MUltseq 2.0
A general purpose finite-valued prover

Angel J. Gil[1], Gernot Salzer[2], and Richard Zach[3]

[1] Universitat Pompeu Fabra. Barcelona, Spain
angel.gil@upf.edu
[2] Technische Universität Wien. Vienna, Austria
gernot.salzer@tuwien.ac.at
[3] University of Calgary. Calgary, Canada
rzach@ucalgary.ca

1 Introduction

The system MUltlog was developed and implemented in Prolog in Vienna and its theoretical foundations can be found in the thesis dissertation of R. Zach ([10] and also [3, 2]). The main motivation behind MUltlog is that from any finite-valued logic first order logic it is always possible to generate a set of rules for m-sequents (similar to Rousseau's approach, [9]) that is sound and complete for this logic and enjoys the cut elimination property.

These systems, restricted to the propositional case, were studied from the algebraic logic point of view in Barcelona (see [6, 8]) and a new set of algorithms called MUltseq was produced and implemented, in such a way that the rules produced by MUltlog were used to generate proofs in the different logical or algebraic systems naturally associated with the original finite-valued logic (see [5, 7]). Later on, due to the high compatibility of the systems, MUltlog and MUltseq were officially "married" ([1]).

In its current state MUltseq is able to produce proofs of formulas (or give counterexamples) and to determine if a consequence relation is valid in an arbitrary finite-valued logic, and also to check if an equation or quasi-equation is valid in an arbitrary finite algebra, but these formulas or equations must be previously introduced to the system by the user.

2 MUltseq 2.0

Recent changes made to MUltlog, now available on GitHub [11], suggested some new and major improvements to MUltseq. We are happy to announce MUltseq 2.0, a new Prolog system that expands the capability of the previous one. We will show how to automatically produce a scientific paper with a comprehensive study of the properties of a given finite-valued logic. The logic is understood as a finite algebra with a set of propositional connectives and a set of designated truth values (and a set of anti-designated values), plus the rules obtained by MUltlog. More precisely, the "paper" will contain:

1. the description of the logic and its connectives, as well as the rules produced by MUltlog;
2. for each connective or subset of connectives, a checklist of the usual properties they may have (commutativity, associativity, idempotency, . . .);
3. a list of valid formulas (tautologies) in the logic;
4. a list of equations (quasi-equations) valid in the algebra and in the variety (quasi-variety) it generates;

5. a checklist of valid entailments in various consequence relations.

The consequence relations that can be considered are preservation of designated truth values and, if the set of truth values is ordered, preservation of degrees of truth. If anti-designated truth values are provided, it can also analyse the corresponding strict/tolerant and tolerant/strict relations [4]. Examples of entailments to check are Modus Ponens and De Morgan laws, and other properties relevant for the algebraic study of the logic.

In the conference we will present the main results that make MUltseq work and a live tool demo will be organised to generate papers for different logics.

It goes without saying that in every case there is a limitation on the length/depth of the formulas and on the number of premises that can be easily adapted. Users will still be able to choose special objects (formulas, inferences, equations . . .) that will appear as designated in the paper, possibly with the corresponding proofs or counterexamples like in the previous version of MUltseq. Finally, the results obtained may be stored in a Prolog database for further investigations and comparison of different logics.

We hope this system will help to simplify calculations and serve as a kind of useful general purpose calculator for finite-valued logics in the propositional case. (See [12] for a recent example of such an application.)

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