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Review

Ant colony optimization: A bibliometric review

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A R T I C L E I N F O

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A B S T R A C T

This paper is a follow-up of one of the most-cited articles published in the first 20 years of the existence of *Physics of Life Reviews*. The specific topic is "ant colony optimization", which is a metaheuristic for solving challenging optimization problems. Due to its inspiration from natural ant colonies' shortest path-finding behavior, this optimization technique forms part of a larger field known as swarm intelligence. After a short introduction to ant colony optimization, we first provide a chronology focusing on algorithmic developments rather than applications. The main part of the paper deals with a bibliometric study of the ant colony optimization literature. Interesting trends concerning, for example, the geographic origin of publications and the change in research focus over time, can be learned from the presented graphs and numbers.

1. Introduction

Ant colony optimization (ACO) is a nature-inspired metaheuristic for solving hard optimization problems. In particular, ACO was inspired by the ability of natural ant colonies to find short paths between their nest and food sources. In nature, ants use pheromones, which are chemical substances they deposit on the ground, to communicate with one another. As ants move, they leave pheromone trails, and other ants tend to follow these trails, with a higher probability of choosing paths with stronger pheromone concentrations. Over time, shorter paths get reinforced more quickly because ants traveling on shorter paths complete their trips faster, which leads to a faster accumulation of pheromones on shorter paths. This, in turn, leads to more ants following shorter paths. This process leads to the collective discovery of short—even though not necessarily optimal—paths.

The natural inspiration described above was utilized by Marco Dorigo in his doctoral dissertation titled "Optimization, Learning, and Natural Algorithms", completed in 1991 at the Politecnico di Milano, Italy [1]. In particular, Dorigo's thesis introduced the first ACO variant called Ant System (AS), which was published, for example, in [2,3]. AS was the first instance of what would later evolve into the broader ACO framework as described, for example, in [4]. The work on AS is nowadays widely recognized as fundamental to the development of ACO. It formalized the algorithm and demonstrated its application to the Traveling Salesman Problem (TSP), showcasing its potential in solving combinatorial optimization problems.

This paper is a follow-up paper of [5]. However, instead of simply extending the previous paper with the research from the last 20 years, we decided to provide a critical reflection of these 20 years in terms of a bibliometric review. Another reason for this decision was that we noticed a lack of bibliometric information on ACO in the literature. The latest bibliometric review concerning the complete ACO literature can be found in [6]. However, this paper only considers the literature up to 2010. Otherwise, the literature only offers a bibliometric study of ACO publications from the specific field of dynamic optimization [7].

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Table 1
Ant Colony Optimization Chronology.

1991	Doctoral thesis of Dorigo (published in 1992) [1].
1991	The first ACO variant called "Ant System" (AS) is introduced [2].
1995	Development of the first algorithm variant for continuous optimization [8].
1996	Introduction of "Ant Colony System" (ACS) [9].
1997	Hybridization of ACO with local search [10].
1998	Study of parallel implementations of ACO [11].
1999	First use of negative learning in ACO [12].
1999	Proposal of the "Rank Based Ant System" variant [13].
2000	First journal special issue dedicated to ACO [14].
2000	Introduction of the "Max-Min Ant System" (MMAS) [15].
2000	First convergence proof for a simple ACO variant [16].
2001	Study of first multi-objective ACO variants [17].
2001	Proposal of an ACO variant for dynamic optimization [18].
2002	Introduction of an ACO variant for stochastic optimziation [19].
2002	Study of the relation to Stochastic Gradient Descent (SGD) [20].
2002	First presentation of population-based ACO [21].
2004	Dorigo and Stützle publish the first book on ACO [4].
2004	Introduction of the "Hyper-Cube Framework" (HCF) for ACO [22].
2004	Hybridization between ACO and Constraint Programming (CP) [23].
2005	First Combination of ACO with Dynamic Programming (DP) [24].
2005	Introduction of Beam-ACO, a hybrid of ACO and beam search [25].
2007	Implementation of ACO for GPUs [26].
2008	Hybridization of ACO with Lagrangian Relaxation (LR) [27].
2009	First ACO for mixed integer non-linear optimization problems [28].
2010	Use of linear programming relaxation for guiding ACO [29].
2014	Interactive ACO for problems requiring human ratings [30].
2015	Study of ACO variants under a heavily limited computational budget [31].
2017	Assisting feasibility in ACO using integer programming [32].
2021	Learning ACO's heuristic function with Reinforcement Learning [33].
2022	Machine learning for producing heuristic guidance within ACO [34].

The remainder of the paper is organized as follows. Section 2 provides a concise chronology of important algorithmic developments in ACO research. Next, Section 3 presents a bibliometric study of the ACO literature. Finally, Section 4 offers conclusions and an outlook to the future.

2. Chronology of developments concerning ACO research

Table 1 shows the main developments since the beginning of research on ACO. However, note that this is a publication-based chronology focusing on the development of ACO variants, both generally applicable variants and algorithm variants for important families of optimization problems, and the development of ACO hybrids with other optimization techniques.

As mentioned already in the introduction, work on ACO started in 1991 with Marco Dorigo's PhD thesis and the development of AS, the first ACO variant for solving discrete (combinatorial) optimization problems. Even before the development and publication of the second ACO variant for discrete optimization called Ant Colony System (ACS) [9,10] in 1996, some researchers proposed an ACO-inspired algorithm for continuous optimization already in 1995 [8]. However, even though since then there have been other attempts to adapt ACO to continuous optimization—see, for example, [35,36]—this line of research has never really prospered. Instead, an immensely successful trend—the hybridization with other techniques for optimization—started, in the context of ACO, already in 1997 in [10] by combining the ACO variant ACS with a local search technique for improving the solutions generated by the artificial ants. Apart from AS and ACS, other often-used ACO variants are the Rank-Based Ant System [13] (introduced in 1999), the Max-Min Ant System (MMAS) [15] (introduced in 2000), population-based ACO [21] (introduced in 2002), and the Hyper-Cube Framework for Ant Colony Optimization [22] (introduced in 2004). In addition, the line of research dealing with introducing "negative learning" to existing ACO variants also aimed at a general improvement of the algorithm. Apart from the first such attempt in 1999 [12], other papers in this line of research include [37–39].

In addition to the development of generally applicable ACO variants, researchers have also focused on ACO variants tailored to solve important families of optimization problems. This is the case, for example, for multi-objective optimization (first publication in 2001 [17]), for dynamic optimization (first publication in 2001 [18]), for stochastic optimization (first publication in 2002 [19]), for integer non-linear optimization problems (first publication in 2009 [28]), and for problems where solutions require a human rating (first publication in 2014 [30]). After the initial publications for each of the mentioned optimization problem families, much additional work was done, especially in multi-objective optimization. Specific application examples can be found in [40,41], while [42] provides a recent review on this specific topic. Moreover, researchers even investigated the automatic design of multi-objective ACO algorithms [43].

Before researchers considered combining and hybridizing ACO with other techniques for optimization, important research work studied the relation of ACO algorithms to existing (more classical) techniques. The pioneers of ACO were already aware, for example, of its relation to reinforcement learning [44]. Another example concerns a study from 2002 that uncovers the similarities between ACO and Stochastic Gradient Descent [20]. Additionally, ACO was studied from a theoretical point of view. The first proof of convergence (concerning a simple graph-based ACO variant) was published in 2000 [16]. Another theoretical topic of interest was the runtime analysis of (simplified) ACO variants. Examples of work on this topic can be found in [45–47].

Interesting work on ACO hybrids—going beyond simply adding local search or combining the method with another metaheuristic—started with combining ACO with constraint programming in 2004 [23]. Other examples of this hybridization type include [48,49]. Even a book was published on this topic in 2010 [50]. A specific ACO hybrid using dynamic programming in order to find the best solution in a larger object was proposed [24]. Another generally applicable hybrid ACO variant is Beam-ACO, introduced in 2005 [25]. Instead of generating a single solution, in Beam-ACO each artificial ant executes a probabilistic beam search algorithm (incomplete tree search) based on the current pheromone information. For example, other applications and variants of Beam-ACO can be found in [51,52]. Next, a hybrid of ACO with Lagrangian relaxation was introduced in 2008 [27], and another one with linear relaxation in 2010 [29]. In both cases, the information obtained from the relaxations is used to bias the solution construction process of ACO. For example, a similar hybrid can be found in [53]. In a similar way, integer programming was used in 2017 by [32] to guide ACO's solution construction to areas in the search space with feasible and high-quality solutions. In [54], ACO is used to improve the lower bounds within the Lagrangian relaxation heuristic. In recent years, researchers have studied machine learning (ML) models rather than operations research (OR) techniques to guide the construction of solutions in ACO by making use of learned heuristic information. In 2021 [33], reinforcement learning was used for the first time for this purpose. Deep learning methods such as graph neural networks (GNNs) were used for the first time in 2022 [34] for the same purpose. For example, a similar approach can be found in [55].

As was the case for all metaheuristics, research on parallelization strategies has been conducted rather early. In the context of ACO, this happened for the first time in 1999 [11]. Later, research was undertaken in order to be able to execute ACO algorithms beneficially on GPUs, which was done for the first time in 2007 [26]. Another attempt to speed-up ACO was done in [31].

Finally, it is worth mentioning that the first journal special issue dedicated to ACO was published in the journal Future Generation Computer Systems in 2000 [14], and the first book on ACO was published in 2004 [4].

3. Bibliometric study of the ACO literature

The graphics shown and discussed in the following were generated on September 5, 2024, based on data obtained from `Scopus` (www.scopus.com) with the following search term:

```
( TITLE-ABS-KEY("ant colony optimization") OR TITLE-ABS-KEY("ant algorithm") OR
                          TITLE-ABS-KEY("ant colony system") )
```

More specifically, the complete `Scopus` database was searched for documents with at least one of the terms "ant colony optimization", "ant algorithm", or "ant colony system" in the document title, abstract, or keywords. We are aware that this might not capture all possible documents on ACO variants. However, the vast majority of documents are considered in this way. The body of 25,993 documents discovered in this way will henceforth be called the *body of literature on ACO*.

Fig. 1 shows how the body of literature on ACO distributes over different scientific areas, document types, countries of origin, and authors/editors. Not surprisingly, more ACO-related documents were published in Computer Science than in any other scientific area[1]; see Fig. 1a. Computer Science if followed by Engineering and Mathematics. Other scientific areas such as Decision Sciences, Physics and Astronomy, and Materials Science are trailing with considerably fewer ACO-related documents published. Regarding the document type distribution shown in Fig. 1b, it might initially seem surprising that the body of literature on ACO contains more journal articles than conference papers. However, in the author's opinion, this reflects the trend seen in the last 20 years, in which

[1] In this context, note that documents may be attributed to more than one scientific area.

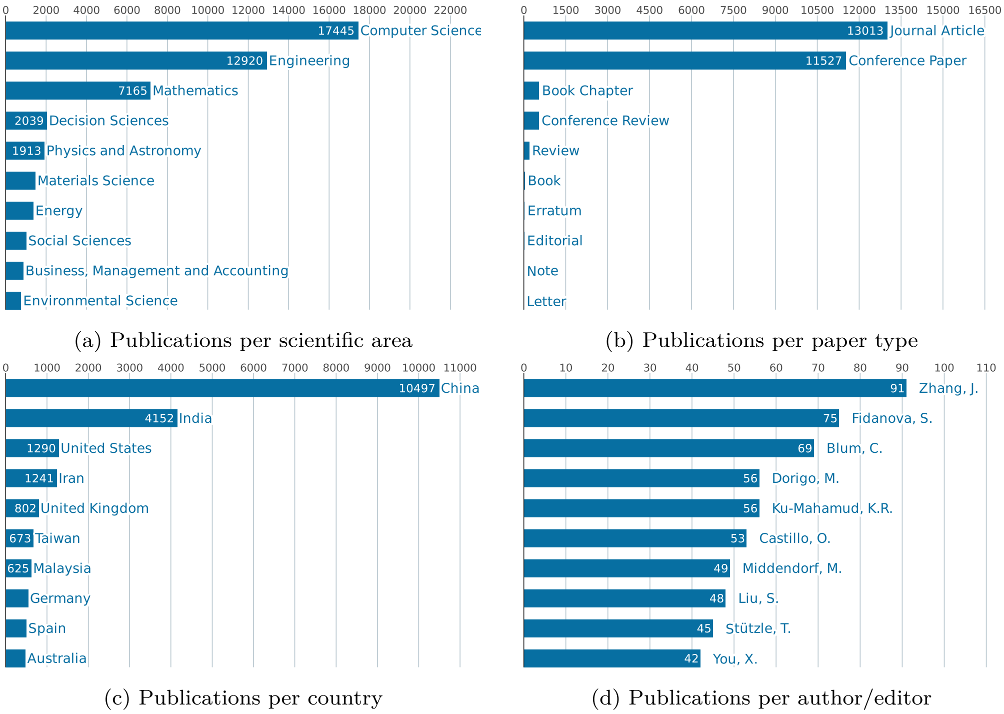


Fig. 1. Bar charts that show how the body of literature on ACO distributes over (a) scientific areas, (b) document types, (c) countries of origin, and (d) authors/editors. Note that only the ten most relevant entries are shown for each category.

funding agencies from numerous countries have attributed much more value to journal articles than to conference papers. In fact, this assumption is confirmed by Figs. 2b and 2c that show, respectively, the evolution over the years of the percentage that journal articles and conference papers occupy concerning the body of literature on ACO. While the percentage of journal articles was growing over the years, the percentage of conference articles was decreasing.

Another interesting aspect concerns the distribution of documents according to countries of origin; see Fig. 1c. Interestingly, authors from China have published more than double the number of documents on ACO compared to authors from any other country. In fact, Fig. 2e indicates that the percentage of ACO publications from China was growing strongly starting in 1999. Currently, nearly 50% of all ACO publications per year are (co-)authored by Chinese authors. This is in contrast to the percentage of ACO publications authored by scientists from Europe and North America (see Fig. 2d), which has been strongly decreasing since 1993, currently reaching a level of approx. 15%. Finally, the ten most prolific authors for what concerns publications on ACO are shown in Fig. 1d.

Next we study the evolution of the ACO publication numbers over the years in comparison to some of the most well-known metaheuristics from the literature:

1. Particle Swarm Optimization (PSO) [56]: PSO is a metaheuristic inspired by the flocking behavior of birds. It maintains a population of solutions throughout the search process. These solutions change at each iteration, depending on themselves and other solutions from their neighborhood. This technique is, together with ACO, among the most well-known nature-inspired metaheuristics.[2] The data for PSO was obtained on September 5, 2024, from `Scopus` with the following search term:

```
TITLE-ABS-KEY("particle swarm optimization")
```

2. Simulated Annealing (SA) [59]: This metaheuristic is arguably the oldest among the ones based on local search, that is, the move to neighbors of the incumbent solution obtained by applying a rather small change to the incumbent solution. Such a neighbor solution is accepted as a new incumbent solution if it is either of better quality or with a probability that decreases with an increasing quality difference with the incumbent solution. The data for SA was obtained on September 5, 2024, from `Scopus` with the following search term:

```
TITLE-ABS-KEY("simulated annealing")
```

[2] Note that this comment does not take into account so-called new bio-inspired metaheuristics such as grey wolf optimizer (GWO) or cuckoo search (CS), as many of them have been shown in the literature to re-invent the wheel rather than adding new algorithmic ideas to the literature on metaheuristics; see, for example, [57,58].

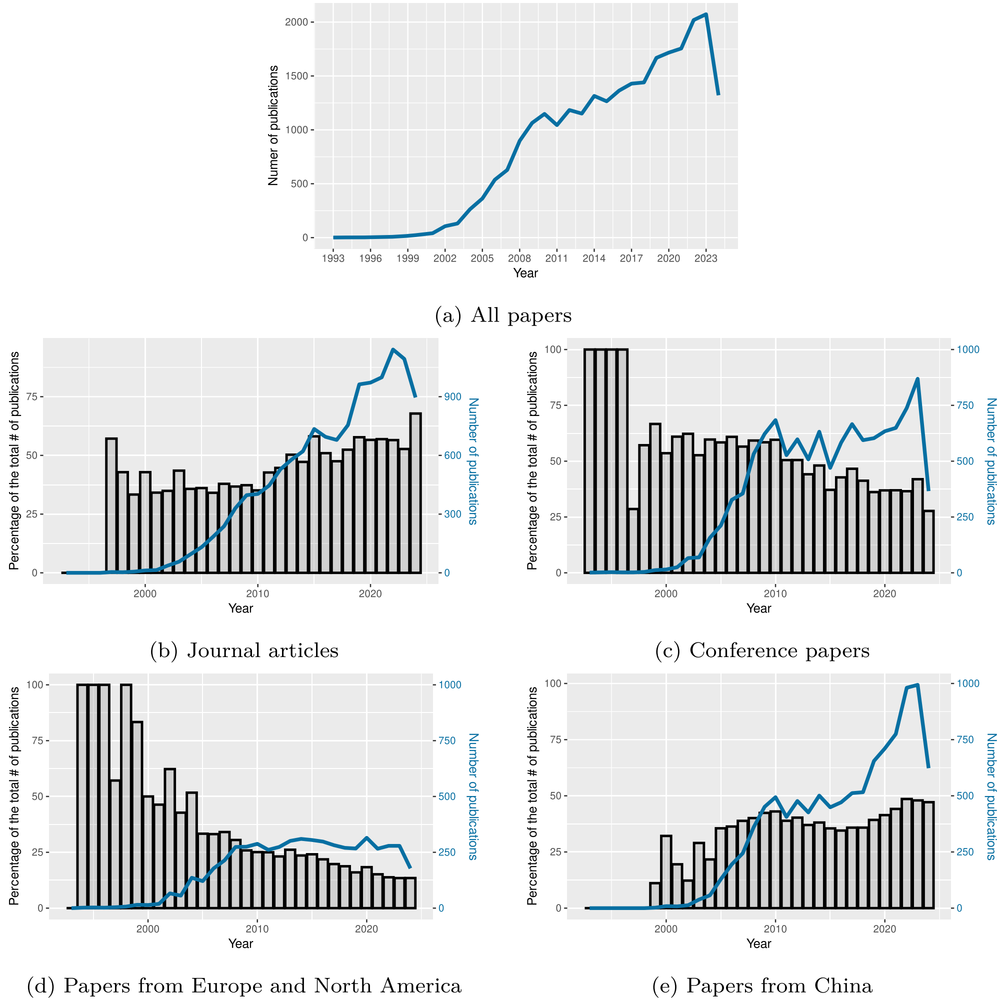


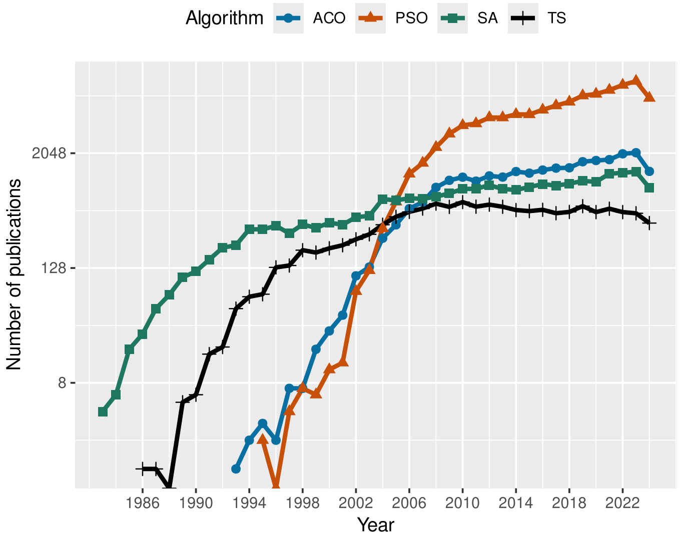
Fig. 2. Plots that show the evolution of (a) ACO publications in general, (b) journal articles, (c) conference papers, (d) papers written by authors from Europe and North America, and (e) papers written by authors from China.

3. Tabu Search (TS) [60]: While SA is able to escape from local optima by means of probabilistic decisions on the acceptance of neighbor solutions worse than the incumbent solution, TS is able to do so by explicitly forbidding to return to recently visited solutions by means of so-called tabu lists that store attributes of recently seen solutions. TS, just like SA, is one of the prominent metaheuristics based on local search. The data for TS was obtained on September 5, 2024, from `Scopus` with the following search term:

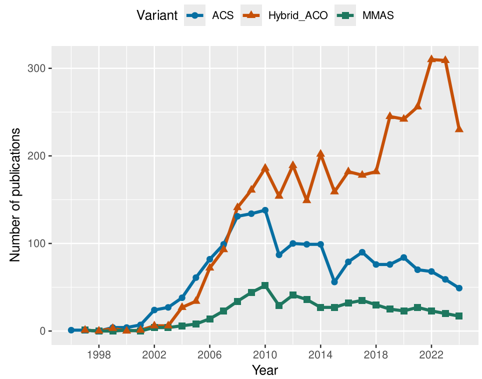
```
( TITLE-ABS-KEY("tabu search") OR TITLE-ABS-KEY("taboo search") )
```

While ACO and PSO were introduced in the early/mid-1990s, SA and TS were already introduced in the 1980s, as indicated by the start of the different lines in Fig. 3a. When ACO and PSO were introduced, both SA and TS already enjoyed more than 100 publications per year. However, the publication numbers of ACO and PSO quickly started to rise. In 2006, PSO—for the first time—surpassed both SA and TS regarding the number of publications. The same happened for ACO in 2008. Both events coincide with a boom around research on metaheuristics that started in the early 2000s, which is indicated in Fig. 3c.[3] Comparing ACO and PSO, it can be noticed that, after the year 2008, the publication numbers of PSO continued a steep increase, while the increase of ACO's publication numbers continued in a more moderate way. This might be attributed to several possible reasons. First, while well-working PSO variants exist for both continuous and discrete optimization problems, ACO is predominantly applied to discrete optimization problems, even though several ACO variants for continuous optimization exist (see, for example, [35,61]). Second, the main inventor

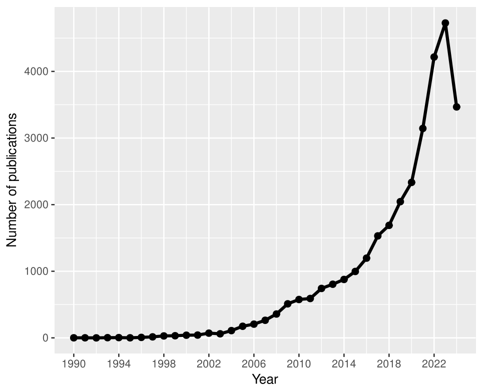
[3] The data for this graphic was obtained on September 5, 2024, from `Scopus` with the search term `TITLE-ABS-KEY("metaheuristic")`. Even though many authors do not mention the term "metaheuristic" in the title/abstract/keywords of their paper when dealing with a specific metaheuristic, this graphic gives a good idea of the sharp rise of research on metaheuristics starting in the early 2000s.



(a) Different metaheuristics



(b) Two most common ACO variants



(c) Mentioning the term metaheuristic

Fig. 3. Evolution of the publication numbers of (a) different metaheuristics (including ACO), (b) the two most common ACO variants (MMAS and ACS), and (c) publications mentioning the term metaheuristic in title/abstract/keywords. **Attention:** the y-axis in (a) is plotted in log-scale.

of the ACO metaheuristic (Marco Dorigo) gradually turned his scientific attention to other research topics (mainly swarm robotics), starting in the early/mid-2000s. This did not happen to the same extent in PSO. Many of the researchers involved in early works on PSO are still heavily involved nowadays. Finally, a third reason might be found in the slightly stronger adoption of PSO in Asia and the South-Pacific region. In terms of numbers, approximately 79% of all publications on PSO originated there, while this is only the case for approximately 71% of the publications in the case of ACO.

Next, we take a look at a noticeable change in research on ACO that has been observed during the last 15-20 years. In the first approximately ten years after the introduction of ACO, one of the important research lines concerned the development of well-working variants of the algorithm. Most of the ACO variants used today have been introduced in these years. The two most important and most-used ACO variants are undoubtedly ACS [10] and MMAS [15]. However, when looking at the evolution of the number of publications that explicitly mention either ACS or MMAS in the title/abstract/keywords—see Fig. 3b—it can be seen that the publication numbers, in both cases, start to decrease after the year 2010. Interestingly, the number of publications on hybrids between ACO algorithms and other techniques for optimization has been growing since then; see the orange line in Fig. 3b. This coincides with a general growth of literature on hybrid metaheuristics since the early 2000s. In other words, work on specific ACO variants has gradually been replaced by work on hybridizing ACO with other optimization techniques, a tendency that can be seen in the same way in the context of other metaheuristics.

Finally, we show how the trend towards open-access publishing is reflected in the ACO literature; see Fig. 4. Disregarding the first few years with some open-access publications (due to a low statistical significance) the graphic provided in this figure shows that the percentage of open-access publications has been steadily growing from below 10% to currently above 30%. It is worth mentioning, however, that—in the last six years—this number has rather remained constant. This leaves room for speculation. A possible explanation could be that all researchers/institutions that can afford open-access publishing are already making use of it,

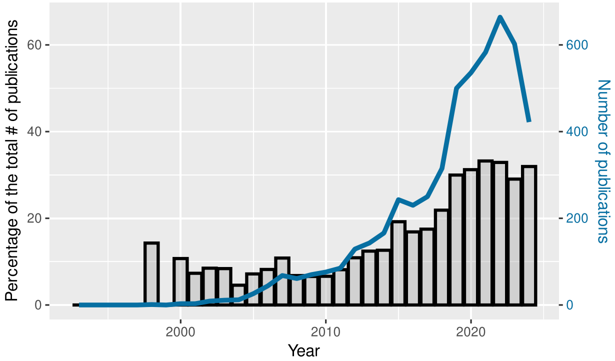


Fig. 4. Number and percentage of ACO publications published under an open access scheme.

while those researchers/institutions that have not yet adopted open-access publishing will not be doing so in the near future due to economic reasons. In any case, the author of this paper argues that a new publishing model is necessary to free academic publishing from questionable economic interests.

4. Conclusion

In this paper we first provided a chronology of algorithmic developments in ACO research. Clearly, the focus in the early years was on the development of different ACO variants, either generally applicable or focused on a specific family of optimization problems. Moreover, researchers tried to gain a better understanding of ACO by studying its relationship to other (more classical) techniques for optimization. In the last 20 years, the trend to hybridize different optimization algorithms is clearly seen in the chronology and the subsequent bibliometric study. Moreover, the bibliometric study shows an interesting geographic shift of the research on ACO from Europe and North America in the early years to China and India in the later years. Moreover, the bibliometric data also indicates a clear shift from publishing in conference proceedings to publishing in journal papers. Even though there might be several reasons for this, a significant one is surely the importance given to journal publications by funding agencies in different countries. Other aspects seen in the bibliometric data are the growing share of open-source publishing and the popularity of nature-inspired optimization techniques in recent years.

Finally, there is room for some speculation about future directions in ACO research and, more generally, in research on metaheuristics (in the sense of approximate, iterative, stochastic optimization algorithms). As indicated already by the trend to hybridize different techniques, there is probably no point in continuing to do research on different metaheuristics separately from each other. In the opinion of this author, as a research community, we should come up with a catalog of different algorithmic components both from metaheuristics and from exact techniques, and we should work towards a (automized) methodology to combine these components in a convenient way when faced with a new optimization problem to be solved. There is already recent research in this line (see, for example, [62]), and the author believes that even more ambitious works of similar nature will appear during the next years.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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