Author Proof

Bargaining Through Amalgamation

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Abstract. This paper presents a bargaining framework grounded in the methods of conceptual blending and amalgamation to support automated negotiation. The bargaining scenario is represented using a content-rich and descriptive formal language, featuring a subsumption hierarchy among negotiation terms. Bargainers generate proposals or counter-proposals within the amalgam of their initial offers, incorporating unifications of all possible generalizations of these offers. Under this framework, we prove that any bargaining sequence converges to an agreement when each bargainer makes minimal concessions according to their individual preferences. The resulting agreement satisfies individual rationality, collective rationality, Pareto optimality, and contract independence. Furthermore, we show that this solution is uniquely characterized by these properties.

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Keywords: automated negotiation · bargaining · amalgamation · feature logic

1 Introduction

Bargaining is a typical form of human interaction. John Nash defines bargaining as "*two individuals who have the opportunity to collaborate for mutual benefit in more than one way*" [21]. "*Under such a definition, nearly all human interactions can be seen as bargaining of one form or another.*" [3]. Computer scientists further extend the definition of bargaining as the interactions among autonomous agents, including computer systems [12,25,29]. As such, bargaining, has been one of the most fundamental AI research topics, known as *automated negotiation*, in the realm of multi-agent systems, with numerous applications across various domains [1,9,13,14,17,22,24,25].

Traditionally, a bargaining situation is abstracted as a numerical game. The game-theoretic model of bargaining provides quantitative methods to facilitate bargaining analysis [21,26]. This highly abstract model allows us to directly apply the approach to computing-related applications [20]. Most existing work on automated negotiation is built upon game-theoretic models due to their well-established framework and simplicity in implementation. However, numerical

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models of bargaining have limited expressive power. Real-world bargaining or negotiations involve much more complex interactions, making purely numerical models difficult to apply [25,27,29]. The primary challenge in developing and applying automated negotiation technology lies in effectively describing negotiation environments and domains, and designing appropriate negotiation mechanisms for autonomous systems across various applications [2,13,15,17,19,33]. To illustrate this challenge, let's consider the following real-life scenario.

Assume that there are two agents, *Geek* and *Artist*, who share a room and must agree on its configuration, *i.e.*, assigning favorite furniture to each available position. Figure 1 shows the initial room design proposals from each agent. Specifically, the agents must negotiate which large piece of furniture to place against the North, South, East, and West walls, and which small piece of furniture to place in the NW, NE, and SE corners (the SW corner is occupied by a door). The agreement must specify both the type of furniture and its location. The only consensus in their initial proposals is to place the sofa against the north wall.

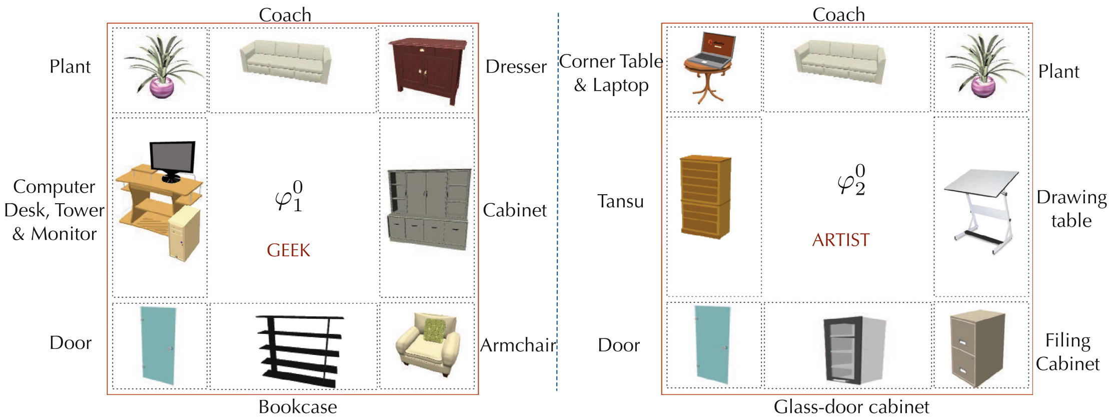

Fig. 1. A room design bargaining scenario: Geek vs Artist.

These types of negotiation scenarios are common in everyday life. Applying the game-theoretic model of bargaining to such situations requires enumerating all possible options (in this case, all potential furniture assignments) and determining the utility for each agent for every alternative. While this is feasible, it is not realistic and does not align with how humans typically negotiate.

The real challenge for the implementation of automated negotiation with real-world scenarios is how to describe the complicated bargaining domains and then facilitate the negotiation between autonomous agents based on the domain-specific criteria, rather than universally quantifying players' preferences. Several approaches have been proposed for describing negotiation domains, using verity of formal languages, such as propositional/first-order logic, game description language, description logic and other specialised formal languages [11,16,32]. However, these logic-based languages often lead to highly complex domain descriptions and sophisticated mechanisms, resulting in substantial computational costs for automated negotiation, which does not seem necessary. This paper aims to introduce an abstract language to describe bargaining domains. We borrow the notion of amalgam from the case-base reasoning literature to express the blending process [4,7,23]. Basically, an amalgam of two terms is a unification of generalisations of these two terms [10]. In such a way, an amalgam merges the information of the two terms but, due to the different degree of generalisation from different terms, may take more information from one than the other. By restricting players' next proposals to the amalgams of their initial proposals, we can then simplify the representation of individual preferences from numerical to ordinal, which means that a player only has to compare alternatives rather than quantify the alternatives. Based on the new preference representation, we propose in this paper a bargaining model and develop a solution concept with characterisation.

While we begin by describing a bargaining situation using order-sorted feature (OSF) logic, our bargaining model is not dependent on OSF logic. Any formal language with the structure of a bounded distributive lattice can be used to describe a bargaining situation within our framework. We specify the language requirements in Sect. 2 meanwhile introduce the concept of amalgam and develop a theorem that is fundamental the approach of this paper. Section 3 introduces our bargain model. We then develop a solution concept for our bargaining model and discuss the desired properties of a bargaining solution. The following section investigates what kind of bargaining procedures can lead to a solution that satisfied all desired properties and how to construct such a bargaining procedure. Finally we conclude the paper with a discussion of future work.

2 Preliminaries

In this section, we introduce the concept of amalgams and basic assumptions for the language we use to describe bargaining domains.

2.1 Feature Terms

For automated negotiation, a bargaining situation needs to be described in a formal language[1]. We assume that there is a set of atom terms described in a meta language, such as the terms in first-order logic, description logic or feature term logic. For simplicity, we use the concept of terms from the order-sorted feature logic (OSF), a mini segment of first-order logic, capable of describing any objects in a formal object-oriented languages [10].

An OSF term can be defined by its signature: $\Sigma = \langle \mathcal{S}, \mathcal{F}, \leq, \mathcal{V} \rangle$, where $\mathcal{S}$ is a set of sort symbols, which includes $\perp$ (also called "any", representing the most general sort), and $\top$ (also called "none", representing the most specific sort). $\leq$ is a complete partial order among the sorts in $\mathcal{S}$ (representing the *is-a* relation

[1] Although automated negotiation can potentially support natural language through natural language processing technology, bargaining terms still need to be formally represented internally.

common to object oriented languages) such that $\bot \leq s \leq \top$ for any $s \in \mathcal{S}$. $\mathcal{F}$ is a set of feature symbols and $\mathcal{V}$ is a set of variable names. An OSF term, or called a feature term, φ is an expression in the form

$$X : s[f_1 \doteq \varphi_1, ..., f_n \doteq \varphi_n]$$

which can be dissolved into the following OSF clause:

$$\varphi ::= X : s \ \& \ X.f_1 \doteq X_1 \ \& \ ... \ \& \ X.f_n \doteq X_n \tag{1}$$

where X is a variable in $\mathcal{V}$, representing the root variable of the term φ and each value X_i is the root of another feature term. s is a sort in $\mathcal{S}$, $f_1, \cdots, f_n$ are features in $\mathcal{F}$ (see [10] for more formal definitions). All feature terms with respect to an OSF signature Σ are denoted as the set $\mathcal{T}$.

In the remainder of this paper we will use the clausal form as shown in (1), where $vars(\varphi)$ represents the set of variables in a given feature term φ, $features(X)$ to denote the set of features defined for a variable X, and $sort(X)$ to denote the sort defined for a variable X.

Example 1. Consider the room design problem we mentioned in the previous section. The scenario shown in Fig. 1 can be described in feature terms as follows:

– Geek's design, denoted by φ_1^0:
  $(X_1 : Room) \wedge$
  $(X_1.NE \doteq X_2 : Dresser) \wedge (X_1.NW \doteq X_3 : PlantPot) \wedge$
  $(X_1.SE \doteq X_4 : Armchair) \wedge (X_1.E \doteq X_5 : Cabinet) \wedge$
  $(X_1.W \doteq X_6 : ComputerDesk) \wedge (X_1.N \doteq X_7 : Coach) \wedge$
  $(X_1.S \doteq X_8 : Bookcase) \wedge (X_6.supports \doteq X_9 : TowerComputer)$
– Artist's design, denoted by φ_2^0:
  $(X_1 : Room) \wedge$
  $(X_1.NE \doteq X_2 : PlantPot) \wedge (X_1.NW \doteq X_3 : CornerDesk) \wedge$
  $(X_1.SE \doteq X_4 : FilingCabinet) \wedge (X_1.E \doteq X_5 : DrawingTable) \wedge$
  $(X_1.W \doteq X_6 : Tansu) \wedge (X_1.N \doteq X_7 : Coach) \wedge$
  $(X_1.S \doteq X_8 : GlassDoorCabinet) \wedge (X_3.supports \doteq X_{10} : Netbook)$
  where N, S, W, and E denote north, south, west and east, respectively. Accordingly, for instance, NE meaning north east.

Given an OSF signature, $\Sigma = \langle \mathcal{S}, \mathcal{F}, \leq, \mathcal{V} \rangle$, the ordering of sorts induces a new ordering over all terms $\mathcal{T}$, called *subsumption*, denoted by $\sqsubseteq$.

Definition 1. *A feature term φ_1 subsumes another one φ_2, denoted by $\varphi_1 \sqsubseteq \varphi_2$, if there is a mapping m: $vars(\phi_1) \rightarrow vars(\varphi_2)$ such that $root(\varphi_2) = m(root(\varphi_1))$ and for all $X \in vars(\varphi_1)$,*

1. *$sort(X) \leq sort(m(X))$ and*
2. *$features(X) \subseteq features(m(X))$*

We write $\varphi_1 \sqsubset \varphi_2$ to mean $\varphi_1 \sqsubseteq \varphi_2$ but $\varphi_2 \not\sqsubseteq \varphi_1$.

Given the subsumption relation, for any two terms φ_1 and φ_2, we can define their *anti-unification* $(\varphi_1 \sqcap \varphi_2)$ as their *least general generalisation*:

Definition 2 (Anti-unification). *The* anti-unification *of two terms* φ_1 *and* φ_2*, noted as* $\varphi_1 \sqcap \varphi_2$*, is the most specific term that subsumes both, i.e.,* $\varphi_1 \sqcap \varphi_2 = \varphi$ *such that:*

1. $\varphi \sqsubseteq \varphi_1$ *and* $\varphi \sqsubseteq \varphi_2$*;*
2. *there is no* φ' *such that* $\varphi' \sqsupset \varphi$*,* $\varphi' \sqsubseteq \varphi_1$ *and* $\varphi' \sqsubseteq \varphi_2$*.*

According to the definition, the anti-unification of the two terms in Example 1 is:

$\varphi_1^0 \sqcap \varphi_2^0 =$

$(X_1 : Room) \wedge (X_1.NE \doteq X_{11} : SF) \wedge (X_1.NW \doteq X_{11} : SF) \wedge$
$(X_1.SE \doteq X_{12} : Furniture) \wedge (X_1.E \doteq X_{13} : LF) \wedge$
$(X_1.W \doteq X_{13} : LF) \wedge (X_1.N \doteq X_7 : Coach) \wedge (X_1.S \doteq X_{13} : LF)$

where LF denotes "*Large Furniture*" and SF for "*Small Furniture*".

It is easy to see that the anti-unification owns all the features that are in common with the two terms. In general, anti-unification give the most specific term that contains all features shared by two given terms. In case of two terms have nothing in common, then $\varphi_1 \sqcap \varphi_2 = \bot$.

A complementary operation to the anti-unification is that of *unification*, or *most general specialisation*:

Definition 3 (Unification). *The* unification *(or most general unifier MGU) of two terms* φ_1 *and* φ_2*, denoted as* $\varphi_1 \sqcup \varphi_2$*, is the most general term that is subsumed by both, i.e.,* $\varphi_1 \sqcup \varphi_2 = \varphi$ *such that:*

1. $\varphi_1 \sqsubseteq \varphi$ *and* $\varphi_2 \sqsubseteq \varphi$*;*
2. *there is no* φ' *such that* $\varphi' \sqsubset \varphi$*,* $\varphi_1 \sqsubseteq \varphi'$ *and* $\varphi_2 \sqsubseteq \varphi'$*.*

When two terms have contradictory information then they have no unifier - which is equivalent to say that their unifier is "none": $\varphi_1 \sqcup \varphi_2 = \top$. Obviously it is the case for the two terms in Example 1.

Definition 4. *Let* φ_1 *and* φ_2 *be two terms in* $\mathcal{T}$*.* φ_1 *and* φ_2 *are inconsistent if* $\varphi_1 \sqcup \varphi_2 = \top$*; otherwise, they are consistent.*

2.2 Amalgams

As we mentioned in the introduction, we need some controlled way to "merge" or "blend" bargainers' proposals We call such a merging process *amalgamation*. The notion of amalgams was introduced in the context of Case-based Reasoning (CBR) [4,7,23], where new problems are solved based on previously solved problems (or cases, residing on a case base). Solving a new problem often requires more than one case from the case base, so their content has to be combined in some way to solve the new problem. The notion of amalgam of two cases (two descriptions of problems and their solutions) is a proposal to formalize the ways in which cases can be combined to produce a new, coherent case.

Definition 5 (Amalgam). *Given two terms ψ_a and ψ_b in $\mathcal{T}$ with anti-unification $\psi_\sqcap = \psi_a \sqcap \psi_b$, a term, φ, of $\mathcal{T}$ is an* amalgams *of ψ_a and ψ_b if there are φ_a and φ_b in $\mathcal{T}$ such that $\psi_\sqcap \sqsubseteq \varphi_a \sqsubseteq \psi_a$, $\psi_\sqcap \sqsubseteq \varphi_b \sqsubseteq \psi_b$, and $\varphi = \varphi_a \sqcup \varphi_b$. Here, φ_a and φ_b are called the transfers of the amalgam.*

Conventionally, the set of all amalgams of φ_1 and φ_2 is called the *space of amalgams* of φ_1 and φ_2, and will hereby be noted as $\mathbb{A}(\varphi_1, \varphi_2)$. Note that we exclude $\top$ from the set of amalgams of any pair of terms because $\top$ contains no information from any player.

Back to our room design example, it is easy to verify that both the following terms are amalgams of φ_1 and φ_2:
$\varphi'_1 =$
$(X_1 : \mathit{Room}) \wedge (X_1.NE \doteq X_2 : \mathit{Dresser}) \wedge$
$(X_1.NW \doteq X_3 : \mathit{PlantPot}) \wedge (X_1.SE \doteq X_4 : \mathit{Armchair}) \wedge$
$(X_1.E \doteq X_5 : \mathit{Table}) \wedge (X_1.W \doteq X_6 : \mathit{ComputerDesk}) \wedge$
$(X_1.N \doteq X_7 : \mathit{Coach}) \wedge (X_1.S \doteq X_8 : \mathit{Bookcase} \wedge (X_6.\mathit{supports} \doteq X_9 : \mathit{TowerComputer})$

$\varphi'_2 =$
$(X_1 : \mathit{Room}) \wedge (X_1.NE \doteq X_2 : \mathit{PlantPot}) \wedge (X_1.NW \doteq X_3 : \mathit{CornerDesk}) \wedge$
$(X_1.SE \doteq X_4 : \mathit{FilingCabinet}) \wedge (X_1.E \doteq X_5 : \mathit{DrawingTable}) \wedge$
$(X_1.W \doteq X_6 : \mathit{Tansu}) \wedge (X_1.N \doteq X_7 : \mathit{Coach}) \wedge (X_1.S \doteq X_8 : \mathit{Bookcase} \wedge$
$(X_3.\mathit{supports} \doteq X_{10} : \mathit{Netbook})$

By observing these two terms we can see that φ'_1 is the same as φ_1 but "*leans to the other side*" by agreeing on placing any table along the east wall. φ'_2, on the other side, agrees on *putting a bookcase along the south wall* but not on other features.

Based on the definitions, it is easy to see that the binary operations $\sqcap$ and $\sqcup$ comply with the commutative, associative and absorption laws. Moreover the following properties hold for all terms in $\mathcal{T}$:

1. $\psi \sqcap \top = \psi$ and $\psi \sqcup \bot = \psi$ (bounded lattice)
2. $\psi \sqsubseteq \psi_1 \sqcap \psi_2$ iff $\psi \sqsubseteq \psi_1$ and $\psi \sqsubseteq \psi_2$
3. $\psi_1 \sqcup \psi_2 \sqsubseteq \psi$ iff $\psi_1 \sqsubseteq \psi$ and $\psi_2 \sqsubseteq \psi$

Therefore, for any OSF signature $\Sigma = \langle \mathcal{S}, \mathcal{F}, \leq, \mathcal{V} \rangle$, the set of terms $\mathcal{T}$ of Σ forms a structure $\langle \mathcal{T}, \sqsubseteq, \sqcap, \sqcup, \bot, \top \rangle$, which satisfies the following conditions for a *bounded distributive lattice* (BDL) [5]:

1. $\bot \sqsubseteq \varphi \sqsubseteq \top$ for any $\varphi \in \mathcal{T}$;
2. $\psi_1 \sqcap \psi_2$ is the maximum of all lower bounds (in $\sqsubseteq$) of $\{\psi_1, \psi_2\}$ for every pair of terms $\psi_1, \psi_2 \in \mathcal{T}$;
3. $\psi_1 \sqcup \psi_2$ is the minimum of all upper bounds (in $\sqsubseteq$) of $\{\psi_1, \psi_2\}$ for every pair of terms $\psi_1, \psi_2 \in \mathcal{T}$;
4. $\psi_1 \sqcup (\psi_2 \sqcap \psi_3) = (\psi_1 \sqcup \psi_2) \sqcap (\psi_1 \sqcup \psi_3)$ (distributivity of $\sqcup$ over $\sqcap$);
5. $\psi_1 \sqcap (\psi_2 \sqcup \psi_3) = (\psi_1 \sqcap \psi_2) \sqcup (\psi_1 \sqcap \psi_3)$ (distributivity of $\sqcap$ over $\sqcup$)).

In the remaining of the paper we will use $\mathcal{T}$ to refer a language $\langle \mathcal{T}, \sqsubseteq, \sqcap, \sqcup, \bot, \top \rangle$ with the BDL structure.

Lemma 1. *For any term $\varphi \in \mathcal{T}$, $\varphi \in \mathbb{A}(\varphi_1, \varphi_2)$ if and only if $\varphi \neq \top$ and $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi \sqsubseteq \varphi_1 \sqcup \varphi_2$. Therefore, $\mathbb{A}(\varphi_1, \varphi_2)$ is the interval $[\varphi_1 \sqcap \varphi_2, \varphi_1 \sqcup \varphi_2] \setminus \{\top\}$.*

Proof. "$\Leftarrow$" Given an amalgam $\varphi \in \mathbb{A}(\varphi_1, \varphi_2)$, there are ψ_1 and ψ_2 in $\mathcal{L}$ such that $\varphi = \psi_1 \sqcup \psi_2$ with $\varphi_1 \sqcap \varphi_2 \sqsubseteq \psi_1 \sqsubseteq \varphi_1$ and $\varphi_1 \sqcap \varphi_2 \sqsubseteq \psi_2 \sqsubseteq \varphi_2$. From the last two facts, we have $(\varphi_1 \sqcap \varphi_2) \sqcup (\varphi_1 \sqcap \varphi_2) \sqsubseteq \psi_1 \sqcup \psi_2 \sqsubseteq \varphi_1 \sqcup \varphi_2$; therefore $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi \sqsubseteq \varphi_1 \sqcup \varphi_2$.

"$\Rightarrow$" If $\varphi \in [\varphi_1 \sqcap \varphi_2, \varphi_1 \sqcup \varphi_2] \setminus \{\top\}$ then $\varphi \sqsubseteq \varphi_1 \sqcup \varphi_2$. Now, $(\varphi_1 \sqcup \varphi_2) \sqcap \varphi = \varphi$, and by distributivity we know $\varphi = (\varphi \sqcap \varphi_1) \sqcup (\varphi \sqcap \varphi_2)$. Moreover, since $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi$, it holds that $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi \sqcap \varphi_1 \sqsubseteq \varphi_1$ and $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi \sqcap \varphi_2 \sqsubseteq \varphi_2$. Thus φ is an amalgam with transfers $\varphi \sqcap \varphi_1$ and $\varphi \sqcap \varphi_2$. □

The following theorem shows that an amalgam space is monotonic.

Theorem 1. *For any $\varphi'_1, \varphi'_2 \in \mathbb{A}(\varphi_1, \varphi_2)$, $\mathbb{A}(\varphi'_1, \varphi'_2) \subseteq \mathbb{A}(\varphi_1, \varphi_2)$.*

Proof. We want to proof that if $\varphi \in \mathbb{A}(\varphi'_1, \varphi'_2)$ then $\varphi \in \mathbb{A}(\varphi_1, \varphi_2)$, i.e. that $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi \sqsubseteq \varphi_1 \sqcup \varphi_2$ holds. Since $\varphi \in \mathbb{A}(\varphi'_1, \varphi'_2)$, it holds that $\varphi'_1 \sqcap \varphi'_2 \sqsubseteq \varphi \sqsubseteq \varphi'_1 \sqcup \varphi'_2$. Morever, it is easy to see that

$$\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi'_1 \sqcap \varphi'_2 \sqsubseteq \varphi \sqsubseteq \varphi'_1 \sqcup \varphi'_2 \sqsubseteq \varphi_1 \sqcup \varphi_2$$

holds, since

1. $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi'_1 \sqcap \varphi'_2$ holds because $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi'_1$ and $\varphi_1 \sqcap \varphi_2 \sqsubseteq \varphi'_2$, and
2. $\varphi'_1 \sqcup \varphi'_2 \sqsubseteq \varphi_1 \sqcup \varphi_2$ holds because $\varphi'_1 \sqsubseteq \varphi_1 \sqcup \varphi_2$ and $\varphi'_2 \sqsubseteq \varphi_1 \sqcup \varphi_2$. □

Although the classical propositional logic can be abstracted as a boolean lattice thus is a bounded distributive lattice, other non-classical logics are not necessarily satisfies the language requirement. In this case, we need to investigate which segment of the language satisfies the conditions. Otherwise, only part of the results presented in the paper are applicable. See more discussion in the last section.

3 The Bargaining Model

In this section we develop a bargaining model using the notion of amalgam. For the sake of simplicity, we consider the bargaining situations in which there are only two agents.

3.1 Bargaining Structures

Bargaining outcomes are mostly determined by individual preferences. Most traditional bargaining models represent players' preferences in cardinal utility as it is well known that ordinal utility does not contain enough information for a rational bargaining solution [28]. Unlike the traditional bargaining models, we express bargainers' preferences with a richer structure, which contains three basic components. The first component is the bargaining terms expressing demands, proposals or offers of players. For instance, Geek wants the *Armchair* to be in North East corner while Artist wants a *Filing Cabinet* to be there. Such a content-rich representation allows bargainers to combine demands from each other and make more constructive proposals in order to reach a possible agreement. This actually reflects the cooperative aspect of bargaining.

The second component is the "collective preference" of all bargaining parties. This information is encoded by the partial order $\sqsubseteq$ of the language we use. If the order stands for the logical implication, it means that all players agree that an agreement with stronger statements is more preferable than one with weaker statements. If the order represents "subsumption" relation among feature terms, the order will mean that every party prefer specific terms to generic terms. Note that the subsumption ordering may be induced from another order structure, say the sort hierarchy shown in Fig. 2 for our room design problem.

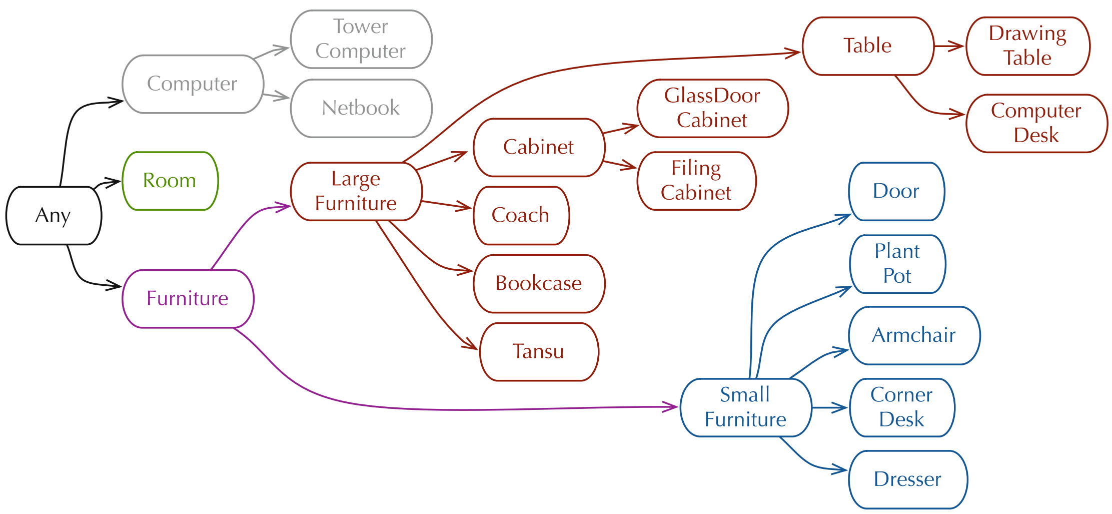

Fig. 2. The sort hierarchy of the room design problem.

The three component of our preference structure is the "individual preference" of each player. As we will define below, this individual preference has to be aligned with the collective preference but mostly refine the common preference. For instance, a player may prefer a bookcase to other particular large furniture to be placed on the South wall even they are in the same level of the sort hierarchy. We assume that individual preference is ordinal because it is only used when a player has to make a decision which term should be chosen against the terms.

Definition 6. *Let* $\langle \mathcal{T}, \sqsubseteq, \sqcap, \sqcup, \bot, \top \rangle$ *be a bounded distributive lattice. A two-player* bargaining structure *is tuple* $(\mathcal{T}, \prec_1, \prec_2)$*, where* $\prec_i$ $(i = 1, 2)$ *is a strict total order over* $\mathcal{T}$ *and satisfies the following properties:*

$$\forall \varphi, \psi \in \mathcal{T} \text{ if } \varphi \sqsubseteq \psi, \text{ then } \varphi \prec_i \psi$$

Note that we require the individual preference to be a strict total order because we assume that whenever a player chooses a term as her new proposal from the available options, she always chooses her most preferred one[2].

Following the standard game-theoretic terminology, for any two terms $\varphi, \psi \in \mathcal{T}$, we say that φ *dominates* ψ if $\psi \prec_i \varphi$ for all $i \in \{1, 2\}$. Note that "dominate" here means "strictly dominate".

Given a bargaining structure $\mathbb{B} = (\mathcal{T}, \prec_1, \prec_2)$, any pair (φ_1, φ_2) of terms in $\mathcal{T}$ is called a *bargaining situation*. Each term φ_i of the bargaining situation is called a *proposal* or an *offer* of player i. We denote all the bargaining situations over the bargaining structure $\mathbb{B}$ by $\mathcal{S}^{\mathbb{B}}$.

Intuitively, a bargaining situation represents the initial state of a negotiation where all the preference information is encoded in the bargaining structure and the proposals of each player. For instance, Fig. 1 represents a bargaining situation when Geek and Artist meet for negotiation with the the initial proposals from Geek and Artist φ_1^0 and φ_2^0 respectively as shown in Fig. 1.

Example 2. *Consider the room design scenario again to show how individual preferences take effect in player's decision-making. Let* φ_1^0 *and* φ_2^0 *be the initial proposals from Geek and Artist respectively as shown in Fig. 1. If Geek chooses the concession not asking for an* Armchair *in SW (let's call it* φ_1^1*) after an amalgamation, he is able to offer the room design in Fig. 3. This amalgamation is the result of unifying two term* $\psi_1^1 \sqcup \psi_2^1$*, where* $\psi_1^1 \sqsubset \varphi_1^0$ *and* $\psi_2^1 \sqsubset \varphi_2^0$*. Similar proposals can be made by the Artist.*

3.2 Solution Concept and Desired Properties

Any bargaining theory is to develop a solution concept that can help predict the agreement bargainers would reach with any bargaining situation [31]. Within our model, an agreement is a term that can be described in the underlying language. Therefore a bargaining solution should tell which term is reached in each bargaining situation as formalised in the following:

Definition 7. *Given a bargaining structure* $\mathbb{B} = (\mathcal{T}, \prec_1, \prec_2)$*, a* bargaining solution $f : \mathcal{S}^{\mathbb{B}} \to \mathcal{T}$ *is a function that maps each bargaining situation* (φ_1, φ_2) *to a term* $f(\varphi_1, \varphi_2) \in \mathcal{T}$*.*

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[2] If somehow two items are not distinguishable for a player, we may remove it from the language or using the reverse of the other player's preference.

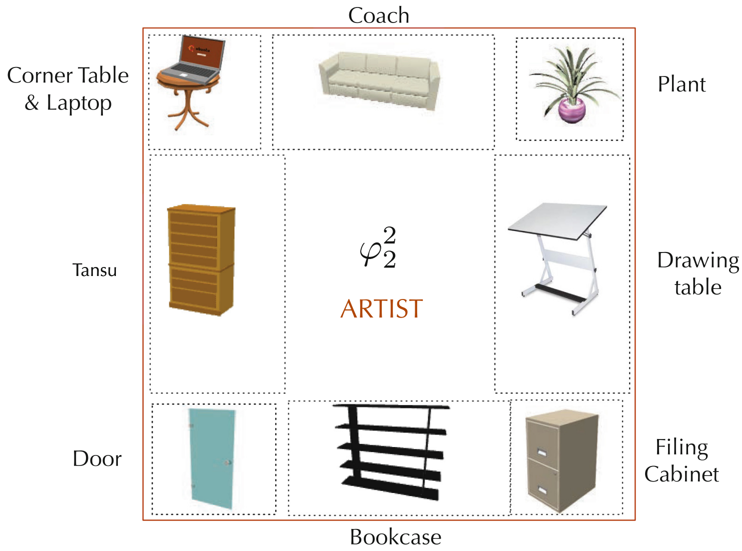

Fig. 3. Artist's counter proposal of removing *Glass-door cabinet* to incorporate Geek's proposal.

Obviously not every such a function is what we want. We want to know which functions give a right prediction for bargaining among rational agents. For his purpose, consider a set of properties, or axioms, we desire a bargaining solution to satisfy.

The first property sets up a bottom line for any bargaining solutions is

Individual Rationality: $f(\varphi_1, \varphi_2) \succeq_i \varphi_1 \sqcap \varphi_2$ for each $i \in \{1, 2\}$.

That is to say, an agreement should never be worse than the minimum of the initial proposals for each player. For instance, if two proposal has something in common, at least these common features should be included in the agreement. With the situation shown in Fig. 1, at least both parties should agree on putting the Coach on the North wall. Note that $f(\varphi_1, \varphi_2)$ can be $\perp$ if $\varphi_1 \sqcap \varphi_2 = \perp$ even though it is not necessary.

The second property deals with an extreme bargaining situation in which there is no conflicting between the initial proposals of two parties. In this case, we assume that the outcome of bargaining is the maximum of their initial proposals.

Collective Rationality: If $\varphi_1 \sqcup \varphi_2 \neq \top$, $f(\varphi_1, \varphi_2) = \varphi_1 \sqcup \varphi_2$.

The property looks quite promising intuitive but we cannot take it for granted without considering other properties. Although $\varphi_1 \sqcup \varphi_2$ is the maximum under the collective preference $\sqsubseteq$, there is no guarantee that $\varphi_1 \sqcup \varphi_2$ is Pareto optimal to the players under the individual preferences. To deal with this issue, let us introduce the most important assumption we propose in this paper. As we have briefly described in the previous sections, we require any proposal from each player must be an amalgam of the initial proposals. Based on the assumption, it is natural to expect that the agreement of bargaining can be represented as an amalgam of the initial proposals. This property can be simply described as follows:

Amalgamation: $f(\varphi_1, \varphi_2) \in \mathbb{A}(\varphi_1, \varphi_2)$.

Now let's introduce the property of Pareto optimality, which is one of the most important properties of a bargaining solution.

Definition 8. *Given a bargaining situation* (φ_1, φ_2), *the* Pareto set *of* (φ_1, φ_2) *is defined as:*

$$P(\varphi_1, \varphi_2) = \{\phi \in \mathbb{A}(\varphi_1, \varphi_2) : \text{ there is no } \varphi' \in \mathbb{A}(\varphi_1, \varphi_2) \text{ such that } \varphi' \text{ dominates } \varphi\}$$

A bargaining solution f being Pareto optimal then means:

Pareto Optimality: $f(\varphi_1, \varphi_2) \in P(\varphi_1, \varphi_2)$.

As mentioned above, these postulates are related each other. The following propositions show Individual Rationality and Collective Rationality can be derived from the other postulations.

Proposition 1. *Any bargaining solution that satisfies Amalgamation is Individual Rational.*

Proof. Since $f(\varphi_1, \varphi_2) \in \mathbb{A}(\varphi_1, \varphi_2)$, by the definition of amalgam, we have $\varphi_1 \sqcap \varphi_2 \sqsubseteq f(\varphi_1, \varphi_2)$. By Definition 6, it follows that $\varphi_1 \sqcap \varphi_2 \succeq_i f(\varphi_1, \varphi_2)$ for each $i \in \{1, 2\}$. □

Proposition 2. *Any bargaining solution that satisfies Amalgamation and Pareto Optimality is Collective Rational.*

Proof. Let f be a bargaining solution that satisfies Amalgamation and Pareto Optimality. On the one hand, for any $\varphi \in \mathbb{A}(\varphi_1, \varphi_2)$, we have $\varphi \sqsubseteq \varphi_1 \sqcup \varphi_2$; thus $\varphi \preceq_i \varphi_1 \sqcup \varphi_2$ for each $i \in \{1, 2\}$. By Amalgamation we know $f(\varphi_1, \varphi_2) \preceq_i \varphi_1 \sqcup \varphi_2$. On the other hand, since f is Pareto optional, there is no term in $\mathbb{A}(\varphi_1, \varphi_2)$, including $\varphi_1 \sqcup \varphi_2$, strictly dominates $f(\varphi_1, \varphi_2)$. Therefore $f(\varphi_1, \varphi_2) = \varphi_1 \sqcup \varphi_2$. □

Finally we introduce a special property similar to Nash's *Independence of Irrelevant Alternatives (IIA)*. The idea is inspired by a similar property in [32]. A bargaining situation (φ_1', φ_2') is a *simultaneous concession* of a bargaining situation (φ_1, φ_2) if $\varphi_i' \prec_i \varphi_i$ and $\varphi_i' \in \mathbb{A}(\varphi_1, \varphi_2)$ for each $i \in \{1, 2\}$; it is *minimal* if there is no $\psi_i \in \mathbb{A}(\varphi_1, \varphi_2)$ such that $\varphi_i' \prec_i \psi_i \prec_i \varphi_i$ for either $i = 1$ or $i = 2$.

Concession Independence: *Let* (φ_1', φ_2') *be a minimal simultaneous concession of* (φ_1, φ_2). If $\varphi_1 \sqcup \varphi_2 \neq \top$, *either* $f(\varphi_1, \varphi_2) = f(\varphi_1', \varphi_2)$, $f(\varphi_1, \varphi_2) = f(\varphi_1, \varphi_2')$ or $f(\varphi_1, \varphi_2) = f(\varphi_1', \varphi_2')$.

Unlike the game-theoretic models of bargaining in which the domain of alternatives is mostly assumed to be continuous and convex thus the solution can be unique if exists, the domain of alternatives in our model is finite and discrete and therefore Pareto optimal solutions may not be unique. For instance, assume that a couple, husband and wife, negotiates for choosing either Thai or Chinese restaurant for dinner. If their preferences are exactly opposite, both the alternatives "Thai restaurant" and "Chinese restaurant" are Pareto optimal.

4 Solution Construction and Characterisation

In this section, we construct a concrete bargaining solution which is lead by a natural procedure of bargaining between rational agents. We show that this solution satisfies all the desired properties we listed in the previous section and also can be and is fully characterised by these properties.

4.1 Bargaining Procedures

We assume that a bargaining procedure consists of a number of rounds. In each round, two players make a proposal simultaneously[3]. The proposal of a player for each round can be the same as the last round (named stand-still) or make a new proposal by picking up a different term from the set of amalgams of the proposals from the last round. Once both players choose to stand still, the procedure terminates. Formally, we can describe a bargaining procedure as a sequences of bargaining situations.

Definition 9. *A* bargaining sequence $q = \{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$, *where n can be any natural number, is a finite sequence of bargaining situations such that $\varphi_i^t \in \mathbb{A}(\varphi_1^{t-1}, \varphi_2^{t-1})$ for any $t > 0$ and $i \in \{1, 2\}$. We call φ_i^t the proposal of player i at round t.*

$(\varphi_1^0, \varphi_2^0)$ is say that the sequence starts from of the sequence. We also say that this sequence starts from $(\varphi_1^0, \varphi_2^0)$. By Theorem 1, $\mathbb{A}(\varphi_1^t, \varphi_2^t) \subseteq \mathbb{A}(\varphi_1^{t-1}, \varphi_2^{t-1})$ for each $t(1 \leq t \leq n)$; therefore the negotiation space monotonically decreasing with the advance of bargaining.

A bargaining sequence records only a possible bargaining procedure without assuming the rationality of players' strategic behaviour. In fact, we may assume that whenever a player decide to make a new proposal, instead of stand-still, she would choose the most preferred one from the space of amalgams of the previous proposal (excluding the previous proposals). In addition, it is reasonable to assume that any bargaining procedure terminates whenever a player finds that the proposal of her opponent is as preferable as the one she is proposing or even better. These rationality assumptions can be formally described as follows:

Definition 10. *A bargaining sequence $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ is* rational *if*

1. for each $t > 0$ and each $i \in \{1, 2\}$, $\varphi_i^t \succeq_i \varphi$ for all $\varphi \in \mathbb{A}(\varphi_1^{t-1}, \varphi_2^{t-1})$.
2. $\varphi_2^t \succeq_1 \varphi_1^t$ and $\varphi_1^t \succeq_2 \varphi_2^t$ for all $t \geq 0$.

This definition plus Theorem 1 immediately implies the following property, which says that any rational bargaining sequence is a monotonic concession procedure.

Proposition 3. *Let $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ be a rational bargaining sequence. Then for each $t > 0$ and $i \in \{1, 2\}$, $\varphi_i^t \preceq_i \varphi_i^{t-1}$.*

[3] Since we allow a player to repeat her previous proposal, bargaining with alternating offers can be viewed as a special case of this simultaneous model.

We say a bargaining sequence $q = \{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ *complete* if there are two proposals $\varphi_1^{t_1}$ and $\varphi_2^{t_2}$, one from each player, such that $0 \leq t_1 \leq n$, $0 \leq t_2 \leq n$ and $\varphi_1^{t_1} \sqcup \varphi_2^{t_2} \neq \top$. $\varphi_1^{t_1} \sqcup \varphi_2^{t_2}$ is called an *agreement* of the sequence. Note that $\varphi_1^{t_1}$ and $\varphi_2^{t_2}$ are not necessarily proposed at the same round (i.e., it is possible that $t_1 \neq t_2$). Also there might be multiple agreements reached by a sequence.

A complete bargaining sequence $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ is *minimal concessive* if for any agreement φ of the sequence, $\varphi_i^t \succeq_i \varphi$ for all $t > 0$ and $i \in \{1, 2\}$. A minimal concessive sequence represents an ideal bargaining procedure in which no body made unnecessary concession. The following lemma shows that such an ideal outcome may be achieved if one player can effectively use the "stand-still" strategy.

A complete bargaining sequence $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ is a *shortest minimal concessive sequence* for $(\varphi_1^0, \varphi_2^0)$ if there is no other minimal concessive sequence starting from $(\varphi_1^0, \varphi_2^0)$ that is shorter than n.

Lemma 2. *Let $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ be a shortest minimal concessive bargaining sequence. Then $\varphi_1^n = \varphi_2^n$ and it is also the unique agreement of the sequence.*

Proof. Let $\varphi = \varphi_1^{t_1} \sqcup \varphi_2^{t_2}$ is an agreement of the sequence. According to Theorem 1 and Definition 9, both $\varphi_1^{t_1}$ and $\varphi_2^{t_2}$ belong to $\mathbb{A}(\varphi_1^0, \varphi_2^0)$. Therefore $\varphi_1^{t_1} \sqcup \varphi_2^{t_2} \in \mathbb{A}(\varphi_1^0, \varphi_2^0)$ because $\varphi_1^{t_1} \sqcup \varphi_2^{t_2} \neq \top$. Note that $\varphi_i^{t_i} \sqsubseteq \varphi_1^{t_1} \sqcup \varphi_2^{t_2}$ for each $i \in \{1, 2\}$; thus $\varphi_i^{t_i} \preceq_i \varphi_1^{t_1} \sqcup \varphi_2^{t_2} = \varphi$ for all $i \in \{1, 2\}$. By the definition of amalgam, it is easy to know that $\varphi \in \mathbb{A}(\varphi_1^t, \varphi_2^t)$ where $t = \min\{t_1, t_2\}$. According to Proposition 3, there is t_i' for each $i \in \{1, 2\}$ such that $1 \leq t_i' \leq t$ and $\phi = \varphi_i^{t_i'}$. By the condition of minimal concession, $\varphi_i^k \succeq_i \phi$ for all $k > 0$. By Proposition 3 again we know $\varphi_i^k = \phi$ for all k $(t_i' \leq k \leq n)$. Specifically, $\phi = \varphi_1^n = \varphi_2^n$, which is the unique agreement of the sequence. $\square$

This lemma lead the following definition.

Definition 11. *A bargaining function f is called a* minimal concessive solution *if for each bargaining situation (φ_1, φ_2), there is a shortest minimal concessive bargaining sequence $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ starting from (φ_1, φ_2) such that $f(\varphi_1, \varphi_2)$ is the agreement of the sequence.*

The following theorem shows that a minimal concessive solution satisfies all the desired properties listed in the previous section and are also fully characterised by these properties.

Theorem 2. *A bargaining solution satisfies Pereto Optimality, Amalgamation and Concession Independence if and only if it is a minimal concessive solution.*

Lemma 3. *Given any bargaining situation (φ_1, φ_2), there are at most two shortest minimal concessive bargaining sequences starting with (φ_1, φ_2).*

Proof. Let $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ be a bargaining sequence starting with (φ_1, φ_2) that satisfies the condition: for each $t > 0$, $(\varphi_1^t, \varphi_2^t)$ is the minimal simultaneous concession of $(\varphi_1^{t-1}, \varphi_2^{t-1})$. Since we assume that individual preferences are strictly total orders, the above sequence is unique for (φ_1, φ_2). Furthermore, we also assume that our language contains only finite number of terms, therefore each term has a chance to appear in both sides of the sequence. Assume that $\varphi_1^{t_1}$ and $\varphi_2^{t_2}$ are the earliest ones that appear in both sides, where t_1 and t_2 are the indexes of their second occurrences for each side, respectively. If $t_1 \neq t_2$, say $t_1 < t_2$, let t_2' be the index such that $\varphi_1^{t_1} = \varphi_2^{t_2'}$. For each $k (0 \leq k \leq t_2')$, let $\psi_2^k = \varphi_2^k$; for each $k (t_2' < k \leq t_1)$, let $\psi_2^k = \varphi_2^{t_2'}$. Then $\{(\varphi_1^t, \psi_2^t)\}_{t=0}^{t_1}$ is a shortest minimal concessive sequence. Since this process is constructive, the shortest minimal concessive sequence is unique for (φ_1, φ_2). If $t_1 = t_2$, we can construct with the above approach two shortest minimal concessive sequences with exactly the same length. Therefore there are at most two shortest minimal concessive bargaining sequences starting from (φ_1, φ_2). □

Lemma 4. *Any minimal concession solution satisfies Pareto Optimality.*

Proof. let f be a minimal concessive solution. For any bargaining situation (φ_1, φ_2), let $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ be the shortest minimal concessive sequence such that $f(\varphi_1, \varphi_2)$ is the agreement of the sequence. According to Lemma 2, $f(\varphi_1, \varphi_2) = \varphi_1^n = \varphi_2^n$. If $f(\varphi_1, \varphi_2)$ is not Pareto optimal, there is $\varphi \in \mathbb{A}(\varphi_1, \varphi_2)$ such that φ dominates $f(\varphi_1, \varphi_2)$. It turns out that $\varphi \succ_i \varphi_i^n$ for each $i \in \{1, 2\}$. Therefore φ appears in the sequence. This implies that φ is an agreement of the sequence, which contradicts uniqueness of agreement proved by Lemma 2. □

Lemma 5. *Any minimal concession solution satisfies Concession Independence.*

Proof. Let (φ_1', φ_2') be the minimal simultaneous concession of (φ_1, φ_2). By Lemma A2 there are at most two shortest minimal concessive sequences for (φ_1, φ_2). Let $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ be one of them. Obviously $\varphi^1 = \varphi_1'$ and $\varphi^2 = \varphi_2'$. If neither φ_1^0 nor φ_2^0 is an agreement of the sequence, then the sequence after remove the initial bargaining situation is a shortest minimal concessive sequence for (φ_1', φ_2'). If one of φ_1^0 and φ_2^0, say the former, is an agreement of the sequence, the sequence after remove φ_2^0 from the original sequence is a shortest minimal concessive sequence for $(\varphi_1^0, \varphi_2')$. Since there are at most two shortest minimal sequences for $(\varphi_1^0, \varphi_2')$, $f(\varphi_1, \varphi_2)$ will have to choose one of them. Therefore $f(\varphi_1, \varphi_2) = f(\varphi_1', \varphi_2)$, $f(\varphi_1, \varphi_2) = f(\varphi_1, \varphi_2')$ or $f(\varphi_1, \varphi_2) = f(\varphi_1', \varphi_2')$. □

Now let's prove the main theorem.

Proof. "⇐". Obviously any minimal concessive solution satisfies Amalgamation. Lemma A3 and A4 shown that Pareto Optimality and Concession Independence are satisfied.

"⇒". Let f be a bargaining solution that satisfies Amalgamation, Pareto Optimality and Concession Independence. For any bargaining situation (φ_1, φ_2), if $\varphi_1 \sqcup \varphi_2 \neq \top$, $\{(\varphi_1, \varphi_2)\}$ is a shortest minimal concessive sequence for (φ_1, φ_2).

Assume now that $\varphi_1 \sqcup \varphi_2 = \top$. Let $\{(\varphi_1^t, \varphi_2^t)\}_{t=0}^n$ be a bargaining sequence starting with (φ_1, φ_2) that satisfies the condition: for each $t > 0$, $(\varphi_1^t, \varphi_2^t)$ is the minimal simultaneous concession of $(\varphi_1^{t-1}, \varphi_2^{t-1})$. Since our language is finite,

$f(\varphi_1, \varphi_2)$ appears in both sides of the sequence. Similar to the proof of Lemma A1, we can construct a minimal concessive sequence that ends with $f(\varphi_1, \varphi_2)$. Since f is pareto optimial, such a sequence is a shortest minimal concessive sequence. □

5 Conclusion

In this paper, we proposed a new bargaining framework based on the notion of amalgam. We use a "content-rich" language to describe bargainers' proposals or offers and use two types of orderings to express bargainers' collective preferences and individual preferences. We assume that any bargain proceeds upon the protocol that any new proposal of a bargainer must be an amalgam of the previous proposals which in fact takes into account the concession of one bargainer and the requests of the other bargainer.

As we have mentioned, our bargaining framework is not only applicable to the language of OSF logic but also any formal language that contains connectives that form a bounded distributive lattice (BDL). Nevertheless, not all formal languages, especially logical languages, are a BDL. For instance, the description logic or sort-ordered logic is not a DBL in general, which blocked our attempt to directly apply our approach to certain domains, such as logic-based bargining theory, GDL-based negotiation and ontology-based negotiation [16,30,32]. It is interesting to know whether any segments of these logics satisfy our language requirements. Nevertheless, the definition of amalgam does not reply on BDL structure therefore our approach is still applicable for the domains that cannot be described by a BDL language (especially when solution characterisation is not a major concern). It is also interesting if we reduce the language requirement from BDL to bounded lattice, what are the results we can still have? We leave these questions for the future.

We also intend to explore the relation of our approach with concept blending and creativity. Concept blending [4,7,8,23] is a cognitive model of creative thinking by which new concepts are created by combining ("blending") two existing concepts (called also *mental spaces*). Bargaining through amalgamation can be seen as a related concept applied to negotiation: a bargainer not only makes concessions on prior demands but explores creative proposals, by making combinations of elements existing in the proposals of both bargainers, while maintaining the sequence of creative proposals guided by the individual preferences.

Nearly all existing bargaining models, including this work, assume static preferences. However, preferences can be dynamic and may evolve during negotiation. Developing a negotiation framework that accommodates the dynamic measurement of bargainers' preferences would be a promising direction for future research [6,18].

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