Bipolar representation and fusion of preferences in the possibilistic logic framework

Salem Benferhat, Didier Dubois, Souhila Kaci and Henri Prade
Institut de Recherche en Informatique de Toulouse (I.R.I.T.)–C.N.R.S.
Université Paul Sabatier, 118 route de Narbonne 31062 TOULOUSE Cedex 4, FRANCE
E-mail:{benferhat, dubois, kaci, prade}@irit.fr

Abstract

Preferences are naturally bipolar: positive aspirations and rejections are complementary in order to express agents wishes. Possibility theory framework allows a simple bipolar representation, using the "classical" possibility and necessity functions, and also the less usual "guaranteed possibility" function. This paper first shows how to deal syntactically with positive aspirations as well as rejections. We then discuss the problem of merging multiple agents preferences in this bipolar framework both at semantic and syntactic levels. Lastly, we discuss the problem of the consistency between positive aspirations and rejections, and how to restore it when necessary.

1 Introduction

The problem of the representation of the preferences of agents has been considered by various researchers in Artificial Intelligence in the recent past years [23, 10, 5, 1, 18, 3, 6]. Indeed this issue is important when we have to represent the desires of users in information systems (e.g. recommender systems), or to reason about them and to solve conflicts between inconsistent goals, as e.g., in multi-agent systems.
Preferences are often expressed in two forms: positive aspirations and rejections. Indeed, on the one hand, an agent expresses what he considers as (more or less) impossible because it is unacceptable for him, and on the other hand he expresses what he considers as really satisfactory. For example assume that we have a one month summer school, and we ask some invited speaker to express his preference for scheduling his talk. We assume that the talks can be either given in the morning, or in the afternoon. The invited speaker may provide two kinds of preferences. In the first one, he specifies satisfactory slots, with satisfaction levels. This is positive preference. In the second one, he describes unacceptable slots with levels of tolerance. For instance, he may strongly refuse to participate one day (e.g., because it is the birthday of her daughter), and weakly refuse to speak on the last day. This is negative preferences or rejections.
This bipolar representation is supported by recent studies in cognitive psychology which have shown that positive and negative preferences are processed separately in the mind, and are felt as independent and different dimensions by people [8, 7]. Note that in general there is no symmetry between positive aspirations and rejections in the sense that positive aspirations do not just mirror what is not rejected.

The idea of bipolar representations of preferences has been considered in different frameworks, such as multicriteria decision making [16], or in Artificial Intelligence works oriented towards qualitative decision [22, 17]. However these works usually assume that the positive and negative parts of the preferences, once they have been specified and represented can be combined and processed together, rather than separately as it is done in this paper.

Section 2 gives the necessary background on possibility theory and possibilistic logic. Section 3 specifies the bipolar representation of preferences. Sections 4 and 5 discuss the possibilistic encoding of rejections and positive aspirations. Sections 6 and 7 present the problems of fusing of agents preferences and restoring the coherence when they conflict.

2 Background on possibility theory and possibilistic logic

We consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$ of atoms. Ω denotes the set of all classical interpretations. $[\![\psi]\!]$ denotes the set of models of the proposition ψ.

The basic concept in possibilistic theory is the notion of possibility distribution, denoted by π, which is simply a function from the set of interpretations to the unit interval $[0,1]$, or to any totally ordered scale, finite or not. Thus we can use only a finite set of qualitative levels if necessary.

Given a posibility distribution π, two standard measures are defined for formulas:

- the possibility (or consistency) measure of a formula ϕ:
$$\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\},$$
which evaluates the extent to which ϕ is consistent with the available information expressed by π, and
- the necessity (or certainty) measure of a formula ϕ:
$$N(\phi) = 1 - \Pi(\neg\phi),$$
which evaluates the extent to which ϕ is entailed by information expressed by π.

Given a possibility distribution π, we define the core of π as the set of interpretations having the highest possibility degree in π. Formally,

Definition 1 *The core of a possibility distribution π, denoted by $core(\pi)$, is defined by:*
$$core(\pi) = \{\omega : \omega \in \Omega, \nexists \omega' \in \Omega, \pi(\omega') > \pi(\omega)\}.$$

We now define the contextual core as follows:

Definition 2 *The contextual core of a possibility distribution π given a formula φ (φ represents the context), denoted by $core_\varphi(\pi)$, is defined by:*
$$core_\varphi(\pi) = \{\omega : \omega \models \varphi, \nexists \omega', \omega' \models \varphi \text{ and } \pi(\omega') > \pi(\omega)\}.$$

Indeed, two kinds of inference can be defined from π, in the same spirit of [20]:

Definition 3 *Plausible and preferential inferences. Let π be a possibility distribution. The formula ψ is said to be a* **plausible** *consequence of π, denoted by $\pi \models_P \psi$, iff*
$$core(\pi) \subseteq [\![\psi]\!].$$
ψ is said to be a **preferential** *consequence of π given the formula φ, denoted by $\pi \models_\varphi \psi$, iff*
$$core_\varphi(\pi) \subseteq [\![\psi]\!].$$

Prioritized information are represented in the possibilistic logic framework by means of a set of weighted formulas, called *a possibilistic logic base*, of the form $\Sigma = \{(\phi_i, a_i) : i = 1, \cdots, n\}$, where ϕ_i is a propositional formula and a_i belongs to a totally ordered scale such as $[0,1]$. (ϕ_i, a_i) means that the necessity degree of ϕ_i is at least equal to a_i, i.e. $N(\phi_i) \geq a_i$. Given a possibilistic logic base Σ, we can generate a unique possibility distribution, denoted by π_Σ, defined by [12]:

Definition 4 $\forall \omega \in \Omega$,
$$\pi_\Sigma(\omega) = \begin{cases} 1 & \text{if } \forall (\phi_i, a_i) \in \Sigma, \omega \models \phi_i \\ 1 - max\{a_i : (\phi_i, a_i) \in \Sigma \text{ and } \omega \not\models \phi_i\} & \text{otherwise,} \end{cases}$$

where π_Σ is the largest possibility distribution such that the necessity measure N associated with π_Σ is such that $N(\phi_i) \geq a_i$ holds for all i.
Besides, $N(\phi \vee \psi) \geq a, N(\neg\phi \vee \xi) \geq b \vdash N(\psi \vee \xi) \geq min(a, b)$, which is the basis of the possibilistic inference machinery [12].

3 Specifying bipolar preferences

In this paper, we propose a bipolar representation of preferences which can be expressed both at the syntactic and at the semantic levels. We introduce the syntactic specification of these preferences in this section. Preferences of an agent will be represented by two different sets of equality constraints.
The first set corresponds to what is not unacceptable, what is more or less acceptable, tolerable for the agent. It is of the form $\{\mathcal{R}(\phi_i) = a_i : i = 1, \cdots, n\}$ where ϕ_i is a propositional formula, $\mathcal{R}$ stands for rejection, and a_i is a real in the interval $[0,1]$. $\mathcal{R}(\phi_i) = a_i$ expresses the rejection strength of ϕ_i by the agent. $\mathcal{R}(\phi_i) = 1$ means that the agent strongly rejects ϕ_i, and any solution where ϕ_i is true is considered as not feasible by the agent. $\mathcal{R}(\phi_i) = 0$ simply means that ϕ_i is not rejected, and hence solutions satisfying ϕ_i are not at all unacceptable, or are feasible. For $\mathcal{R}(\phi_i) = a_i$ with $a_i \in (0,1)$, the higher a_i is, the less acceptable are the solutions satisfying ϕ_i. It turns out that the set of rejections can be easily handled using standard possibilistic logic based on the two classical functions of possibility theory: possibility and necessity measures. This will be developed in Section 4.
The second set expresses "positive" goals, or agent's aspirations. It is of the form $\{\mathcal{G}(\psi_j) = b_j : j = 1, \cdots, m\}$, where $\mathcal{G}$ stands for positive goals, ψ_j is a propositional formula, and b_j is a real in the interval $[0,1]$. $\mathcal{G}(\psi_j) = b_j$ expresses the minimal level of satisfaction which is guaranteed if ψ_j is true. $\mathcal{G}(\psi_j) = 1$ means that the agent is fully satisfied as soon as ψ_j

is true. $\mathcal{G}(\psi_j) = 0$ means that learning that ψ_j is satisfied brings no satisfaction to the agent who is indifferent. When $\mathcal{G}(\psi_j) = b_j$ for $b_j \in (0,1)$ the larger b_j is, the more satisfied is the agent if ψ_j is true. This kind of positive goals cannot be directly handled by the possibilistic logic machinery. In fact, positive goals can be represented using the so-called function of "guaranteed possibility", denoted by Δ, in possibility theory. We will show that the syntactic representation of uncertain information using Δ is dual in some sense of the one used in standard possibilistic logic. This will be developed in Section 5.

4 Modelling rejections in possibilistic logic

This section shows that possibilistic logic provides a natural framework for modelling rejections. Rejections can be represented, at the semantical level, by a total pre-order on the set of all possible outcomes, from what is feasible to what is considered as unacceptable by an agent. Outcomes are called here interpretations, since we use a logic-based representation. The total pre-order can be encoded, in possibility theory framework, using the notion of a possibility distribution, denoted by $\pi_{\mathbb{R}}$ ($\mathbb{R}$ for rejections), which is a function from the set of interpretations to the unit interval $[0,1]$. $\pi_{\mathbb{R}}(\omega)$ represents the degree of acceptability of ω given agent's preferences. $\pi_{\mathbb{R}}(\omega) = 1$ means that ω is fully acceptable, $\pi_{\mathbb{R}}(\omega) = 0$ means that ω is completely unacceptable (rejected), and more generally, $\pi_{\mathbb{R}}(\omega) > \pi_{\mathbb{R}}(\omega')$ means that ω' is more unacceptable than ω.

In practice an agent cannot provide the whole possibility distribution $\pi_{\mathbb{R}}$, but only a set of specifications of negative desires, and their level of rejection. Let $\mathbb{R} = \{\mathcal{R}(\phi_i) = a_i : i = 1, \cdots, n\}$ be this set of rejections, where by default, formulas which are not in $\mathbb{R}$ are assumed to have a rejection level equal to 0, namely they are not explicitely rejected at all.
The question is then how to compute $\pi_{\mathbb{R}}$. Let us illustrate the construction of $\pi_{\mathbb{R}}$ when we only have one strong rejection constraint $\mathcal{R}(\phi) = 1$. Let ω be an interpretation. Intuitively, if ω falsifies ϕ then ω is fully acceptable by the agent, i.e. $\pi_{\mathbb{R}}(\omega) = 1$, and if ω satisfies ϕ then ω is fully unacceptable, i.e. $\pi_{\mathbb{R}}(\omega) = 0$. Now, assume that ϕ is weakly rejected, i.e. $\mathcal{R}(\phi) = a < 1$. Again, if ω falsifies ϕ then it is fully acceptable. If ω satisfies ϕ then the higher is a, the less acceptable is ω. One way to achieve this constraint is to assign the value $1 - a$ to $\pi_{\mathbb{R}}(\omega)$. More formally, the possibility distribution associated with $\mathbb{R} = \{\mathcal{R}(\phi) = a\}$ is:

$$\pi_{\mathbb{R}}(\omega) = \begin{cases} 1 & \text{if } \omega \not\models \phi \\ 1-a & \text{if } \omega \models \phi. \end{cases}$$

Now assume that the agent both rejects ϕ and ψ with $\mathcal{R}(\phi) = a$ and $\mathcal{R}(\psi) = b$ respectively. Then, we have three cases:

- $\omega \not\models \phi$ and $\omega \not\models \psi$, then ω is fully acceptable. Hence $\pi_{\mathbb{R}}(\omega) = 1$.
- $\omega \models \neg\phi \wedge \psi$ (resp. $\omega \models \phi \wedge \neg\psi$) then the higher is b (resp. a), the less acceptable is ω. Hence $\pi_{\mathbb{R}}(\omega) = 1 - b$ (resp. $1 - a$).
- $\omega \models \phi \wedge \psi$ then the higher is a or b, the less acceptable is ω. Hence $\pi_{\mathbb{R}}(\omega) = 1 - max(a, b)$.

More generally, we have:

Definition 5 *The possibility distribution $\pi_{\mathbb{R}}$ associated with a set of rejections $\mathbb{R} = \{\mathcal{R}(\phi_i) = a_i : i = 1, \cdots, n\}$ is:*
$\pi_{\mathbb{R}}(\omega) = 1 - max\{a_i : \omega \models \phi_i, \mathcal{R}(\phi_i) = a_i \in \mathbb{R}\}$,
with $max\{\emptyset\} = 0$.

Clearly, this definition is very close to the one used in standard possibilistic logic, for inducing a possibility distribution π_Σ from a possibilistic knowledge base Σ, viewed as expressing constraints in terms of necessity measure. Indeed, it is enough to replace ϕ_i by $\neg\phi_i$ in $\mathbb{R}$ in order to obtain Σ. Hence, a set of rejections can be directly encoded by means of a possibilistic knowledge base:

Proposition 1 *Let $\mathbb{R} = \{\mathcal{R}(\phi_i) = a_i : i = 1, \cdots, n\}$. Let $\Sigma = \{(\neg\phi_i, a_i) : \mathcal{R}(\phi_i) = a_i \in \mathbb{R}\}$. Then, $\pi_{\mathbb{R}} = \pi_\Sigma$, where π_Σ is the semantic counterpart of the set Σ of possibilistic logic formulas.*

Proofs of Propositions are provided in Appendix.

This result is important since it means that the classical possibilistic logic machinery can be used for handling negative desires and drawing inferences from them. Moreover, the complexity of possibilistic logic is only slightly higher than the one of classical logic [12].

Example 1 *Let co, be and su be three propositional symbols which stand respectively for "country", "beach" and "sun". Assume that the agent provides the following set of rejections:*
$\mathbb{R} = \{\mathcal{R}(be \wedge \neg su) = 1, \mathcal{R}(co \wedge su) = 1,$
$\mathcal{R}(\neg be \wedge su) = \frac{1}{2}\}$.

ω	$\pi_{\mathbb{R}}(\omega)$
$\neg co \wedge \neg be \wedge \neg su$	1
$\neg co \wedge be \wedge su$	1
$co \wedge \neg be \wedge \neg su$	1
$\neg co \wedge \neg be \wedge su$	$\frac{1}{2}$
other interpretations	0

Table 1: The possibility distribution associated with $\mathbb{R}$.

This describes the rejections of an agent regarding sunny places: a total refusal of beach without sun, a total refusal of a country place where there is sun. The last rejection expresses a partial refusal to go to a sunny place where there is no beach.
Table 1 gives the possibility distribution associated with $\mathbb{R}$ applying Definition 5.

Note that the possibilistic base associated with $\mathbb{R}$ is the following $\Sigma = \{(\neg be \vee su, 1), (\neg co \vee \neg su, 1), (be \vee \neg su, \frac{1}{2})\}$. We can check that Σ generates the same possibility distribution as given in Table 1, using Appendix.

5 Representing positive goals

5.1 Semantic representation

The positive goals of an agent can also be described, at the semantical level, in terms of a possibility distribution also. Let $\pi_{\mathbb{G}}$ ($\mathbb{G}$ for goals) be this distribution. The range of $\pi_{\mathbb{G}}$ is also $[0, 1]$ (or any totally ordered set finite or not, and may be different from the one used for $\pi_{\mathbb{R}}$). $\pi_{\mathbb{G}}(\omega) > \pi_{\mathbb{G}}(\omega')$ means that ω is more satisfactory than ω'.
The meaning of $\pi_{\mathbb{G}}(\omega)$ is different from $\pi_{\mathbb{R}}(\omega)$. Indeed $\pi_{\mathbb{G}}(\omega)$ evaluates to what degree ω is satisfactory for the agent, while $\pi_{\mathbb{R}}(\omega)$ evaluates to what degree ω is acceptable, feasible by the agent. In particular $\pi_{\mathbb{G}}(\omega) = 1$ means that the agent is fully satisfied, while $\pi_{\mathbb{G}}(\omega) = 0$ simply means that the agent is indifferent (while $\pi_{\mathbb{R}}(\omega) = 0$ means that ω is impossible).

As for $\mathbb{R}$, in practice, an agent provides a set of positive goals of the form $\mathbb{G} = \{\mathcal{G}(\psi_j) = b_j : j = 1, \cdots, m\}$, where by default it is assumed that the agent is indifferent with respect to the truth of propositions which are not explicitely stated in $\mathbb{G}$. $\mathcal{G}(\psi_j) = b_j$ means that the agent will be satisfied to a degree b_j if ψ_j alone is satisfied.

In a similar way, let us see how to build the possibility distribution, denoted by $\pi_{\mathbb{G}}$, associated with $\mathbb{G}$. We first consider the case where we only have one constraint $\mathcal{G}(\psi) = b$. Then if a given ω satisfies ψ, the associated level of satisfaction will be equal to b. Otherwise, this level will be equal to 0, namely the agent is indifferent (and not to $1 - b$ which corresponds to the rejection of $\neg\psi$). In the general case, given a set of positive goals, the level of satisfaction associated with ω is equal to the highest level of a formula appearing in $\mathbb{G}$ satisfied by ω:

ω	$\pi_{\mathbb{G}}(\omega)$
$\neg co \wedge be \wedge su$	1
$co \wedge \neg be \wedge \neg su$	$\frac{1}{4}$
other interpretations	0

Table 2: The possibility distribution associated with $\mathbb{G}$.

Definition 6 *The possibility degree $\pi_{\mathbb{G}}$ associated with a set of positive goals $\mathbb{G} = \{\mathcal{G}(\psi_j) = b_j : j = 1, \cdots, m\}$ is:*
$\pi_{\mathbb{G}}(\omega) = max\{b_j : \omega \models \psi_j \text{ and } \mathcal{G}(\psi_j) = b_j \in \mathbb{G}\}$, *with $max\{\emptyset\} = 0$.*

Note that the addition of positive goals in $\mathbb{G}$ can only lead to the increasing of the satisfaction level associated with ω. Let us emphasize that this contrasts with the behaviour of $\pi_{\mathbb{R}}$ which is monotonically decreasing with respect to the number of constraints in $\mathbb{R}$.

Example 2 *Let $\mathbb{G}$ be the following set of goals $\mathbb{G} = \{\mathcal{G}(\neg co \wedge be \wedge su) = 1, \mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{4}\}$. The first expression means that the agent is fully satisfied when there is a sun, a beach and he is not in the country. The second expression means that the agent is weakly satisfied when he is in the country (without beach or sun).*
The possibility distribution $\pi_{\mathbb{G}}$ associated with $\mathbb{G}$ is given in Table 2 following Definition 6.

5.2 Properties of the representation

The set of positive goals, contrarily to the rejections, cannot be directly handled by standard possibilistic logic. This is due to the fact that $\mathcal{G}(\psi) = b$ cannot be directly expressed using possibility and necessity measures as $\mathbb{R}$. Rather constraints like $\mathcal{G}(\psi) = b$ have to be represented using a third function called *guaranteed possibility* function, denoted by Δ [15]. The expression $\Delta(\psi) = b$ means that any interpretation where ψ is true, has a satisfactory degree at least equal to b. This is exactly what is intended by the information encoded in $\mathbb{G}$, indeed $\Delta(\psi) = min_{\omega \models \psi} \pi_{\mathbb{G}}(\omega)$[1].

[1]Hence $\Delta(\phi \vee \psi) = min(\Delta(\phi), \Delta(\psi))$, so Δ is decreasing with respect to entailment. Indeed, the semantic en-

Moreover, there exists few works where the logical machinery of the Δ-weighted formulas is described. It is governed by a cut rule of the form $\Delta(\phi \wedge \psi) \geq a, \Delta(\neg\phi \wedge \xi) \geq b \vdash \Delta(\psi \wedge \xi) \geq min(a, b)$ [11] which is the counterpart of the possibilistic resolution changing Δ into a necessity measure and the conjunctions into disjunctions.

Let us now give some properties of this representation:

Lemma 1 *Let $\mathbb{G}$ be a set of positive goals containing ψ_1 and ψ_2 at the level b. Let $\mathbb{G}'$ be a set of positive goals obtained from $\mathbb{G}$ by replacing $\mathcal{G}(\psi_1) = b$ and $\mathcal{G}(\psi_2) = b$ by $\mathcal{G}(\psi_1 \vee \psi_2) = b$. Then, $\mathbb{G}$ and $\mathbb{G}'$ are semantically equivalent, i.e. $\pi_{\mathbb{G}} = \pi_{\mathbb{G}'}$.*

This means that two sets of goals having the same strength can be replaced by their disjunction with a same strength. The second lemma concerns redundant information:

Definition 7 *$\mathcal{G}(\phi) = a \in \mathbb{G}$ is said to be subsumed if there exists $\mathcal{G}(\psi) = b \in \mathbb{G}$ such that $b \geq a$ and $\phi \vdash \psi$.*

Lemma 2 *Let $\mathcal{G}(\phi) = a$ be a subsumed goal in $\mathbb{G}$. Let $\mathbb{G}' = \mathbb{G} - \{\mathcal{G}(\phi) = a\}$. Then, $\mathbb{G}$ and $\mathbb{G}'$ are semantically equivalent, i.e. $\pi_{\mathbb{G}} = \pi_{\mathbb{G}'}$.*

In the following, we define the consistency degree of a set of goals, which is the dual of the inconsistency degree of a possibilistic knowledge base:

Definition 8 *Let $\mathbb{G}$ be a set of positive goals. The consistency degree of $\mathbb{G}$, denoted by $Cons(\mathbb{G})$, is defined by:*

$$Cons(\mathbb{G}) = max\{a_i : \mathcal{G}(\phi_i) = a_i \in \mathbb{G} \text{ and } \phi_i \text{ is consistent}\}.$$

Indeed, we define the consistent subbase of $\mathbb{G}$ for making inferences as follows:

Definition 9 *Let $\mathbb{G}$ be a set of positive goals. The preferred consistent subbase of $\mathbb{G}$, denoted by $\rho_\Delta(\mathbb{G})$, is:*

$$\rho_\Delta(\mathbb{G}) = \{\phi_i : \mathcal{G}(\phi_i) = a_i \in \mathbb{G} \text{ and } a_i = Cons(\mathbb{G})\}.$$

Given these two definitions, we are now able to provide the syntactic counterpart of the plausible and preferential inference:

tailment goes here in a way which is the opposite of the situation in the classical logical representation framework. Namely, if all the models of ϕ are feasible, we can conclude from this piece of information that all the worlds in ψ are feasible only if the entailment $\psi \models \phi$ holds, and nothing is said about the interpretations outside the models of ϕ.

Proposition 2

1. *Let $\mathbb{G}$ be a set of positive goals and $\pi_{\mathbb{G}}$ be the possibility distribution associated with $\mathbb{G}$. Then,* $$\pi_{\mathbb{G}} \models_P \psi \text{ iff } \bigvee_{\phi_i \in \rho_\Delta(\mathbb{G})} \phi_i \vdash \psi.$$

2. *Let $\mathbb{G}$ be a set of goals and φ be a formula. Let $\mathbb{G}' = \{\mathcal{G}(\phi_i \wedge \varphi) = a_i : \mathcal{G}(\phi_i) = a_i \in \mathbb{G}\}$. Then,* $$\pi \models_\varphi \psi \text{ iff } \bigvee_{\phi'_i \in \rho_\Delta(\mathbb{G}')} \phi'_i \vdash \psi.$$

6 Merging multiple agents preferences in a bipolar representation

This section discusses the problem of merging agents preferences from the semantic and the syntactic points of view. The result of the merging process will also be a pair $(\mathbb{R}_{\oplus_\mathbb{R}}, \mathbb{G}_{\oplus_\mathbb{G}})$, where $\mathbb{R}_{\oplus_\mathbb{R}}$ is the result of merging agents rejections, and $\mathbb{G}_{\oplus_\mathbb{G}}$ is the result of merging agents positive goals. These two merging steps are processed separately, using generally different merging operators. They are described in the next two subsections.

6.1 Fusion of negative desires

Let $\{\mathbb{R}_1, \cdots, \mathbb{R}_n\}$ be a set of rejection bases provided by n agents to be merged with some merging operator $\oplus_\mathbb{R}$. $\oplus_\mathbb{R}$ is a function from $[0, 1]^n$ to $[0, 1]$. Since rejections have an immediate encoding in terms of possibilistic knowledge bases, we can apply the merging procedures of possibilistic knowledge bases [4] for merging rejection bases. In particular, this allows to have the syntactic counterpart of any semantic merging operator, satisfying minimal properties. See [4] for details.

In this section, we define a class of operators which seem to be appropriate for merging rejections. The idea is that if some piece of information is rejected by some agent, then it should be explicitely rejected after the merging process. Such kind of behaviour is captured by conjunctive operators. Let ϕ be a classical formula. To see if a formula ϕ should be explicitely stated as rejected (or not) and with what degree of rejection in $\mathbb{R}_{\oplus_\mathbb{R}}$ (the result of merging), we compute from each $\mathbb{R}_i$ its level of rejection a_i. Then, ϕ will be explicitely stated in $\mathbb{R}_{\oplus_\mathbb{R}}$ with a degree $\oplus_\mathbb{R}(a_1, \cdots, a_n)$.

Natural properties of $\oplus_\mathbb{R}$ are:

i) $\oplus_\mathbb{R}(0, \cdots, 0) = 0$.
If a piece of information is not explicitely rejected by any agent then it should not be explicitely rejected in $\mathbb{R}_{\oplus_\mathbb{R}}$.

ii) If $\forall i = 1, \cdots, n$, $a_i \geq b_i$ then
$\oplus_{\mathbb{R}}(a_1, \cdots, a_n) \geq \oplus_{\mathbb{R}}(b_1, \cdots, b_n)$
(monotonicity property).

iii) $\oplus_{\mathbb{R}}(0, \cdots, 0, a, 0, \cdots, 0) = a$.
Namely, if a formula is only explicitely rejected by one agent then its level of rejection should not increase in the result of merging. In fact, it will be rejected with the same level.

Note that $i), ii)$ and $iii)$ imply:

$$\text{if } a_i > 0 \text{ for some } i \text{ then } \oplus_{\mathbb{R}} (a_1, \cdots, a_n) > 0,$$

which corresponds to say that if a formula is rejected by at least one agent then it should be explicitly rejected in the result of merging.
It is also easy to check that $\oplus_{\mathbb{R}}(a_1, \cdots, a_n) \geq max(a_1, \cdots, a_n)$. Indeed, from $ii)$ and $iii)$, we have $\oplus_{\mathbb{R}}(a_1, \cdots, a_n) \geq \oplus_{\mathbb{R}}(0, \cdots, 0, a_i, 0, \cdots, 0) = a_i$. Hence, $\oplus_{\mathbb{R}} = max$ represents the most cautious merging operator, in the sense that a formula is not rejected more than what is stated by the more exigent agent (who more rejects ϕ). In this case, we simply have (for two bases):

$$\mathbb{R}_{max} = \mathbb{R}_1 \cup \mathbb{R}_2.$$

Note that combining rejection strengths with the maximum operation leads to combine possibility distributions associated with agents rejections with the minimum operator, namely $\pi_{\mathbb{R}max} = min(\pi_{\mathbb{R}1}, \pi_{\mathbb{R}2})$.
Since strengths are associated with weights, it is also possible to express reinforcement. For instance, a formula which is weakly rejected by different agents can be strongly rejected in the merging result. The operator $\oplus_{\mathbb{R}}(a, b) = a + b - ab$, which corresponds to the product of the distributions, allows to go beyond $max(a, b)$ without reaching 1 (strong rejection) if a and b differ from 1 (i.e., it is not strongly rejected by any agent).

Example 3 *Let $\mathbb{R}_1$ be the base given in Example 1, namely $\mathbb{R}_1 = \{\mathcal{R}(be \wedge \neg su) = 1, \mathcal{R}(co \wedge su) = 1, \mathcal{R}(\neg be \wedge su) = \frac{1}{2}\}$. Let $\mathbb{R}_2 = \{\mathcal{R}(be \wedge \neg su) = 1, \mathcal{R}(co \wedge su) = 1, \mathcal{R}(mo) = \frac{1}{2}\}$, where "mo" stands for mountain. Let $\oplus_{\mathbb{R}} = max$. Then,*
$\mathbb{R}_{max} = \{\mathcal{R}(be \wedge \neg su) = 1, \mathcal{R}(co \wedge su) = 1, \mathcal{R}(\neg be \wedge su) = \frac{1}{2}, \mathcal{R}(mo) = \frac{1}{2}\}$.

6.2 Fusion of positive desires

We now discuss the merging of positive goals. We first provide a general result on the syntactic fusion of goals, similar to the one in [4] for classical possibilistic knowledge bases.
Let $\mathbb{G}_1, \cdots, \mathbb{G}_m$ be m bases of goals and $\pi_{\mathbb{G}1}, \cdots, \pi_{\mathbb{G}m}$ be their associated possibility distributions given by Definition 6. Let $\oplus_{\mathbb{G}}$ be a merging operator satisfying the following requirements:

- $\oplus_{\mathbb{G}}(0, \cdots, 0) = 0$.
- If $\forall j = 1, \cdots, m$, $a_j \geq b_j$ then
$\oplus_{\mathbb{G}}(a_1, \cdots, a_m) \geq \oplus_{\mathbb{G}}(b_1, \cdots, b_m)$.

The first requirement expresses that if a solution is not satisfactory for any agent then it should not be satisfactory in the result of the merging. The second property is simply the monotonicity property.
Let us restrict ourselves, for the sake of simplicity, to the case of two bases. Then, the following proposition gives the positive goals base associated with $\oplus_{\mathbb{G}}(\pi_{\mathbb{G}1}, \pi_{\mathbb{G}2})$:

Proposition 3 *Let $\mathbb{G}_1 = \{\mathcal{G}(\phi_i) = a_i : i = 1, \cdots, n\}$ and $\mathbb{G}_2 = \{\mathcal{G}(\psi_j) = b_j : j = 1, \cdots, m\}$ be two bases of positive goals. Let $\pi_{\mathbb{G}1}$ and $\pi_{\mathbb{G}2}$ be their associated possibility distributions. Let $\oplus_{\mathbb{G}}$ be a merging operator. Then, the base of positive goals associated with $\oplus_{\mathbb{G}}(\pi_{\mathbb{G}1}, \pi_{\mathbb{G}2})$ is:*

$$\begin{aligned}\mathbb{G}_{\oplus_{\mathbb{G}}} = &\{\mathcal{G}(\phi_i) = \oplus_{\mathbb{G}}(a_i, 0) : \mathcal{G}(\phi_i) = a_i \in \mathbb{G}_1\}\\ &\cup\{\mathcal{G}(\psi_j) = \oplus_{\mathbb{G}}(0, b_j) : \mathcal{G}(\psi_j) = b_j \in \mathbb{G}_2\}\\ &\cup\{\mathcal{G}(\phi_i \wedge \psi_j) = \oplus_{\mathbb{G}}(a_i, b_j) :\\ &\quad \mathcal{G}(\phi_i) = a_i \in \mathbb{G}_1 \text{ and } \mathcal{G}(\psi_j) = b_j \in \mathbb{G}_2\}.\end{aligned}$$

The choice of the merging operator for combining $\pi_{\mathbb{G}1}$ and $\pi_{\mathbb{G}2}$ is less contrained since several merging operators, can be considered, such as conjunctive, disjunctive and also "intermediary" operators which reinforce what is common and discount the goals which only concern one agent (see [14] for a representation of such operators).
If the agents are highly cooperative then we can say that an agent adds to its goals those of the other agent provided that they do not contradict what is acceptable for him. In this case $\oplus_{\mathbb{G}} = max$ is recommended.

Example 4 *Let $\mathbb{G}_1$ be the base given in Example 2, namely*
$\mathbb{G}_1 = \{\mathcal{G}(\neg co \wedge be \wedge su) = 1, \mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{4}\}$.
Let $\mathbb{G}_2 = \{\mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{2}, \mathcal{G}(\neg be \wedge \neg su) = \frac{1}{4}\}$.
Let $\mathbb{G}_{max}$ be the result of combining $\mathbb{G}_1$ and $\mathbb{G}_2$ with $\oplus_{\mathbb{G}} = max$. Then,
$\mathbb{G}_{max} = \{\mathcal{G}(\neg co \wedge be \wedge su) = 1, \mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{4},$
$\cup\{\mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{2}, \mathcal{G}(\neg be \wedge \neg su) = \frac{1}{4}\}$
$\cup\{\mathcal{G}(\bot) = 1, \mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{2}\}$
which is semantically equivalent to
$\mathbb{G}_{max} = \{\mathcal{G}(\neg co \wedge be \wedge su) = 1,$
$\mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{2}, \mathcal{G}(\neg be \wedge \neg su) = \frac{1}{4}\}$
since $\mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{4}$ is subsumed by $\mathcal{G}(\neg be \wedge \neg su) = \frac{1}{4}$ (see Definition 7).

Note that this mode of merging corresponds to considering that what is satisfactory by one agent is also satisfactory by the other.

7 Coherence of positive and negative desires and how to restore it

Merging agents preferences can lead to conflicts. This section first defines the notion of coherence between rejections and positive goals, and shows how to restore the coherence in case of inconsistency.
Let $(\mathbb{R}, \mathbb{G})$ be preferences for an agent (or a group of agents). Intuitively, if $\mathbb{R} = \{\mathcal{R}(\phi_i) = 1 : i = 1, \cdots, n\}$ and $\mathbb{G} = \{\mathcal{G}(\psi_j) = 1 : j = 1, \cdots, m\}$ are classical logic bases (without rejection or satisfaction weights) then $\mathbb{R}$ and $\mathbb{G}$ are coherent if

$$\bigvee_{i=1,\cdots,m} \psi_j \vdash \bigwedge_{i=1,\cdots,n} \neg\phi_i,$$

namely any solution satisfying at least one goal of $\mathbb{G}$ should falsify all formulas in $\mathbb{R}$. More generally, any interpretation which is satisfactory to a degree a (w.r.t. $\mathbb{G}$) should be at least feasible to a degree a (w.r.t. $\mathbb{R}$).

Definition 10 *Let $\pi_\mathbb{G}$ and $\pi_\mathbb{R}$ be the two possibility distributions representing respectively the positive goals and rejections of an agent. Then, $\pi_\mathbb{G}$ and $\pi_\mathbb{R}$ are said to be coherent iff*

$$\forall\omega, \pi_\mathbb{G}(\omega) \leq \pi_\mathbb{R}(\omega).$$

The coherence checking can also be done syntactically using the bases $\mathbb{G}$ and $\mathbb{R}$.

Proposition 4 *Let $\mathbb{G}$ and $\mathbb{R}$ be respectively the sets of goals and rejections of an agent. Then, $\mathbb{G}$ and $\mathbb{R}$ are said to be coherent (in the sense of Definition 10) iff: $\forall a \geq 0$,*

$$\bigvee_{\mathcal{G}(\psi_j)=a_j\in\mathbb{G}, a_j\geq a}\psi_j \vdash \bigwedge_{\mathcal{R}(\phi_i)=a_i\in\mathbb{R}, a_i>1-a}\neg\phi_i.$$

In the merging process, the consistency of each pair $(\mathbb{R}_i, \mathbb{G}_i)$ does not guarantee the coherence of the pair $(\mathbb{R}, \mathbb{G})$, where $\mathbb{R}$ (resp. $\mathbb{G}$) is the result of merging $\mathbb{R}_i$'s (resp. $\mathbb{G}_i$'s), for most of the operators $(\oplus_\mathbb{R}, \oplus_\mathbb{G})$.

Example 5 *Let $\mathbb{R}$ and $\mathbb{G}$ be respectively the sets of merging rejections and merging goals computed in Examples 3 and 4. We have*
$\mathbb{R} = \{\mathcal{R}(be \wedge \neg su) = 1, \mathcal{R}(co \wedge su) = 1,$
$\mathcal{R}(\neg be \wedge su) = \frac{1}{2}, \mathcal{R}(mo) = \frac{1}{2}\}$, *and*
$\mathbb{G} = \{\mathcal{G}(\neg co \wedge be \wedge su) = 1,$
$\mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{2}, \mathcal{G}(\neg be \wedge \neg su) = \frac{1}{4}\}$.
At the semantic level, consider the interpretation $\omega_0 = su \wedge \neg co \wedge be \wedge mo$. Then, we can check that $\pi_\mathbb{R}(\omega_0) = \frac{1}{2}$ while $\pi_\mathbb{G}(\omega_0) = 1$.
Observe that $\pi_\mathbb{G}(\omega_0) \not\leq \pi_\mathbb{R}(\omega_0)$, then $\pi_\mathbb{G}$ and $\pi_\mathbb{R}$ are not coherent.
At the syntactic level, let $a = 1$. Then,
$\bigvee\{\psi_j : \mathcal{G}(\psi_j) = a_j \in \mathbb{G}, a_j \geq 1\} = \neg co \wedge be \wedge su$, *and*
$\bigwedge_{\mathbb{R}(\phi_i)=a_i\in\mathbb{R}, a_i>0}\neg\phi_i = (\neg be \vee su) \wedge (\neg co \vee \neg su) \wedge (be \vee \neg su) \wedge \neg mo$.
We have $\neg co \wedge be \wedge su \nvdash (\neg be \vee su) \wedge (\neg co \vee \neg su) \wedge (be \vee \neg su) \wedge \neg mo$. Then, $(\mathbb{R}, \mathbb{G})$ is not coherent.

When the coherence condition is not satisfied by the results of the merging, this means that the set of goals resulting from merging the goals of the agents is not compatible with what is acceptable by the agents. A way to restore the coherence in the sense of Definition 10 is to revise either the possibility distribution $\pi_\mathbb{G}$ or the possibility distribution $\pi_\mathbb{R}$. We choose to revise $\pi_\mathbb{G}$ since in practice it is more difficult to question $\pi_\mathbb{R}$ which expresses rejections. The revision of $\pi_\mathbb{G}$ in this case consists in decreasing the possibility degree of each interpretation in $\pi_\mathbb{G}$ to the possibility degree of this interpretation in $\pi_\mathbb{R}$. In other terms, restoring the coherence leads to revise $\pi_\mathbb{G}$ into $\pi_{\mathbb{G}rev}$ as follows:

$$\forall\omega, \pi_{\mathbb{G}rev}(\omega) = min(\pi_\mathbb{G}(\omega), \pi_\mathbb{R}(\omega)).$$

At the syntactic level, this leads to weaken goals and to decrease the level of satisfaction associated with goals in $\mathbb{G}$, in the following way: let $a_1, \cdots, a_n$ be the weights of $\mathbb{R}$ such that $a_1 > \cdots > a_n > 0$ i.e., $\mathbb{R} = \{\mathcal{R}(\phi_1) = a_1, \cdots, \mathcal{R}(\phi_n) = a_n\}$. This is always possible by gathering formulas of the same weight in a unique formula.
Let $\mathcal{G}(\psi) = b \in \mathbb{G}$, and let a_i be the minimal weight such that $1 - a_i < b$. Then, $\mathbb{G}_{rev}$ is obtained by replacing each $\mathcal{G}(\psi) = b$ in $\mathbb{G}$ by:
$\{\mathcal{G}(\neg\phi_1 \wedge \cdots \wedge \neg\phi_i \wedge \psi) = b\}$
$\cup\{\mathcal{G}(\neg\phi_1 \wedge \cdots \wedge \neg\phi_k \wedge \psi) = 1 - a_{k+1} : k = 1, \cdots, i-1\}$
$\cup\{\mathcal{G}(\psi) = 1 - a_1\}$.

Intuitively this means that each solution satisfying the goal ψ and at least the formulas $\neg\phi_1, \cdots, \neg\phi_i$ is considered as satisfactory to at least the degree b. However we decrease the satisfaction degree of solutions which satisfy ψ but falsify at least one formula from $\{\neg\phi_1, \cdots, \neg\phi_i\}$.
This approach leads to the following result:

Proposition 5 *Let $\mathbb{G}$ and $\mathbb{R}$ be the sets of goals and rejections respectively. Let $\pi_\mathbb{G}$ and $\pi_\mathbb{R}$ be their associated possibility distributions respectively. Let $\pi_{\mathbb{G}rev}$ be defined as follows: $\forall\omega, \pi_{\mathbb{G}rev}(\omega) = min(\pi_\mathbb{G}(\omega), \pi_\mathbb{R}(\omega))$. Let $\mathbb{G}_{rev}$ be the base constructed from $\mathbb{G}$ and $\mathbb{R}$ above. Then, $\pi_{\mathbb{G}rev}$ is the possibility distribution associated with $\mathbb{G}_{rev}$ following Definition 6.*

Example 6 *Let us consider again the possibility distributions $\pi_{\mathbb{R}}$ and $\pi_{\mathbb{G}}$ associated with $\mathbb{R}$ and $\mathbb{G}$ computed in Examples 3 and 4 respectively. We have*
$\mathbb{R} = \{\mathcal{R}(be \wedge \neg su) = 1, \mathcal{R}(co \wedge su) = 1,$
$\mathcal{R}(\neg be \wedge su) = \frac{1}{2}, \mathcal{R}(mo) = \frac{1}{2}\}$ *and*
$\mathbb{G} = \{\mathcal{G}(\neg co \wedge be \wedge su) = 1,$
$\mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{2}, \mathcal{G}(\neg be \wedge \neg su) = \frac{1}{4}\}.$
Let $\pi_{\mathbb{G}rev} = min(\pi_{\mathbb{G}}, \pi_{\mathbb{R}})$.
The goals base associated with $\pi_{\mathbb{G}rev}$ is obtained by weakening (when necessary) the goals of $\mathbb{G}$. For example the goal $\mathcal{G}(\neg co \wedge be \wedge su) = 1$ is replaced by two goals $\mathcal{G}(\neg co \wedge be \wedge su \wedge \neg mo) = 1$ and $\mathcal{G}(\neg co \wedge be \wedge su) = \frac{1}{2}$ because the situation $\neg co \wedge be \wedge su \wedge \neg mo$ (induced by $\neg co \wedge be \wedge su$) does not satisfy any formula in $\mathbb{R}$, while the situation $\neg co \wedge be \wedge su \wedge mo$ satisfies mo so it is decreased to the degree $\frac{1}{2}$. With a similar treatment of other goals we get:
$\mathbb{G}_{rev} = \{\mathcal{G}(\neg co \wedge be \wedge su \wedge \neg mo) = 1, \mathcal{G}(\neg co \wedge be \wedge su) = \frac{1}{2}, \mathcal{G}(co \wedge \neg be \wedge \neg su) = \frac{1}{2}, \mathcal{G}(\neg be \wedge \neg su) = \frac{1}{4}\}.$

8 Conclusion

This paper has advocated a bipolar framework for representing preferences accurately under the form of two sets of formulas having different semantics, both of them being encoded in the framework of possibilty theory. The representation framework remains simple (although each of the two sets could be equivalently represented under a graphical form, or as a set of conditionals [2]). Besides, the proposed model remains qualitative since only the ordering between the satisfaction levels or the rejection levels is meaningful.
The idea of a bipolar representation framework could accommodate more quantitative frameworks, like penalty logic [9], as well as when the weights are thought as penalities (e.g., as in [22, 13]).

In the penalty logic framework, one can also distinguish between two penalty bases representing rejections and positive goals respectively. The weights (which are then integer values) associated to formulas in the rejection base express the price to pay if the rejection is not respected. While the weights associated with formulas in the base of goals express the bonus to win if the desires are satisfied. In both cases, the weight associated with a given solution is the sum of the weights of falsified (in case of rejection), or satisfied (for the goals) formulas.

Another quantitative framework is the evidence theory framework [19, 21]. Belief functions have already been used for representing rejections. One can also use the so-called "commonality" function Q for representing positive goals. Indeed, a commonality function plays a role similar to the one of the guaranteed possibility measure Δ. Particularly when focal elements are nested, then the commonality function simply coincides with a guaranteed possibility measure.

Such a bipolar representation is also of interest when representing knowledge rather than preferences, as is discussed in [11], where the negative parts correspond to what is known as being (more or less impossible), while the positive parts gather worlds wich are guaranteed to be feasible because they have been observed or reported. This is why integrity constraints which are pieces of knowledge of the first kind, can be added when necessary to the negative part of the representation of the preferences, leading to the specification of what is acceptable because it is not impossible by integrity constraints or because of the taste of the user. Beyond the interest of representing preferences of agents for fusing them, and restoring coherence when necessary, the inference machinery of the possibilistic logic framework (extended to formulas weighted in terms of the function Δ) enables us to reason about preferences.

In the long range, such a representation scheme for preferences should turn to be useful for elicitating user's wishes in information or recommender systems, and handling preferences conflict between agents. In the full paper, we will also provide a comparative study with recent related works on preferences, e.g. [22].

References

[1] F. Bacchus and A. J. Grove. Utility Independence in a Qualitative Decision Theory. In *Proceedings of the 5th International Conference on Principles of Knowledge Representation, (KR'96)*, pages 542–552, 1996.

[2] S. Benferhat, D. Dubois, S. Kaci, and H. Prade. Bridging logical, comparative and graphical possibilistic representation frameworks. In *6th European Conference on Symbolic and Quantitative Approaches to Reasoning and Uncertainty (ECSQARU'01), LNAI 2143*, pages 422–431, 2001.

[3] S. Benferhat, D. Dubois, and H. Prade. Towards a possibilistic logic handling of preferences. *Applied Intelligence*, 14(3):303–317, 2001.

[4] S. Benferhat, D. Dubois, H. Prade, and M. Williams. A practical approach to fusing and revising prioritized belief bases. In *Proc. of EPIA 99, LNAI n° 1695, Springer Verlag*, pages 222–236, 1999.

[5] C. Boutilier. Toward a Logic for Qualitative Decision Theory. In *Proceedings of the 4th Inter*

national Conference on Principles of Knowledge Representation, (KR'94), pages 75–86, 1994.

[6] C. Boutilier, R.I. Brafman, H. Hoos, and D. Poole. Reasoning with conditional ceteris paribus preference statements. In *Proceedings of 15th Conference on Uncertainty in Artificial Intelligence (UAI'99)*, pages 71–80, 1999.

[7] J.T. Cacioppo and G.G. Bernston. The affect system: Architecture and operating characteristics. *Current Directions in Psychological Science*, 8, 5:133–137, 1999.

[8] J.T. Cacioppo, W.L. Gardner, and G.G. Bernston. Beyond bipolar conceptualizations and measures: The case of attitudes and evaluative space. *Personality and Social Psychology Review*, 1, 1:3–25, 1997.

[9] F. Dupin de Saint-Cyr, J. Lang, and T. Schiex. Penalty logic and its link with dempster-shafer theory. In *Proc. of 10th International Conference on Uncertainty in Artificial Intelligence (UAI'94)*, pages 204–211, 1994.

[10] J. Doyle, Y. Shoham, and M. P. Wellman. A Logic of Relative Desire. In *Proceedings of the 6th International Symposium on Methodologies for Intelligent Systems (ISMIS'91)*, pages 16–31, 1991.

[11] D. Dubois, P. Hajek, and H. Prade. Knowledge-driven versus data-driven logics. *Journal of Logic, Language, and Information*, 9:65–89, 2000.

[12] D. Dubois, J. Lang, and H. Prade. Possibilistic logic. *In Handbook of Logic in Artificial Intelligence and Logic Programming*, 3, Oxford University Press:pages 439–513, 1994.

[13] D. Dubois and H. Prade. Toll sets and toll logic. In *FUZZY LOGIC, State of the Art, (édité par R. Lowen and M. Roubens)*, pages 169–177. Kluwer Academic Publishers, 1993.

[14] D. Dubois and H. Prade. Possibility theory and data fusion in poorly informed environments. In *Control Engineering Practice*, volume 2(5), pages 811–823, 1994.

[15] D. Dubois and H. Prade. Possibility theory: qualitative and quantitative aspects. *In Handbook of Defeasible Reasoning and Uncertainty Management Systems. (D. Gabbay, Ph. Smets, eds.), Vol. 1: Quantified Representations of Uncertainty and Imprecision*, (Ph. Smets, ed.) Kluwer, Dordrecht:169–226, 1998.

[16] M. Grabisch and C. Labreuche. The Sipos integral for the aggregation of interacting bipolar criteria. In *8th International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems (IPMU'00), Madrid*, pages 395–409, 2000.

[17] C. Lafage. Représentation de préférences en logique: application à la décision de groupe. In *Thèse de doctorat. Université Paul Sabatier. Toulouse.*, 2001.

[18] J. Lang. Conditional desires and utilities - and alternative logical approach to qualitative decision theory. In *Proceedings of 12th European Conference on Artificial Intelligence (ECAI'96)*, pages 318-322, 1996.

[19] G. Shafer. A mathematical theory of evidence. In *New Jersey: Princeton University Press*, 1976.

[20] Y. Shoham. Reasoning aboutchange. In *MIT Press, MA*, 1988.

[21] P. Smets and R. Kennes. The transferable belief model. *Artificial Intelligence*, 66:191–234, 1994.

[22] L. van der Torre and E. Weydert. Parameters for utilitarian desires in a qualitative decision theory. *Applied Intelligence*, 14:285–301, 2001.

[23] M. P. Wellman, J. Doyle, and Y. Shoham. Preferential semantics for goals. In *Proceedings of the 9th Conference on Artificial Intelligence, (AAAI'91)*. 698-703, 1991.