Categorical Foundations for Fundamental Logic

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Holliday [1] recently introduced a non-classical logic called *Fundamental Logic*, which intends to capture exactly those properties of the connectives $\wedge, \vee$ and $\neg$ that hold in virtue of their introduction and elimination rules in Fitch's natural deduction system for propositional logic. Holliday provides an intuitive semantics for fundamental logic in terms of *fundamental frames* (sets endowed with a relation of openness between its points satisfying some conditions) which generalizes both Goldblatt's semantics for orthologic and Kripke semantics for intuitionistic logic.

The main goal of this talk (based on [3, Chap. 4] and [4]) is to provide some robust categorical foundations for Holliday's semantics for Fundamental Logic. First, we will show how his semantics naturally arises as the discretization of a duality between fundamental lattices (the natural algebraic companions of Fundamental Logic) and a subcategory of the category of Priestley spaces. The main construction, which is of independent technical interest, consists in using Priestley's duality between distributive lattices and Priestley spaces to a obtain a duality between the category of all lattices and a category of binary products of Priestley spaces.

Time permitting, we will also discuss how one can construct natural functors between the category of fundamental lattices and a category of fundamental frames, so as to obtain a version of the Goldblatt-Thomason theorem both for Fundamental Logic and for its modal extension [2].

References

[1] Wesley H. Holliday. A fundamental non-classical logic. *Logics*, 1:36–79, 2023.

[2] Wesley H. Holliday. Modal logic, fundamentally. *Manuscript*, 2024.

[3] Guillaume Massas. *Duality and Infinity*. PhD thesis, University of California, Berkeley, 2024.

[4] Guillaume Massas. Goldblatt-Thomason theorems for fundamental logic. *Manuscript*, 2024.