Hereditarily Structurally Complete Extensions of **R**-mingle

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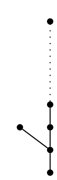
Abstract

The presentation is devoted to structurally complete extensions of the system **R**-mingle. The main theorem states that the set of all hereditarily structurally complete extensions of **RM** is countably infinite and 'almost' forms a chain having only one 'branching' element. As a corollary, we establish that the set of structurally complete **RM**'s extensions which are not hereditary is also countably infinite and forms a chain. We use algebraic methods to provide a full description of both sets. Additionally, we provide a certain characterization of the passive structural completeness among extensions of **RM**. Namely, we prove that a given quasivariety of Sugihara algebras is passively structurally complete iff it does not contain any of the two special algebras. As a corollary, an extra characterization of quasivarieties of Sugihara algebras which are oveflow complete but not structurally complete is given.

Extended Abstract

The presentation will be devoted to structural completeness [11] among consequence relations extending the system **R**-mingle [1]. Results on structural completeness of **RM** has been restricted either to some fragments of $\mathbf{RM}^{\mathbf{t}}$ [9, 10], or just to its axiomatic extensions [8]. We will consider **RM** in its original signature and with respect to its arbitrary (finitary and structural) extensions. Our main theorem states that the set of all hereditarily structurally complete extensions of **RM** is countably infinite and 'almost' forms a chain having only one 'branching' element. Precisely, we will prove that the structure of the poset of all hereditarily structurally complete subquasivarieties of Sugihara algebras is an ω^+ well-ordering with an additional element adjoined above number one:



RM is known to be algebraizable [3] with the quasivariety of Sugihara algebras [5]. Sugihara algebras are locally finite [2] and locally finite quasivarieties are known to be generated by theirs critical members [6]. Thus, our main tool will be critical Sugihara algebras which have been described in in [4]. To prove the main theorem, we will also use the caracterization of the bottom of the lattice of Sughihara subquasivarieties obtained in [7]. On the basis of the main result, we shall establish several corollaries. First, we will show that the set of structurally complete **RM**'s extensions which are not hereditarily structurally complete is also countably infinite and forms a chain. Additionally, we provide a certain characterization of the passive structural completeness [12] among extensions of **RM**. Namely, we prove that a given quasivariety of Sugihara algebras is passively structurally complete iff it does not contain any of the two special algebras. Also, an extra characterization of quasivarieties of Sugihara algebras which are passively structurally complete but not structurally complete is given.

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