

Relational semantics and ordered algebras for monotone propositional logics

Ramón Jansana

I will present part of the general theory of relational semantics for propositional logics developed together with Tommaso Moraschini in the recent past. More specifically, I will expound a relational semantics for monotone logics.

A logic (as a consequence relation) is monotone if every connective is in each coordinate either increasing or decreasing with respect to the pre-order induced in the algebra of formulas by the consequence relation (i.e., the pre-order that declares a formula below another if the second follows from the first.)

The semantics we develop is based on a duality between a class of ordered algebras associated with a monotone logic and a class of general frames for it. The idea we use to turn an ordered algebra into a frame and conversely is inspired by the notion of the relational dual of a function, coming from B. Jónsson and A. Tarski's work on Boolean algebras with operators, as well as from M. Dunn's gaggle theory; it also is inspired by M. Gehrke's work on RS-frames.

The frames we use consist of a polarity—a set of positive states (worlds), a set of negative states (co-worlds), and a relation between them—and for each connective of the language of the logic a suitable relation between worlds and co-worlds in accordance to the logical behavior of the connective.

We will discuss some examples and see that in many well-known cases our relational semantics specializes to the traditional one.