^+ = G_i \tag{8}$$

$$\delta_i^-, \delta_i^+ \geq 0 \tag{9}$$

$$\boldsymbol{x} \in S \tag{10}$$

where each $g_i(\boldsymbol{x})$ is a particular goal defined by decision-makers that ultimately depends on policy $\boldsymbol{x}$ belonging to set S of feasible solutions. Equation (7) expresses the aim of decision makers to minimize the sum of positive (δ_i^+) and negative (δ_i^-) deviations. These deviations are the required slack variables to reach targets G_i in equation (8). Finally, the particular preferences are incorporated to determine the relative importance of each goal by means of a set of positive (w_i^+) and negative weights (w_i^-).

3.2 Goal programming under uncertainty

To deal with uncertainty, we need a method to consider random variables in the optimization problem. A sound approach to deal with uncertainty is chance constrained programming [18, 21, 25]. Thus, we reformulate program encoded from equation (7) to (10) as a SGP model:

$$\min \sum_{i=1}^{q} (w_i^+ \delta_i^+ + w_i^- \delta_i^-) \tag{11}$$

subject to:

$$g_i(\boldsymbol{x}) + \delta_i^- - \delta_i^+ = G_i \tag{12}$$

$$P(h_k(\boldsymbol{x}) \leq 0) \leq 1 - \zeta \tag{13}$$

$$\delta_i^-, \delta_i^+ \geq 0 \tag{14}$$

$$\boldsymbol{x} \in S \tag{15}$$

$$k = 1, 2, \ldots, K \tag{16}$$

where P in equation (13) denotes probability and $h_k(\boldsymbol{x})$ denotes any type of constraint depending on policy $\boldsymbol{x}$. Parameter ζ is a threshold set by decision-makers that typically takes values of 0.95 or 0.99 establishing a low probability that a given constraint is not satisfied.

3.3 A generalized SGP model for cash management

Next, we adapt the SGP model encoded from equations (11) to (16) to the particular characteristics of the cash management problem with multiple accounts. This generalized stochastic goal programming model for cash management (SGP-CM) includes forecasts as a way to reduce the uncertainty introduced by future cash flows. To this end, we follow the approach initiated by [34] and followed by [35, 36] of transforming a linear program with uncertain data in its robust deterministic counterpart. [34] proposed to use the maximum uncertainty as a way to replace stochastic constraint (13) with a deterministic approximation. This approach is equivalent to set $\zeta = 1$ and presents the drawback of an excessive conservatism. To solve this problem, [35, 36] followed a similar strategy but used parameter ζ as a way to capture the particular risk preferences of decision-makers.

Within the context of the cash management problem, the uncertainty is introduced by future cash flow uncertainty. Ultimately, this uncertainty results in a real cash balance that differs from the expected balance in the cash flow forecasting error. To account for such an uncertainty, we here propose to replace constraint (13) with the following set of linear constraints to capture the particular uncertainty of the cash management problem:

$$\hat{\boldsymbol{b}}_t = A \cdot \hat{\boldsymbol{b}}_{t-1} + B \cdot \boldsymbol{x}_t + C \cdot \hat{\boldsymbol{f}}_t \tag{17}$$

$$\hat{\boldsymbol{b}}_t \geq \zeta \cdot \hat{\boldsymbol{b}}_{min} \tag{18}$$

where $\hat{\boldsymbol{b}}_{min}$ is an $m \times 1$ vector with elements set to minimum cash balances for each of the m accounts in the system. As mentioned above, parameter $\zeta \in [0, 1]$ adjusts the aversion of cash managers to the risk of an overdraft so that the higher the value of ζ, the more averse to risk they are.

However, equation (18) is a hard constraint, namely, one that has to be satisfied whatever the realization of the uncertainty of cash flow process $\{\boldsymbol{f}_t\}$. This fact, may lead to unnecessarily reduce the feasibility space by discarding solutions that result in balances that are close enough to a given reference such as a minimum cash balance set by cash managers. GP provides a suitable approach to overcome this inconvenient since goals can be considered as soft constraints which can be violated without generating unfeasible solutions [17]. Here, we propose to replace equation (18) with the following set of constraints:

$$\hat{b}_{it} + \delta_{it}^{-} - \delta_{it}^{+} = b_{i,ref} \tag{19}$$

for each account $i = 1, 2 \ldots, m$. This transformation implies that, instead of satisfying a hard minimum balance constraint, we aim to minimize the sum of deviations of balances with respect to a predefined target $b_{i,ref}$ as m additional goals. Moreover, the relative importance of the deviations from targets of these soft constraints are set according to the preferences (w_i^+, w_i^-) of cash managers.

Finally, note that equation (19) can be easily casted in the general SGP model described in (11)-(16) to formulate our general SGP-CM as follows:

$$\min \sum_{i=1}^{q} \left[w_i^+ \sum_{t=1}^{\tau} \delta_{it}^+ + w_i^- \sum_{t=1}^{\tau} \delta_{it}^- \right] \tag{20}$$

subject to:

$$g_i(\boldsymbol{x}_t) + \delta_{it}^- - \delta_{it}^+ = G_{it} \tag{21}$$

$$\hat{\boldsymbol{b}}_t = A \cdot \hat{\boldsymbol{b}}_{t-1} + B \cdot \boldsymbol{x}_t + C \cdot \hat{\boldsymbol{f}}_t \tag{22}$$

$$\boldsymbol{x}_t \in S \tag{23}$$

$$G_{it} \in \mathbb{R} \tag{24}$$

$$\delta_{it}^-, \delta_{it}^+ \geq 0. \tag{25}$$

$$t = 1, 2, \ldots, \tau. \tag{26}$$

It is important to highlight that the SGP-CM encoded from equation (20) to equation (26) produces a single solution. Under a multiple criteria decision making context, a solution is called Pareto efficient if no other feasible solution can achieve the same or better performance for all the criteria while being strictly better for at least one criterion. This definition leads to the concept of efficient frontier comprising all Pareto efficient solutions. Although our SGP-CM provides one solution among many efficient solutions, the rest of efficient solutions can be viewed as alternative solutions that could become eligible when the preferences weights of the set of goals considered by cash managers vary due to a change in the economic circumstances. Some examples of these varying circumstances are financial crisis, credit restrictions, or market changes.

We next present an instance of our generalized SGP-CM that aims to minimize three goals: cost, risk and balance variability of policies in cash management systems with multiple accounts. Note that minimizing balance variability is equivalent to maximizing balance stability.

4 An instance of the SGP-CM with three goals

To illustrate the use of the SGP-CM model, we next present an instance of the model that aims to minimize three attributes: cost, risk, and cash balance variability. Later, we solve a numerical example using a linear programming solver.

4.1 Model formulation

Consider a cash management system with m bank accounts and n transactions. A cash manager aims to minimize three attributes: cost (g_1), risk (g_2) and cash balance variability (g_3) for a planning horizon of τ days. First, consider that cost $g_1(\boldsymbol{x}_t)$ is measured by the daily cost computed using equations (3) and (4) as follows:

$$g_1(\boldsymbol{x}_t) = \boldsymbol{\gamma}_0^T \cdot \boldsymbol{z}_t + \boldsymbol{\gamma}_1^T \cdot \boldsymbol{x}_t + \boldsymbol{v}^T \cdot \hat{\boldsymbol{b}}_t. \tag{27}$$

On the other hand, our cash manager uses the CCaR as a measure of risk in finance [23, 37]. The CCaR is defined as the expected loss above a given reference c_0 for a given period of time. Then, to avoid policies with daily costs above some cost reference c_0, our cash manager measures risk $g_2(\boldsymbol{x}_t)$ by means of the following expression:

$$g_2(\boldsymbol{x}_t) = \max(g_1(\boldsymbol{x}_t) - c_0, 0). \tag{28}$$

Finally, our cash manager is interested in minimizing cash balance variability for a subset of accounts within the system with respect to some global daily reference b_{ref}. This reference is established according to the particular context faced by cash managers. If we define a subset of accounts $\mathcal{P} \subseteq \mathcal{M}$ indexed by $\mathcal{I} \subseteq \{1, 2, \ldots, m\}$, we can compute balance variability $g_3(\boldsymbol{x}_t)$ for a given subset of accounts as follows:

$$g_3(\boldsymbol{x}_t) = \sum_{i \in \mathcal{I}} b_{it}. \tag{29}$$

Translated into the goal programming language, our cash manager aims to minimize: 1) the sum of positive deviations of cost (g_1) above zero; 2) the sum of positive deviations of cost (g_1) above a cost reference (c_0) as a measure of risk (g_2); and 3) the sum of both positive and negative deviations of daily balances (g_3) with respect to a reference balance b_{ref}. Then, an instance of the SGP-CM encoded by means of equations (20) to (26) is expressed as follows:

$$\min \left[\frac{w_1}{C_{max}} \sum_{t=1}^{\tau} \delta_{1t}^+ + \frac{w_2}{R_{max}} \sum_{t=1}^{\tau} \delta_{2t}^+ + \frac{w_3}{V_{max}} \sum_{t=1}^{\tau} \sum_{i \in \mathcal{I}} \left(\delta_{3ti}^+ + \delta_{3ti}^- \right) \right] \tag{30}$$

subject to:

$$\hat{\boldsymbol{b}}_t = A \cdot \hat{\boldsymbol{b}}_{t-1} + B \cdot \boldsymbol{x}_t + C \cdot \hat{\boldsymbol{f}}_t \tag{31}$$

$$g_1(\boldsymbol{x}_t) - \delta_{1t}^+ \leq 0 \tag{32}$$

$$g_2(\boldsymbol{x}_t) - \delta_{2t}^+ \leq c_0 \tag{33}$$

$$g_3(\boldsymbol{x}_t) - \sum_{i \in \mathcal{I}} \delta_{3ti}^+ + \sum_{i \in \mathcal{I}} \delta_{3ti}^- = b_{ref} \tag{34}$$

$$\delta_{1t}^+, \delta_{2t}^+, \delta_{3ti}^+, \delta_{3ti}^-, \hat{b}_{ti} \geq 0 \tag{35}$$

$$\boldsymbol{x}_t \in \mathbb{R}^n_{\geq 0} \tag{36}$$

$$\hat{\boldsymbol{b}}_t, \hat{\boldsymbol{f}}_t \in \mathbb{R}^m_{\geq 0} \tag{37}$$

$$w_1 + w_2 + w_3 = 1 \tag{38}$$

$$t = 1, 2, \ldots, \tau \tag{39}$$

where C_{max}, R_{max} and V_{max} are normalization factors to avoid meaningless comparisons. Note also that negative deviations δ^-_{1t} and δ^-_{2t} are set to zero since we are only interested in minimizing positive deviations, i.e., costs and costs above a given reference. This is the reason why equations (32) and (33) are expressed in a simplified form. Moreover, equation (34) is a representation of variability goal as a minimum balance soft constraint, which we think is one of the main advantages of the model. Note also that the previous problem is a mixed integer linear program that can be solved using state-of-the-art solvers such as CPLEX or Gurobi.

4.2 A numerical example

In this section, we solve a numerical example to illustrate our SGP-CM model with three different criteria as described in Section 4. Consider again the cash management system of Figure 1 with two current bank accounts 1 and 2, and an investment account 3. Temporary idle cash balances can be invested in short-term marketable securities and bonds through an investment account 3 with an average return of 3.6% per annum, equivalent to 0.01% per day. This is equivalent to set a holding cost 0.01% per day for both accounts 1 and 2. Transactions are allowed between all three accounts and charged with fixed (γ_0) and variable (γ_1) costs determining the cost structure detailed in Table 1.

Transaction	γ_0 (€)	γ_1 (%)	Account	v (%)
1	50	0	1	0.01
2	50	0	2	0.01
3	100	0.01	3	0
4	50	0.001		
5	100	0.01		
6	50	0.001		

Table 1 Cost structure data for the example.

Assume also that we obtain forecasts for the next five working days with a given accuracy. From this known accuracy, instead of setting a minimum balance hard constraint, we set a balance reference vector of $[6, 6, 0]^T$, where each element is the desired balance for each of the three accounts. In other words, we set a global reference of 12 millions for accounts 1 and 2. Then, in addition to cost and risk, we aim to minimize the total deviation of daily balances for these two accounts with respect to $b_{ref} = 12$.

Given an initial cash balance $\boldsymbol{b}_0 = [5, 8, 12]^T$, for accounts 1, 2 and 3, we look for the best policy for a planning horizon comprising the next five days in terms of cost from equation (27), risk from equation (28), and also balance variability from (29). To this end, we obtain a $\tau \times m$ matrix F of forecasts with time steps in rows, accounts in columns and figures in millions of Euros:

$$F = \begin{bmatrix} 3 & -3 & 0 \\ 1 & -2 & 0 \\ -2 & -3 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 0 \end{bmatrix}. \tag{40}$$

Consider that we have no bias to any of the goals. This fact is equivalent to setting $w_1 = w_2 = w_3 = 0.333$. To allow comparisons with respect to a no-control strategy as a benchmark ($\boldsymbol{x}_t = 0$, for the whole planning horizon), we set normalization factors $C_{max} = 0.7$, $R_{max} = 0.34$ and $V_{max} = 67$, figures in millions of Euros. We obtain these factors from the cost, risk and balance variability that resulted from a no-control strategy when the cash flow is F. Furthermore, cost reference c_0 for the risk measurement is set to the daily holding cost computed with equation (4) for the balance reference vector $[6, 6, 0]^T$. This cost reference is the resulting cost of a no-control strategy. Finally, once an instance of the problem is created introducing all the required input data, we are in a position to derive the best policy by solving (30)-(39), resulting in the balance depicted in Figure 2.

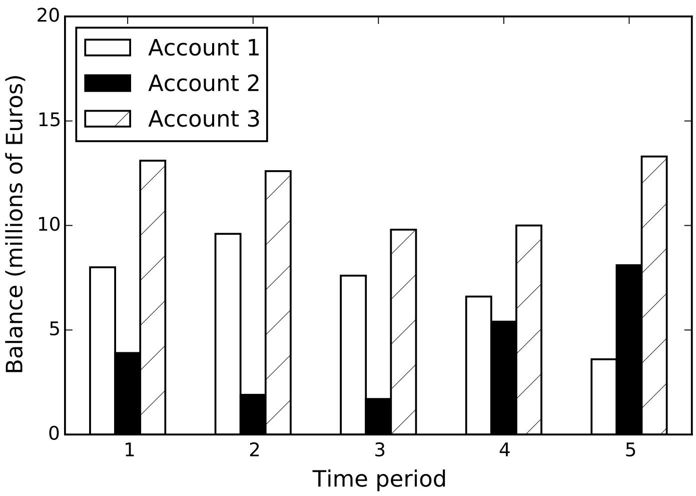

Fig. 2 Optimal balances for an instance of the SGP-CM.

A final comment must be done in the sense that in the case that the either economic context or the quality of the cash flow predictions may change, we can adjust preferences to the new situation by varying weights w_1, w_2 and w_3 and solve the SGP-CM again.

5 Specifying useful SGP-CM model parameters in practice

In Section 4.2, we describe a numerical example for a given cash management system in which the main parameters such as preference weights, normalization factors, cost and balance references are conveniently set for illustrative purposes. In practice, selecting the appropriate values for these parameters can be a more complicated task and it may influence performance. In this section, we experiment on the impact of the SGP-CM model parameters to provide useful guidelines for practitioners.

5.1 Setting preference weights

The notion of preference is a critical issue in multiple criteria decision making. One of the most popular techniques to set preference weights is the Analytic Hierarchy Process proposed by [38, 39]. According to the SGP-CM model described in Section 4, we evaluate cash management policies in terms of the sum of cost, risk and balance variability ratios with respect to normalization factors C_{max}, R_{max} and V_{max}. Since we are dealing with a minimization problem, partial goal achievement can also be viewed as keeping cash management plans at a percentage of maximum cost, risk and variability budgets under the logic the less the better. This approach facilitates the task of cash managers since ratios and percentages are common concepts in business and finance. However, cash managers may have preferences between cost, risk and stability (i.e., the inverse of variability).

Decision-making preferences for the three criteria under consideration are introduced in the SGP-CM model by means of weights w_1, w_2 and w_3. Following the AHP recommendations, we elicit these weights by establishing the pairwise importance of criteria for a hypothetical cash manager. First, let us consider a risky cash manager that is willing accept higher levels of risk in exchange for lower costs. Then, this cash manager states that cost is moderately more important risk, cost is strongly more important than stability, and risk is moderately more important than stability. According to the 1-9 scale described in [39], an example of goal preferences for a risky cash manager is summarized in the first three rows of Table 2, where we approximate weights by adding each row of the matrix and dividing by their total.

Now consider a moderately conservative cash manager stating that risk is moderately more important than cost, stability is also moderately more important than cost, and risk is equally important than stability. An example of goal preferences for a moderately conservative cash manager is summarized in the last three rows of Table 2. The three central rows of the table show the goal preferences for a neutral cash manager. Consistency ratios for all three

Goal	Cost	Risk	Stability	Weights	Case
Cost	1	3	5	$w_1 = 0.61$	Risky
Risk	1/3	1	3	$w_2 = 0.29$	case
Stability	1/5	1/3	1	$w_3 = 0.10$	
Cost	1	1	1	$w_1 = 0.33$	Neutral
Risk	1	1	1	$w_2 = 0.33$	case
Stability	1	1	1	$w_3 = 0.33$	
Cost	1	1/3	1/3	$w_1 = 0.14$	Moderately
Risk	3	1	3	$w_2 = 0.43$	conservative
Stability	3	1	1	$w_3 = 0.43$	case

Table 2 Pairwise comparison of criteria for alternative cash managers.

pairwise comparison matrices computed as described in [39] are below the acceptable threshold of 0.1.

5.2 Cost and balance references

The selection of cost reference c_0 may influence the ability of the CCaR in equation (28) to measure risk. Cost reference c_0 is then a way to penalize policies with costs above a given threshold. Provided there is a data set with past daily costs, we can easily set c_0 to the average daily cost as an intuitive and rational cost reference. Then, $g_2(\boldsymbol{x}_t)$ computes deviations of cost above c_0 as a measure of risk. However, it is important to comment on the implications of setting c_0 to some extreme values. Note that setting $c_0 = 0$ implies that $g_1(\boldsymbol{x}_t)$ and $g_2(\boldsymbol{x}_t)$ are measuring the same. Then, δ_{2t} is no longer a measure of risk. On the other hand, consider setting c_0 to a high enough value such that $g_2(\boldsymbol{x}_t)$ is always below c_0. Then, $\delta_{2t}^{+} = 0$ and the risk of cash management policies are ignored.

The selection of b_{ref} is quite less problematic since both positive and negative deviations with respect to the balance reference are considered in the minimization process. If a low reference is set, then positive deviations will be probably higher than negative ones. Similarly, if a high reference is set, then negative deviations will be probably higher than positive ones. In both cases, however, cash management policies with the lowest total deviation will be preferred ensuring stability.

6 Concluding remarks

Most cash management models in the literature try to solve the cash management problem for both a single bank account and a single objective, namely, cost. In this paper, we propose a generalized stochastic goal programming model to derive stable policies within cash management systems with multiple bank accounts using cash flow forecasts as a key input. Our formulation provides a flexible framework to face cash management problems according to the particular needs of cash managers in terms of: (i) number of bank accounts; (ii) relationship between accounts; (iii) different goals and targets.

In order to deal with the uncertainty introduced by cash flows when managing multiple bank accounts, we provide cash managers with a new SGP-CM model to produce stable policies. More precisely, we derive satisfying policies in terms of three criteria: cost, risk and cash balance stability. From a cash management perspective, the consideration of cash balance stability as a third interesting goal in addition to cost and risk allows to adjust policies to particular preferences and current circumstances. The motivation behind this statement is the possibility to transform hard minimum balance constraints into soft constraints that can be satisfied within a given tolerance set by cash managers in terms of the uncertainty introduced by cash flow uncertainty. In addition, our SGP-CM model allows to handle cash flows predictions within a discrete time framework that we claim to be more appropriate to real-world environments since common planning practices in most organizations are typically performed in discrete intervals.

Our aim to help cash managers in practice motivated us to further elaborate on the settings of the SGP-CM model. The selection of the appropriate values for parameters such as preference weights, normalization factors, cost and balance references can be a more complicated task than it may seem at first glance. As an additional result, we provide guidelines for cash managers to set the main parameters of our model.

Summarizing, our SGP-CM model provides a systematic approach to manage multiple accounts and multiple objectives while considering the uncertainty coped with in practice by cash managers. Within a context of time-varying circumstances, we claim that cash balance stability is an interesting goal to deal with uncertainty. Moreover, the proposed approach is fully applicable to real-world environments in which predictable cash flows are available, at least for a short-term planning horizon. Finally, it is important to highlight that the exploration of alternative (and maybe non-linear) measures of risk and stability, shows that an interesting future research line should consider multiobjective non-linear optimization.

Acknowledgments

Work partially funded by Generalitat de Catalunya (2017 SGR 172), AI4EU (H2020-825619), LOGISTAR (H2020-769142).

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