

Semi-divisible lattices and Modal operators

Chancelle O. Kamga¹, Surdive Atamewoue², Célestin Lele^{1*}, and Lutz Strüingmann³

¹ Department of Mathematics and Computer Science, University of Dschang, Cameroon
kamgachancel@gmail.com and celestinlele@yahoo.com

² Department of Mathematics, University of Yaounde, Cameroon
surdive.atamewoue@univ-yaounde1.cm

³ Mannheim University of Applied Sciences, 68163 Mannheim, Germany
l.struengmann@hs-mannheim.de

Abstract

The class of semi-divisible residuated lattices is a wide class of algebraic structures. In fact, Turunen [6] shows that all divisible residuated lattices (RI-monoids) are semi-divisible residuated lattices but not all semi-divisible residuated lattice are RI-monoids. So, divisible residuated lattices are subclass of semi-divisible residuated lattices. Turunen in [6], while studying states on semi-divisible residuated lattices proved that for some states, study them on semi-divisible residuated lattices or on divisible residuated lattices is the same. This proves that there are notions which, once studied in the divisible residuated lattices setting, there is no longer any point in moving on to the semi-divisible residuated lattices context.

Modal operators were first defined and studied on Heyting algebras in 1981 by Macnab [3]. Since then, many authors have investigated properties of modal operators on other classes of residuated lattices: Harlenderovand Rachunek [1] studied modal operators on MV-algebra, Rachunek and Salounov[4] studied modal operators on RI-monoids, monotone modal operators on bounded integral residuated lattices were studied by Rachunek et al [5] and Kondo [2] studied modal operators on commutative residuated lattices. One of the goals behind the study of modal operators is to build special cases of closure operators which are important for the theoretical study of partial ordered sets. These special cases of closure operators are monotone modal operators. Rachunek and Salounov[4] proved when studying modal operators on RI-monoids, that, all modal operators on divisible residuated lattices are closure operators. It becomes really interesting to know if this is the case in a context of semi-divisible residuated lattices.

We start this paper by showing that not all modal operators on semi-divisible residuated lattices are closure operators. We also investigate some properties of modal operators on a residuated lattice which are not found in literature and state some conditions for a modal operator to be a closure operator on a semi-divisible residuated lattice. Then, we define some operators on a semi-divisible residuated lattice and show that they are strong modal operators. We end the paper by constructing a semi-divisible residuated lattice which is not an RI-monoid from an idempotent element of a semi-divisible residuated lattice and we define a monotone modal operator on it.

References

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*Corresponding author

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